



EFFECTS OF COUPLED GEOMETRIC SCALING AND POLYNOMIAL SURFACE CURVATURE ON AERODYNAMIC DRAG AND ENERGY LOSS IN SIMPLIFIED VEHICLE GEOMETRIES

Michael Handal ^{1,*} and Jana Al Zurba ²

¹ St Christopher's School, Kingdom of Bahrain.

² American School of Bahrain, Kingdom of Bahrain.

* Corresponding Author

International Journal of Science and Research Archive, 2026, 20(03), 645–659

Publication history: Received on 08 August 2026; revised on 13 September 2026; accepted on 16 September 2026

Article DOI: <https://doi.org/10.30574/ijrsra.2026.20.3.1795>

Copyright © 2026 Author(s) retain the copyright of this article. This article is published under the terms of the Creative Commons Attribution License 4.0

ABSTRACT

This study investigates the aerodynamic performance and drag-induced energy loss of three-dimensional parabolic vehicle geometries under controlled flow conditions. Four polynomial profiles, represented by $y = h - ax^2$, were created in CAD with coupled changes in peak height (h) and curvature coefficient (a), ranging from Model 1 ($h = 2.0$, $a = 0.02$) to Model 4 ($h = 5.0$, $a = 0.05$). Computational scripts executed in Google Colab were used to evaluate frontal area, drag coefficient (C_d), aerodynamic drag force, and drag-induced energy loss at a constant Reynolds number of approximately 2.3×10^7 .

Across the four configurations, increases in geometric scale were accompanied by increases in frontal area and drag coefficient. Model 1, with a frontal area of 12.566 m^2 , produced the lowest C_d of 0.162, drag force of 273.80 N and energy loss of 410.69 kJ. Model 4, with a frontal area of 78.540 m^2 , produced the highest C_d of 0.237, drag force of 2500.01 N and energy loss of 3750.01 kJ.

The results show that steeper and larger parabolic profiles within the tested configuration set were associated with greater aerodynamic resistance and energy demand. Because geometric scale, frontal area, and curvature changed simultaneously, their individual effects cannot be fully separated from the data. The study should therefore be interpreted as a preliminary computational investigation of coupled geometric-scaling effects, providing a baseline for future constant-area curvature studies, validated CFD simulations, and early-stage aerodynamic design of energy-efficient transport geometries.

Investigating the Combined Influence of Geometric Scale and Polynomial Surface Curvature on Aerodynamic Drag and Drag-Induced Energy Loss.

Research Question

To what extent do coupled changes in geometric scale and polynomial surface curvature, represented by changes in peak height (h) and curvature coefficient (a) in simplified vehicle profiles ($y = h - ax^2$), influence frontal area, aerodynamic drag coefficient (C_d), aerodynamic drag force, and drag-induced energy loss (E_{loss}) under constant flow conditions?

Objective

The primary objective of this investigation is to determine how coupled changes in geometric scale and polynomial surface curvature, represented by the function $y = h - ax^2$, affect aerodynamic performance. Using CAD and custom computational processing, the study compares vehicle models with different profile heights (h) and curvature coefficients (a) to evaluate their combined influence on frontal area, drag coefficient (C_d), aerodynamic drag force, and drag-induced energy loss (E_{loss}) under constant-flow conditions. Because h and a vary simultaneously between models, the investigation examines their coupled effect rather than treating either variable as independently isolated.

Keywords: Aerodynamic drag; Polynomial geometry; Drag coefficient; Vehicle aerodynamics; Energy loss; Computational modelling

1 INTRODUCTION

A substantial portion of energy in transport systems is dissipated overcoming air resistance. Aerodynamic drag constitutes a primary mechanism of energy loss in contemporary transport infrastructures, influencing the operational efficiency of road vehicles, aircraft, and high-performance cycling. As bodies traverse a fluid medium, a resistive force is generated via the interaction between the surface geometry and the fluid. This force correlates positively with velocity and is highly contingent upon the geometric configuration and surface morphology of the object. Consequently, marginal geometric modifications may yield significant variations in thermodynamic efficiency.

In transport systems, a significant proportion of energy is lost due to aerodynamic drag inefficiencies. At highway speeds (approximately 65 mph or 100 km/h), aerodynamic drag can account for over 50% of total energy expenditure in vehicles.

Aerodynamic drag is a key concept in physics that describes the resistive force acting on an object as it moves through a fluid such as air. This force depends on several variables, including the object's velocity, frontal area, air density, and coefficient of drag, as expressed by the equation. As velocity increases, the magnitude of drag increases significantly, making it the dominant resistive force at higher speeds.

This investigation examines how coupled changes in geometric scale and aerodynamic curvature, modelled using polynomial surface functions, influence frontal area, drag coefficient (C_d), aerodynamic drag force, and the resulting energy loss due to air resistance. The aim is to quantify the aerodynamic response of simplified vehicle-shaped models under controlled flow conditions. Flow velocity, air density, Reynolds number, domain dimensions, and evaluation distance were held constant, while profile height, curvature coefficient, and the resulting frontal area varied between models.

2 PHYSICS THEORIES/COMPONENTS

2.1 Fluid Dynamics and Boundary Layer Behavior

For external aerodynamic bodies, these flow characteristics also determine boundary-layer development, surface pressure distribution, flow separation, and wake formation, all of which contribute to the aerodynamic resistance experienced by the body.

2.2 Drag Coefficient

The drag force acting on an object can be described by [1]:

$$F_d = \frac{1}{2} C_d \rho A v^2$$

F_d = drag force

C_d = drag coefficient

ρ = air density

A = cross-sectional area

v = velocity

A lower drag coefficient generally indicates lower aerodynamic resistance for otherwise comparable conditions. However, total drag force depends on both C_d and frontal area (A), meaning a body with a relatively low C_d can still experience substantial drag if its frontal area is large. In this investigation, the reported C_d values are evaluated computationally rather than being inferred from deceleration or stopping distance.

Aerodynamic drag can arise from multiple physical mechanisms. Pressure drag results from differences in pressure between the front and rear regions of a body and is strongly influenced by flow separation and wake formation. Skin-friction drag results from viscous shear stresses acting along the body's surface. The present computational model evaluates overall drag rather than separately resolving these components; therefore, pressure-drag and friction-drag contributions are discussed conceptually rather than quantified independently.

2.3 Flow separation

Flow separation occurs when the boundary layer of a fluid traversing a surface detaches from the solid contour.

Aerodynamic Consequences of Flow Separation:

- Formation of a turbulent downstream wake
- Reduction in rear or base pressure
- Increased pressure drag
- Recirculation within separated-flow regions
- Increased aerodynamic energy losses

2.4 Reynolds Number:

A dimensionless fluid-mechanics metric defined as the ratio of inertial to viscous forces within a fluid.

$$Re = (\rho \cdot v \cdot L) / \mu$$

ρ = air density (kg/m³)

v = relative fluid velocity (m/s)

L = characteristic linear dimension (m)

μ = dynamic fluid viscosity (Pa·s)

ν = kinematic fluid viscosity (m²/s)

For this investigation, the characteristic length was taken as the fixed vehicle length of 20 m. Because fluid velocity, air density, dynamic viscosity, and this reference length were held constant, the nominal Reynolds number remained approximately 2.3×10^7 for all four models. This provides a consistent reference flow condition for comparison; however, changes in local geometry may still alter boundary-layer development, separation, and wake behaviour even when the nominal Reynolds number is held constant.

3 HYPOTHESIS

It is hypothesized that, within the tested family of polynomially modelled vehicle profiles, coupled increases in peak height (h) and quadratic coefficient (a) in the function $y = h - ax^2$ will be associated with increases in frontal area, drag coefficient (C_d), aerodynamic drag force, and drag-induced energy loss. Because h and a change simultaneously, the hypothesis concerns their combined geometric effect rather than an independently isolated curvature effect. Therefore, Model 1 is expected to demonstrate the lowest aerodynamic drag and energy loss, while Model 4 is expected to demonstrate the highest.

4 VARIABLES

4.1 Controlled Variables

Relative Fluid velocity

v = relative fluid velocity; the rate of air displacement relative to the vehicle model, maintained uniformly in meters per second (m/s)

Fluid velocity = 15 m/s

Air Density (ρ) and Dynamic Fluid Viscosity (μ)

The intrinsic thermodynamic and physical properties of the ambient fluid medium, held constant to ensure experimental repeatability.

Air Density = 1.196 kg/m³

Dynamic Fluid Viscosity = $1.529 \times 10^{-5} \text{ kg/(m}\cdot\text{s)}$

Horizontal Domain Bounds: $(-10 < x < 10)$

Reference vehicle length / characteristic length = 20.00 m

Dimensional convention: For the computational analysis, one geometric model unit was treated as 1 m. Therefore, the fixed 20-unit horizontal model length corresponds to 20 m, while peak-height values of 2.0–5.0 correspond to 2.0–5.0 m. This convention allows frontal area and Reynolds number to be expressed consistently in SI units.

Evaluation Distance (d)

1500 m

NOTE: Vehicle mass: Vehicle mass is omitted from active calculation because aerodynamic forces are derived from steady-state fluid flow boundaries rather than dynamic translational acceleration.

4.2 Coupled Independent Variables:

Geometric profile scaling representation

The experimental geometry is controlled through two coupled parameters: peak height (h) and curvature coefficient (a). Both parameters increase simultaneously between Model 1 and Model 4, meaning the resulting changes in frontal area and local curvature cannot be independently isolated within the present four-model dataset. The investigation therefore evaluates the combined aerodynamic effect of geometric scale and curvature rather than attributing the observed changes to either parameter alone. The geometric parameters used for Models 1–4 are summarized in Table 1.

Equation representation: $y = h - ax^2$

Table 1. Geometric parameters of the four polynomial vehicle models.

Model	Polynomial equation	Polynomial Coefficient (a)	Domain Restrictions (x)	Maximum Height (m)	Total Vehicle Length (m)
Model 1	$y = 2 - 0.02x^2$	0.02	$-10.00 < x < 10.00$	2.00	20.00
Model 2	$y = 3 - 0.03x^2$	0.03	$-10.00 < x < 10.00$	3.00	20.00
Model 3	$y = 4 - 0.04x^2$	0.04	$-10.00 < x < 10.00$	4.00	20.00
Model 4	$y = 5 - 0.05x^2$	0.05	$-10.00 < x < 10.00$	5.00	20.00

4.2.1 Frontal Cross-Sectional Area

For the revolved geometries, frontal area was calculated as $A = \pi r^2$. The calculated frontal areas are presented in Table 2.

Table 2. Frontal cross-sectional areas of the four revolved geometries.

Model	Radius (r)	Frontal Cross-Sectional Area (A)
Model 1	2.00	$A = 12.56637$
Model 2	3.00	$A = 28.2743$
Model 3	4.00	$A = 50.2654$
Model 4	5.00	$A = 78.5398$

4.3 Dependent Variables:

Drag Coefficient (Cd)

A dimensionless indicator of the aerodynamic resistance associated with each model under the defined computational conditions.

The reported Cd values represent overall aerodynamic resistance and do not separately resolve pressure drag and skin-friction drag.

Aerodynamic Drag Force (Fd)

The total resistive aerodynamic force calculated in the direction opposite to the incoming airflow.

Calculated using $F_d = \frac{1}{2} C_d \rho A v^2$.

Drag-Induced Energy Loss (Eloss)

The mechanical energy required to overcome aerodynamic drag over the fixed evaluation distance (d).

Calculated using $E_{loss} = F_d \times d$.

This quantity represents work done against aerodynamic resistance and should not be interpreted as a direct volumetric measurement of turbulent or viscous energy loss within the flow field.

5 METHODOLOGY

To eliminate physical manufacturing tolerances and ensure consistent mathematical reproduction of the polynomial contours, a fully digital computational workflow was used. The workflow was designed to construct the polynomial geometries accurately, convert them into simplified three-dimensional CAD models, and process their aerodynamic parameters under controlled mathematical conditions.

5.1 Digital Tools and Software Infrastructure

To execute the investigation in a fully digital environment, a suite of browser-accessible applications was selected for geometric definition, CAD construction, and numerical processing. This workflow reduced physical fabrication uncertainty and allowed the same polynomial definitions and controlled parameters to be applied consistently across all four models. The software tools served different stages of the investigation and were not treated as substitutes for experimentally validated CFD or wind-tunnel testing.

Polynomial geometry definition and visualization — Desmos Graphing Calculator [2]

Three-dimensional CAD construction — Onshape [3]

Numerical aerodynamic calculations and data processing — Google Colab [4]

5.2 Desmos: Polynomial Geometry Development

Desmos (developing the coupled geometric parameters):

Red — model 1

Blue — model 2

Green — model 3

Purple — model 4

5.2.1 Trial 1:

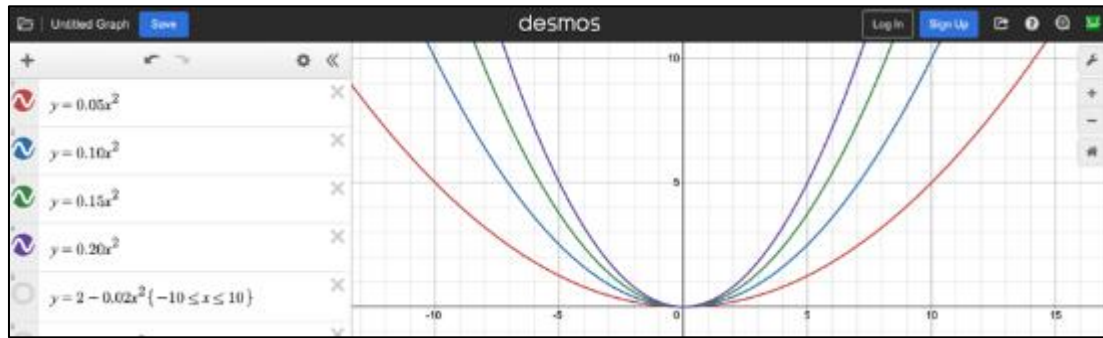


Figure 1: Initial upward-opening polynomial profiles used in Trial 1.

5.2.1.1 Why Trial 1 Was Unsuitable

The initial upward-opening profiles are shown in Figure 1. In the initial design phase, the vehicle profiles were modelled using upward-opening parabolas with the general equation $y = ax^2$. This geometry was unsuitable for the intended simplified vehicle body because the positive quadratic coefficient caused the profiles to open upward, producing a bowl-shaped rather than roof-like contour. If revolved or extruded in CAD, the resulting body would not represent the streamlined external geometry required for the investigation.

5.2.2 Trial 2

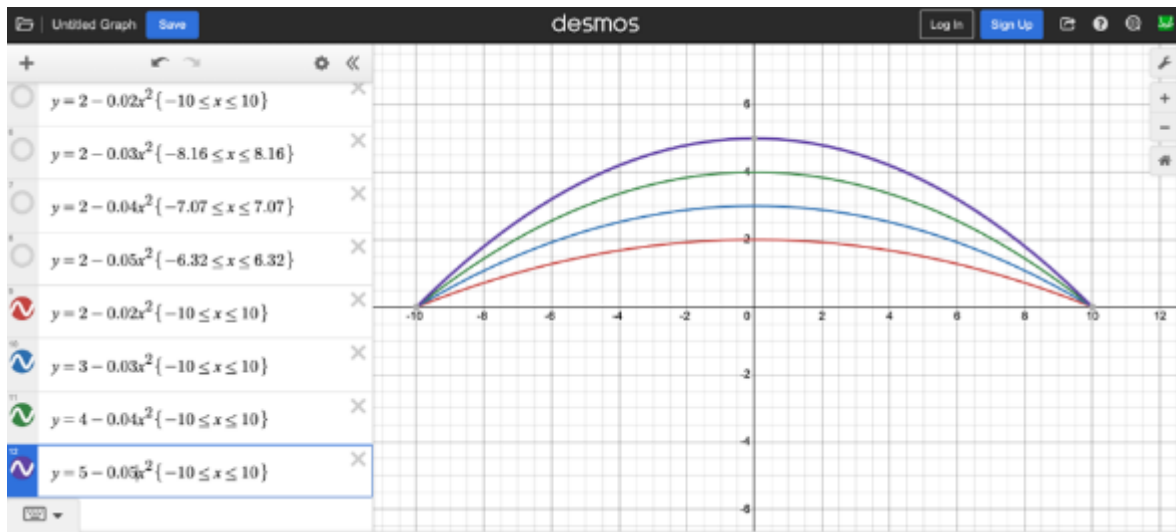


Figure 2: Corrected downward-opening polynomial profiles for Models 1-4.

5.2.2.1 Corrected Polynomial Geometry:

The corrected profiles are shown in Figure 2. To correct the geometry, the polynomial functions were reformulated using the structure $y = h - ax^2$, producing downward-opening profiles suitable for simplified vehicle rooflines. Across Models 1-4, the peak height (h) increased from 2 to 5 m while the curvature coefficient (a) increased simultaneously from 0.02 to 0.05. The horizontal domain remained fixed at $-10 < x < 10$, maintaining a constant reference length of 20 m across all models. These coupled changes increased both the frontal area and the steepness of the polynomial profiles. Therefore, the resulting model set represents a controlled progression in combined geometric scale and curvature rather than an isolated change in either parameter alone.

5.3 CAD Construction and Geometric Modeling

5.3.1 CAD Geometry Construction

The corrected polynomial profiles developed in Desmos were transferred into Onshape to create simplified three-dimensional bodies. Each model retained the same 20 m horizontal reference length, while peak height (h) and curvature coefficient (a) increased together across the four configurations. The resulting two-dimensional profiles and three-dimensional CAD geometries are shown in Figure 3.

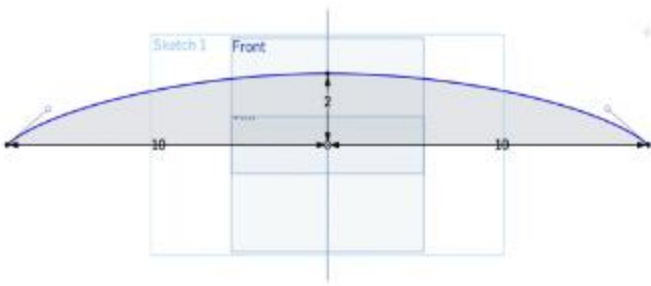
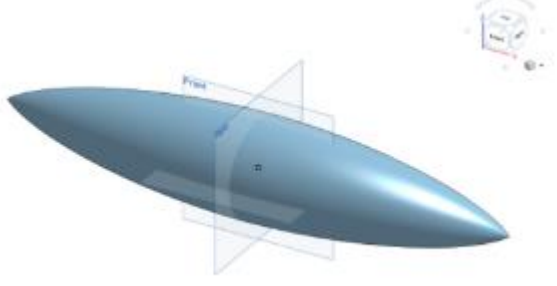
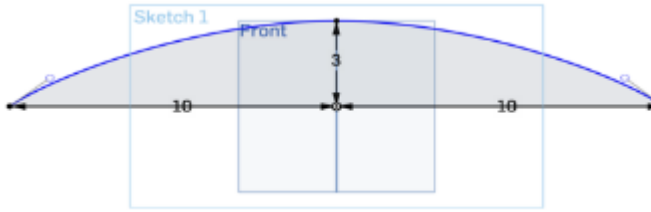
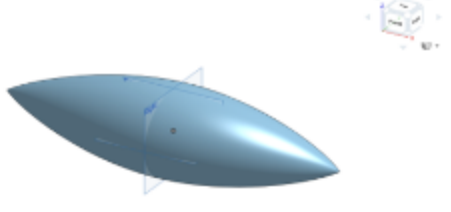
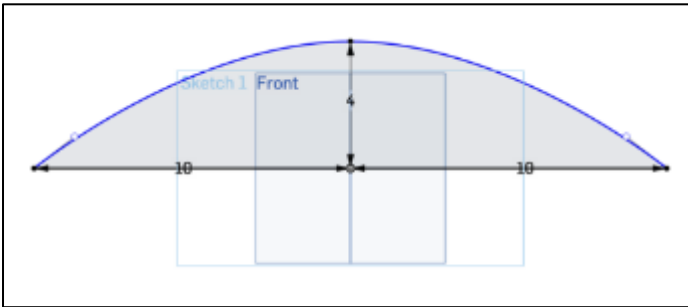
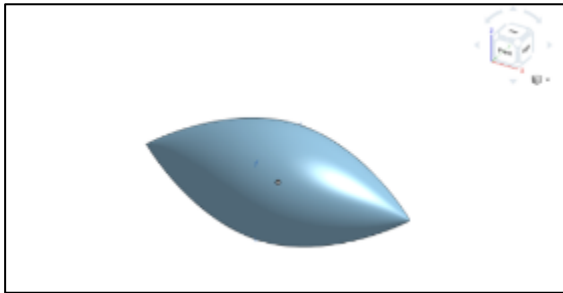
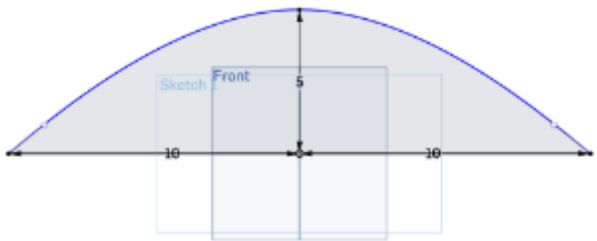
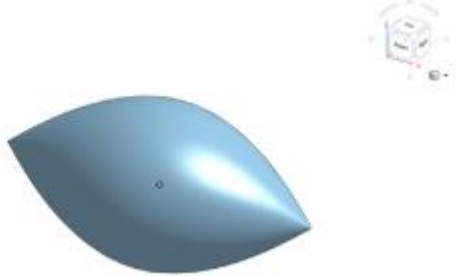
Model 1:	
2D Profile and Dimensions:	3D Shape
	
Model 2:	
2D Profile and Dimensions:	3D Shape
	
Model 3:	
2D Profile and Dimensions:	3D Shape
	
Model 4:	
2D Profile and Dimensions:	3D Shape
	

Figure 3: Two-dimensional polynomial profiles and corresponding three-dimensional CAD geometries for Models 1–4.

5.3.2 What was done

The four polynomial equations were transferred into the CAD environment (Onshape) and used to construct the corresponding three-dimensional models. Each profile was maintained over the same horizontal domain, $-10 < x < 10$,

giving all models a constant reference length of 20 m. Across the four configurations, h and a increased simultaneously, producing systematic changes in peak height, curvature, and frontal cross-sectional area. Consequently, the CAD model set represents a coupled geometric progression rather than an isolated variation in curvature or scale.

The CAD stage was used to verify the relative differences in model height, curvature, and frontal cross-sectional area and to ensure that the mathematical profiles were represented consistently in three dimensions. The CAD models themselves did not calculate aerodynamic flow fields, pressure contours, wake structure, or turbulence behaviour.

5.4 Computational Processing and Aerodynamic Calculations

The aerodynamic outputs for each geometry were processed using a custom Python-based calculation workflow in Google Colab. The model used the defined geometric parameters and controlled flow conditions to calculate frontal area, Reynolds number, drag force, and drag-induced energy loss. The schematic flow visualizations in Figure 4 provide a representation of the incoming flow around each polynomial profile; they should not be interpreted as resolved CFD velocity, pressure, turbulence, or wake fields.

Cd Determination: The drag coefficient (C_d) for each model was estimated using a custom reduced-order geometric relationship based on the local slope of the polynomial profile. For each geometry, the local surface slope was calculated as $-2ax$ and converted into a local angle using $\arctan$. A geometric pressure factor was then calculated by integrating $\sin^2$ of the absolute local angle across the 20 m profile length:

$$\text{Pressure Factor} = (1/L) \int \sin^2(|\arctan(-2ax)|) dx$$

$$C_d = 0.14 + 0.45(\text{Pressure Factor})$$

Using this method, the full-precision C_d values obtained were 0.161932, 0.184688, 0.210461, and 0.236574 for Models 1–4 respectively, which were rounded to 0.162, 0.185, 0.210, and 0.237 in the results tables. These coefficients were then used together with frontal area, air density, and flow velocity in the drag equation to calculate F_d . This C_d relationship is a custom reduced-order approximation used for comparative analysis and should not be interpreted as an experimentally validated or CFD-derived drag coefficient.

The constants 0.14 and 0.45 were used as fixed modelling parameters within the custom computational relationship; therefore, the absolute C_d values should be interpreted cautiously, while the relative trends between the four geometries are the primary focus of the analysis.

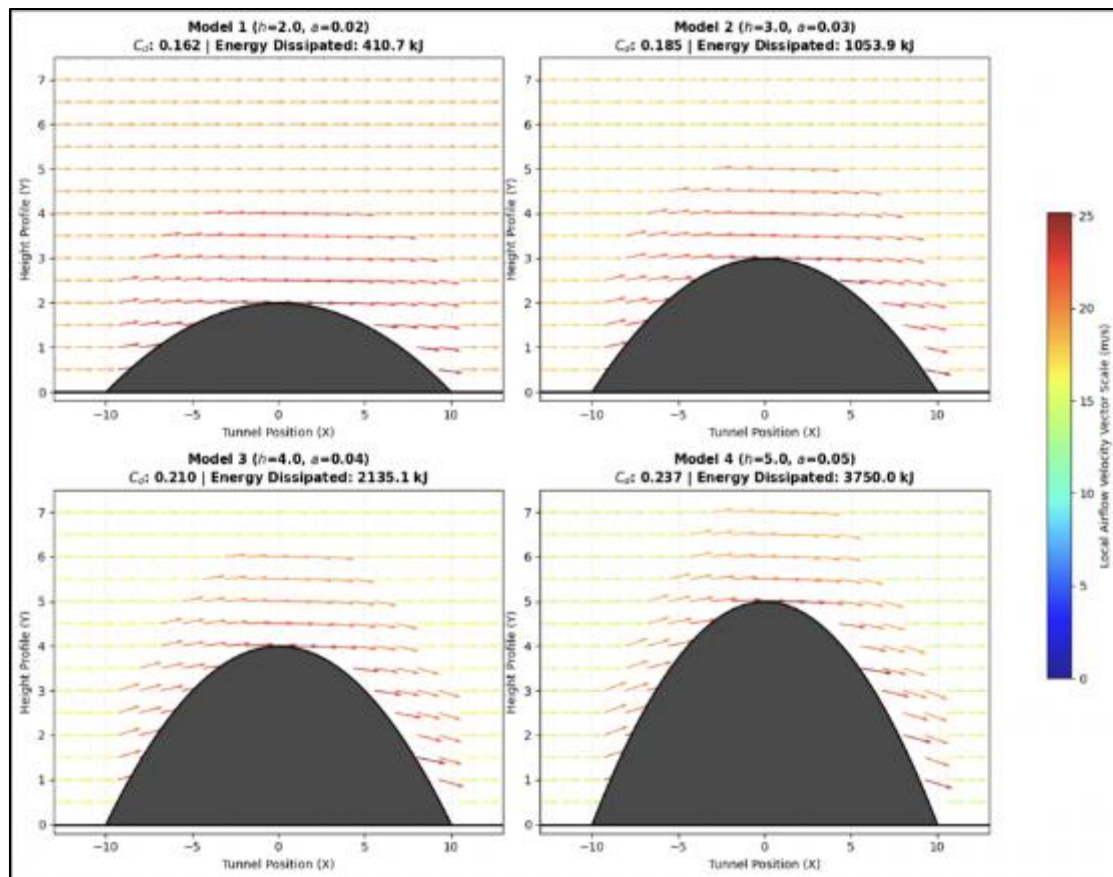


Figure 4: Schematic flow visualization for Models 1–4 under the constant 15 m/s reference flow condition.

The arrows illustrate the imposed flow direction and relative geometric differences between the models; they do not represent a numerically resolved CFD flow field.

Model	Area (A)	Reynolds (Re)	C_d	Drag F_d (N)	Energy (kJ)
Model 1	12.566	$2.3e+07$	0.162	273.80	410.69
Model 2	28.274	$2.3e+07$	0.185	702.61	1053.91
Model 3	50.265	$2.3e+07$	0.210	1423.39	2135.09
Model 4	78.540	$2.3e+07$	0.237	2500.01	3750.01

Figure 5: Computationally processed geometric and aerodynamic outputs for the four tested configurations.

Figure 5 summarizes the outputs used for subsequent analysis. Because the present computational workflow does not independently resolve surface pressure, wall shear stress, turbulent kinetic energy, or downstream recirculation, the reported aerodynamic values are treated as overall model-level outputs. Pressure drag, skin-friction drag, and wake-related losses therefore cannot be quantitatively separated using the present dataset.

Google Colab Custom Code

https://docs.google.com/document/d/1e9M_m2rryxbyaJG1nJsqr8mkdmoT30nleBtRByn3iwQ/edit?usp=sharing

5.5 Computational Method Development

The final workflow was designed to apply the same mathematical relationships and controlled parameters consistently to all four geometries. Equations for frontal area, Reynolds number, aerodynamic drag force and drag-induced energy loss were incorporated into the Google Colab code to standardize the calculations between models.

6 RESULTS

6.1 Geometric results

The geometric data obtained from the CAD models and custom Python processing workflow are presented in Table 3.

Table 3: Geometric parameters and frontal areas of Models 1–4.

Model	(h)	(a)	Frontal Area (m ²)
Model 1	2.0	0.02	12.566
Model 2	3.0	0.03	28.274
Model 3	4.0	0.04	50.265
Model 4	5.0	0.05	78.540

Model 4 had approximately 6.25 times the frontal area of Model 1. Therefore, changes in total drag cannot be attributed to curvature alone.

6.2 Aerodynamic results

The calculated aerodynamic results are presented in Table 4.

Table 4: Calculated aerodynamic results for Models 1–4.

Model	Frontal Area (m ²)	(Cd)	Drag Force (N)
Model 1	12.566	0.162	273.80
Model 2	28.274	0.185	702.61
Model 3	50.265	0.210	1423.39
Model 4	78.540	0.237	2500.01

Note: Cd values are displayed to three decimal places in the results tables, while aerodynamic force calculations were performed using the full-precision computational Cd values.

Across the tested configurations, both Cd and drag force increased progressively from Model 1 to Model 4.

6.3 Energy Loss Results

Drag-induced energy loss was calculated over the fixed evaluation distance of 1500 m, with the results presented in Table 5:

$$E_{\text{loss}} = F_d \times 1500$$

Table 5: Drag-induced energy loss over the fixed 1500 m evaluation distance.

Model	Drag Force (N)	Distance (m)	Energy Loss (kJ)
Model 1	273.80	1500	410.69
Model 2	702.61	1500	1053.91
Model 3	1423.39	1500	2135.09
Model 4	2500.01	1500	3750.01

The calculated drag-induced energy loss increased by approximately 9.13 times between Model 1 and Model 4, directly reflecting the increase in aerodynamic drag force over the constant 1500 m evaluation distance.

7 DATA ANALYSIS AND DISCUSSION

7.1 Coupled Effect of Geometric Scaling on Frontal Area

In this investigation, peak height (h) and curvature coefficient (a) were increased simultaneously across the four polynomial profiles while the horizontal domain remained fixed. Increasing h enlarges the radius of the revolved frontal face, causing frontal cross-sectional area (A) to increase according to $A = \pi r^2$.

The geometric data show a strong increase in frontal area across the model set. Model 1, with $h = 2.0$, has a frontal area of 12.566 m^2 , whereas Model 4, with $h = 5.0$, has a frontal area of 78.540 m^2 . Model 4 therefore presents approximately 6.25 times the frontal area of Model 1 to the incoming flow. Since aerodynamic drag force is directly proportional to frontal area when other terms in the drag equation are held constant, this area increase is expected to make a major contribution to the overall rise in drag. The simultaneous change in C_d provides an additional contribution, which is analysed separately in the following sections.

7.2 Coupled Effect of Geometric Scaling and Curvature on Drag Coefficient

Across the four tested configurations, the reported drag coefficient (C_d) increased progressively as peak height (h) and curvature coefficient (a) increased together. Because these geometric parameters were varied simultaneously, the present dataset demonstrates an association between the coupled geometric progression and C_d rather than proving that curvature alone caused the increase.

From Model 1 to Model 4, C_d increased from 0.162 to 0.237, corresponding to an increase of approximately 46.3%. Steeper or larger profiles may influence pressure distribution, flow separation, and wake behaviour; however, these mechanisms were not directly resolved by the present computational method. Therefore, the observed C_d increase should be interpreted as a result associated with the combined geometric changes rather than as proof that increased curvature universally increases drag.

7.3 Effect of Frontal Area and C_d on Drag Force

Under the constant flow conditions used in this investigation, aerodynamic drag force (F_d) is directly proportional to the product of frontal area (A) and drag coefficient (C_d). Both quantities increased across the model set, meaning their effects compound when calculating total drag force.

Between Model 1 and Model 4, frontal area increased by a factor of approximately 6.25, while C_d increased by a factor of approximately 1.46. Since F_d is proportional to $A \times C_d$ under constant density and velocity, these two factors combine to produce an overall drag-force increase of approximately 9.13 times. This comparison shows that frontal-area growth is the larger numerical contributor to the increase in drag force, while the rise in C_d provides an additional amplifying effect.

7.4 Effect of Drag Force on Energy Loss

Because all models were evaluated over the same distance of 1500 m, drag-induced energy loss increased directly in proportion to drag force according to $E_{\text{loss}} = F_d \times d$:

Model 1 ($F_d = 273.80 \text{ N}$): Energy loss = 410.69 kJ

Model 2 ($F_d = 702.61 \text{ N}$): Energy loss = 1053.91 kJ

Model 3 ($F_d = 1423.39 \text{ N}$): Energy loss = 2135.09 kJ

Model 4 ($F_d = 2500.01 \text{ N}$): Energy loss = 3750.01 kJ

The calculated drag-induced energy loss therefore increased from 410.69 kJ for Model 1 to 3750.01 kJ for Model 4. This increase reflects the higher drag force calculated for the larger, steeper tested geometries and should be interpreted within the coupled geometric conditions of this investigation.

7.5 Relative Contributions of Frontal Area and C_d to Drag

Under the constant density and velocity used in this investigation, drag force is proportional to the product $A \times C_d$. This allows the numerical contributions of frontal-area growth and C_d growth to be compared directly, even though the geometric causes of the C_d change cannot be independently isolated.

From Model 1 to Model 4:

Frontal Area Growth Factor: $78.540 / 12.566 \approx 6.25\times$

Cd Growth Factor: $0.237 / 0.162 \approx 1.46\times$

Observed Drag-Force Growth Factor: $2500.01 / 273.80 \approx 9.13\times$

The frontal-area increase is therefore the larger numerical contributor to the rise in total drag force across the tested model set, while the increase in Cd further amplifies the effect. The Cd increase may be associated with changes in pressure distribution, boundary-layer behaviour, separation, and wake structure; however, these mechanisms were not directly resolved by the present computational method and cannot be attributed to curvature alone.

8 CONCLUSION AND FUTURE WORK

8.1 Hypothesis Verification and Quantitative Findings

The results were consistent with the hypothesis that, within the tested family of polynomial profiles, coupled increases in peak height (h), curvature coefficient (a), and the resulting frontal area would be associated with higher Cd, aerodynamic drag force, and drag-induced energy loss. Because h and a changed simultaneously, the results verify the predicted trend across the four tested configurations but do not independently establish whether curvature or geometric scale caused the observed change in Cd. Under the constant nominal Reynolds number of approximately 2.3×10^7 , the quantitative progression was as follows:

From Model 1 to Model 4, frontal area increased by approximately 6.25 times, Cd increased by approximately 1.46 times, and the combined drag force and calculated Eloss increased by approximately 9.13 times. This decomposition indicates that frontal-area growth was the larger numerical contributor to the increase in drag, while the simultaneous rise in Cd further amplified the aerodynamic resistance within the tested configuration set.

8.2 Answering the Research Question

The numerical comparison indicates that frontal-area growth was the larger contributor to the increase in total drag, while the rise in Cd provided an additional amplifying effect. However, because geometric scale and curvature were varied simultaneously, the present results cannot determine the independent effect of curvature alone. Therefore, the research question is answered specifically for the coupled model set used in this investigation rather than as a universal relationship between curvature and aerodynamic drag.

8.3 Practical Applications and Future Work

Vehicle and Transport Design: The results demonstrate how frontal area and overall profile geometry can strongly influence aerodynamic resistance. In early-stage vehicle, train, or transport design, reducing unnecessary frontal area and optimizing local surface shape may reduce the force required to overcome aerodynamic drag.

Future Parameter-Isolation Research: A stronger follow-up study should vary one geometric parameter at a time. In particular, models should be created with the same frontal area while changing polynomial curvature, allowing the influence of shape to be examined separately from global size.

8.4 Methodological Refinements

The present investigation should be treated as a controlled preliminary computational study. A stronger future methodology would first isolate geometric parameters by varying h and a independently and by introducing a constant-frontal-area curvature series.

For higher-fidelity aerodynamic analysis, future work should use a CFD solver such as SimScale [5], with a validated setup capable of resolving surface pressure, wall shear stress, flow separation, recirculation, wake structure, and turbulent behaviour. Such a study should clearly state the turbulence model, perform a mesh-independence study, and compare at least one configuration with published benchmark data or wind-tunnel measurements.

These additions would allow the geometric causes of Cd variation to be examined more confidently and would provide stronger external validation of the aerodynamic results.

8.5 Limitations

Coupled Geometric Variables: Peak height (h) and curvature coefficient (a) were increased simultaneously, while frontal area also changed as a consequence of the geometry. The present four-model dataset therefore cannot isolate the independent aerodynamic effect of curvature from the effect of global geometric scale and frontal area.

Limited Number of Geometries: Only four configurations were evaluated. Although these models show a consistent trend within the tested range, a larger parametric sweep would provide stronger evidence regarding how aerodynamic outputs vary across intermediate geometries.

Simplified Vehicle Geometry: The investigation used idealized polynomial bodies rather than complete vehicle geometries. The results are therefore most suitable for identifying qualitative and comparative trends within the simplified model family and should not be interpreted as direct predictions for full vehicles.

Computational Method Limitations: The custom Python workflow applies mathematical aerodynamic relationships but does not perform mesh-based CFD. Consequently, surface-pressure distributions, wall shear stress, flow separation, recirculation, turbulent kinetic energy, and detailed wake structures were not independently resolved.

Scale-Transfer Limitations: A nominal Reynolds number of approximately 2.3×10^7 was maintained as a controlled reference condition. However, separation and wake behaviour may vary with scale, meaning trends obtained from simplified geometries should not automatically be assumed to transfer directly to full-scale vehicle conditions.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

REFERENCES

- [1] White FM. Fluid mechanics. 7th ed. New York (NY): McGraw-Hill; 2011.
- [2] Desmos Studio PBC. Desmos graphing calculator [Internet]. 2024. Available from: <https://www.desmos.com/>
- [3] PTC Inc. Onshape computer-aided design (CAD) platform [Internet]. 2024. Available from: <https://www.onshape.com/>
- [4] Google LLC. Google Colaboratory [Internet]. 2024. Available from: <https://colab.research.google.com/>
- [5] SimScale GmbH. SimScale cloud-based computational fluid dynamics (CFD) platform [Internet]. 2024. Available from: <https://www.simscale.com/>

APPENDIX

Appendix A: Google Colab Custom Python Script

The following Python-based data processing script was executed within the Google Colab environment to standardize geometric calculations, compute aerodynamic forces, and calculate drag-induced energy loss consistently across all four configurations. The script provides computational consistency between models but does not independently validate C_d values or resolve aerodynamic flow fields.

Google Colab Custom Code

https://docs.google.com/document/d/1e9M_m2rryxbyaIG1nlsqw8mkdmoT30nleBtRByn3iwQ/edit?usp=sharing

Appendix B: Sample Mathematical Derivations (Model 1)

To verify the arithmetic consistency of the computational processing, this section reproduces the Model 1 calculations manually using the controlled flow parameters of air density (ρ) = 1.196 kg/m³ and velocity (v) = 15 m/s, together with the full-precision Model 1 C_d value of 0.161932.

Frontal Cross-Sectional Area (A)

Because the 2D profile is revolved 360 degrees about its longitudinal axis, the maximum profile height h defines the radius r of the circular frontal cross-section used in the simplified model:

$$\text{Area} = \pi \cdot r^2 = \pi \cdot (2.0)^2 = 4 \cdot \pi = 12.566 \text{ m}^2$$

Aerodynamic Drag Force (F_d)

Using the fundamental **fluid dynamics** drag equation:

$$F_d = \frac{1}{2} \cdot C_d \cdot \rho \cdot A \cdot v^2$$

$$F_d = 0.5 \times 0.161932 \times 1.196 \times 12.56637 \times (15)^2 = 273.80 \text{ N}$$

This manual calculation confirms that the custom code applies the stated drag equation consistently for Model 1; it does not constitute independent validation of the reported C_d value.

Drag-Induced Energy Loss (E_{loss})

Mechanical work required to overcome the calculated aerodynamic drag over the fixed evaluation distance ($d = 1500$ m):

$$E_{\text{loss}} = F_d \times d = 273.795 \text{ N} \times 1500 \text{ m} \approx 410,693 \text{ J} = 410.69 \text{ kJ}$$

Therefore, E_{loss} represents mechanical work against aerodynamic drag over the specified distance rather than a direct measurement of turbulent or viscous energy loss within the airflow.

Appendix C: Geometric Boundary Conditions & Coefficient Derivation

The investigation maintained a strictly **fixed vehicle length of 20.00 m** across a static domain of $-10.00 < x < 10.00$. To ensure the edge profiles naturally intersect the baseline ($y = 0$) at the domain limits, the quadratic coefficient a must explicitly scale as a function of the vertex height h .

$$y = h - ax^2$$

Setting the boundary condition at the outer domain limit ($x = 10, y = 0$):

$$0 = h - a(10)^2$$

$$0 = h - 100a$$

$$100a = h$$

$$a = h / 100$$

This boundary condition establishes a direct mathematical coupling between peak height (h) and curvature coefficient (a), such that $a = h / 100$ for the fixed domain $-10 < x < 10$. Consequently, increasing h necessarily increases a within the present model family, meaning the effects of geometric scale and curvature cannot be independently isolated using these four configurations. The resulting parameter values are:

Model 1 ($h = 2.0$): $a = 0.02$

Model 2 ($h = 3.0$): $a = 0.03$

Model 3 ($h = 4.0$): $a = 0.04$

Model 4 ($h = 5.0$): $a = 0.05$

This relationship was useful for maintaining the same outer boundary points across the original model set, but it also creates an experimental-design limitation. A future parameter-isolation study should relax this coupling and construct geometries in which h and a can be varied independently, including configurations with constant frontal area and different polynomial curvature.

Appendix D: Supplemental Aerodynamic Data Analysis

The analysis below compares the percentage changes in peak height, Cd, frontal area, and calculated Eloss relative to Model 1. These values describe the combined progression across the coupled geometry set and should not be interpreted as evidence that height or curvature independently caused the aerodynamic changes.

Model 1 (Baseline)

Peak Height Increase: 0.0%

Cd Increase: 0.0%

Frontal Area Increase: 0.0%

Eloss Increase: 0.0%

Model 2

Peak Height Increase: +50.0%

Cd Increase: +14.2%

Frontal Area Increase: +125.0%

Eloss Increase: +156.6%

Model 3

Peak Height Increase: +100.0%

Cd Increase: +29.6%

Frontal Area Increase: +300.0%

Eloss Increase: +419.9%

Model 4

Peak Height Increase: +150.0%

Cd Increase: +46.3%

Frontal Area Increase: +525.0%

Eloss Increase: +813.1%