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QUANTUM TIME DILATION: RELATION BETWEEN TIME DILATION AND ENERGY DIFFERENCE OF QUANTUM WAVE FUNCTIONS IN GALILEAN AND LORENTZ FRAMES

Tanujit Mondal ^{1,*}, Hriddhiman Bag ¹ and Arnab Maitra ²

¹ M. Tech Scholar, Department of Civil Engineering, NITTTR Kolkata, PIN - 700106

² M. Tech, Artificial Intelligence & Machine Learning, NITTTR Kolkata, PIN - 700106

* Corresponding Author

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ABSTRACT

This paper investigates the quantum mechanical relationship between time dilation and energy difference of the quantum photon and electron wave functions. The wave functions are studied under two different times t_1 and t_2 and two energy differences E_{n_1} and E_{n_2} . The time-dependent Schrödinger equation (TDSE) is used to analyse the wave functions and time dilation is studied in the Galilean frame of reference. A key relation linking Lorentz time dilation to the energy difference between the photon (relativistic kinetic energy) and electron (classical kinetic energy) wave functions is derived analytically. Python simulation is used to compute and visualise the wave function phase, energy ratio, total phase energy, and time dilation as functions of speed v/c .

Keywords: Quantum Wave Function, Time Dilation, Schrödinger Equation, Galilean Frame, Lorentz Transformation, Photon, Electron, Energy Difference, Relativistic Kinetic Energy

1 INTRODUCTION

The quantum photon wave function and the quantum electron wave function are taken into consideration and studied as two wave functions under two different times t_1 and t_2 , and two energy differences E_{n_1} and E_{n_2} . The two wave functions are time-dependent and are analysed by the time-dependent Schrödinger equation (TDSE), and time dilation is studied.

Two quantum wave functions are studied under the Galilean frame of reference and the time relation between the two quantum wave functions is recorded. This study provides a theoretical bridge between quantum mechanics and special relativistic time dilation through the formalism of time-dependent quantum wave functions.

The motivation for this study arises from the deep connection between quantum phase evolution and relativistic effects. In the Schrödinger picture, the temporal evolution of a quantum state is governed by the energy eigenvalues through the phase factor $e^{-iEt/\hbar}$. When two quantum systems --- a photon (characterised by relativistic kinetic energy) and an electron (characterised by classical kinetic energy) --- are compared in a Galilean frame, the ratio of their wave functions encodes the time dilation between frames.

2 METHODOLOGY

Theoretically, the quantum wave functions of the photon and electron are taken, and the time relation between the two quantum wave functions is calculated. The methodology consists of three steps :

- Writing the time-dependent wave functions of the photon and electron using the TDSE.
- Taking the ratio of the two wave functions in the Galilean frame and computing the logarithmic form.
- Applying Lorentz time dilation to derive the relation between energy difference and time dilation.

Python quantum coding is used to compute and verify the wave function ratios and to plot: (a) time dilation vs speed, (b) relativistic vs classical kinetic energy vs speed, (c) total phase energy vs speed, and (d) energy ratio E_{n_2}/E_{n_1} vs speed.

The setup is defined as :

$$\Psi_1(r_1, t_1): \text{light wave function} \longrightarrow E_{n_1} = (\gamma - 1)mc^2 \quad (\text{relativistic KE})$$

$$\Psi_2(r_2, t_2): \text{electron wave function} \longrightarrow E_{n_2} = \frac{1}{2}m_e v^2 \quad (\text{classical KE})$$

3 RESULTS AND DISCUSSION

Wave Functions of Photon and Electron

Let the quantum wave functions of the photon and electron be expressed as:

$$\Psi_1(r_1, t_1) = \sum_{n_1} C_{n_1} \psi_{n_1}(r_1) e^{-iE_{n_1} t_1/\hbar},$$

$$\Psi_2(r_2, t_2) = \sum_{n_2} C_{n_2} \psi_{n_2}(r_2) e^{-i(E_{n_1} t_1 + E_{n_2} t_2)/\hbar}.$$

Wave Function Ratio in the Galilean Frame

Taking the ratio of the two wave functions:

$$\frac{\Psi_1(r_1, t_1)}{\Psi_2(r_2, t_2)} = \sum \frac{C_{n_1}}{C_{n_2}} \frac{\psi_{n_1}(r_1)}{\psi_{n_2}(r_2)} e^{-(i/\hbar)(E_{n_1} t_1 + E_{n_2} t_2)}.$$

Taking the natural logarithm of both sides :

$$\ln \left[\frac{\Psi_1(r_1, t_1)}{\Psi_2(r_2, t_2)} \right] = \sum \ln \left(\frac{c_1}{c_2} \right) + \ln \left[\frac{\psi_{n_1}(r_1)}{\psi_{n_2}(r_2)} \right] - \frac{i}{\hbar} (E_{n_1} t_1 + E_{n_2} t_2).$$

Energy Relations and Lorentz Time Dilation

The energies are defined as:

$$E_{n_1} = (\gamma - 1) mc^2, \quad E_{n_2} = \frac{1}{2} m_e v^2.$$

Computing the combined energy-time product:

$$E_{n_1} t_1 + E_{n_2} t_2 = (\gamma - 1) mc^2 t_1 + \frac{1}{2} m_e v^2 t_2 = E_{n_1} \left(t_1 + \frac{E_{n_2} t_2}{E_{n_1}} \right).$$

Applying the Lorentz time dilation relation $t_1 = t_2 / \sqrt{1 - v^2/c^2}$ and substituting the energy expressions:

$$t_1 = \frac{t_2}{\sqrt{1 - \frac{\frac{1}{2}m_e v^2(\gamma - 1)}{\frac{1}{2}mc^2(\gamma - 1)}}} = \frac{t_2}{\sqrt{1 - \frac{E_{n_2}(\gamma - 1)}{\frac{1}{2}\gamma E_{n_2}}}}.$$

Derivation of the Time Ratio

$$\frac{t_1}{t_2} = \frac{1}{\sqrt{1 - \frac{2(\gamma - 1)}{\gamma} \frac{E_{n_2}}{E_{n_1}}}}.$$

Solving for the time ratio:

$$\frac{t_1^2}{t_2^2} = \frac{1}{1 - \frac{2(\gamma - 1)}{\gamma} \frac{E_{n_2}}{E_{n_1}}}.$$

Squaring:

$$\text{Rearranging: } \frac{t_1^2}{t_2^2} = 1 + \frac{2(\gamma - 1)}{\gamma} \frac{E_{n_2}}{E_{n_1}}.$$

$$\text{Therefore the energy ratio is: } \frac{E_{n_2}}{E_{n_1}} = \left(\frac{t_1^2}{t_2^2} - 1 \right) \frac{\gamma}{2(\gamma - 1)}.$$

Key Result: Energy E_{n_2} in Terms of Time Variables

Solving for E_{n_2} step by step:

$$\begin{aligned} E_{n_2} &= E_{n_1} \left[t_1 + \left(\frac{t_1^2}{t_2^2} - 1 \right) \frac{\gamma t_2}{2(\gamma - 1)} \right] \\ &= E_{n_1} \left[t_1 + \frac{(t_1^2 - t_2^2)}{t_2^2} \cdot \frac{\gamma t_2}{2(\gamma - 1)} \right] \\ &= E_{n_1} \left[t_1 + \frac{t_1^2 - t_2^2}{t_2} \cdot \frac{\gamma}{2(\gamma - 1)} \right] \\ &= (\gamma - 1) mc^2 t_1 + \frac{\gamma}{2} \frac{t_1^2 - t_2^2}{t_2}. \end{aligned} \quad (1)$$

This yields the fundamental result:

$$\boxed{E_{n_2} = (\gamma - 1) mc^2 t_1 \left[1 + \frac{\gamma}{2} \frac{t_1^2 - t_2^2}{t_1 t_2} \right]} \quad (2)$$

where $\gamma = 1/\sqrt{1 - v^2/c^2} > 1$ is the Lorentz factor. Equation 2 is the fundamental relation between time dilation and energy difference of the photon and electron wave functions in the Galilean frame of reference (TDSE frame).

3.1 Python Simulation Results

The Python simulation (see Figure 1) produces four plots for a proton/electron mass system as a

function of speed v/c :

- **Time Dilation** - shows $t_1 = \gamma t_2$ rising steeply as $v \rightarrow c$, with $t_2 = 1$ s as the proper time.
- **Relativistic vs Classical KE** - $E_{n_1} = (\gamma - 1)mc^2$ diverges as $v \rightarrow c$, while $E_{n_2} = \frac{1}{2}m_e v^2$ remains small.
- **Total Phase Energy** - $E_{n_1} t_1 + E_{n_2} t_2$ diverges as $v \rightarrow c$, confirming the phase accumulation.

- **Energy Ratio** - $E_{n_2}/E_{n_1} \rightarrow 0.5$ in the non-relativistic limit ($v \rightarrow 0$) and $\rightarrow 0$ as $v \rightarrow c$ and as consistent with the classical/relativistic energy correspondence.

These results confirm *Equation 2* numerically and provide visual evidence for the quantum time dilation relation across the full relativistic speed range.

3.2 Quantum Time Dilation: Summary of the Relation

Based on the wave function ratio in the Galilean frame and the Lorentz time dilation, the complete system is summarised as follows.

Set-up :

$$\Psi_1(r_1, t_1): \text{light wave function} \Rightarrow E_{n_1} = (\gamma - 1)mc^2$$

$$\Psi_2(r_2, t_2): \text{electron wave function} \Rightarrow E_{n_2} = \frac{1}{2}m_e v^2$$

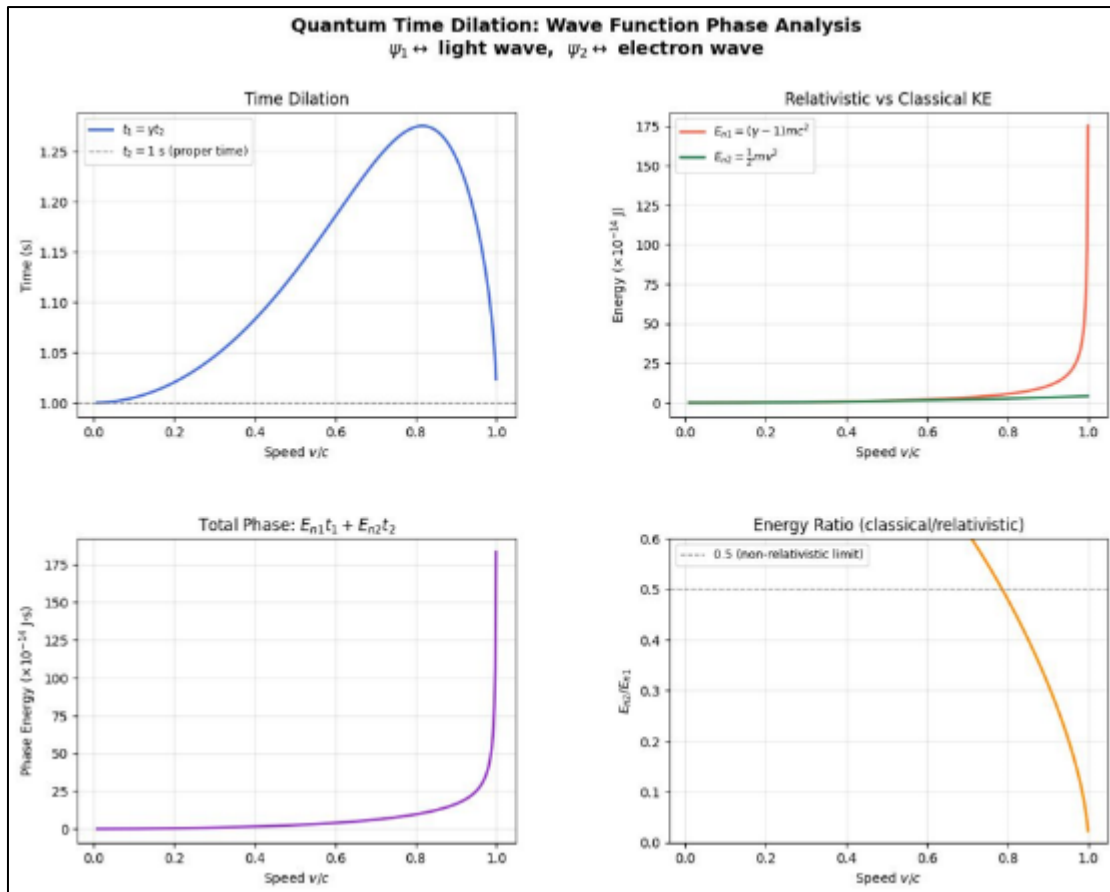


Figure 1: Visualisation for a proton/electron mass system as a function of speed v/c

3.3 Result

$$\frac{t_2}{\sqrt{1 - \frac{2(\gamma-1)}{\gamma} \frac{E_{n_2}}{E_{n_1}}}} \left[1 + \frac{\gamma}{2} \frac{t_1^2 + t_2^2}{t_1 t_2} \right] = \frac{E_{n_2}}{(\gamma - 1)mc^2}.$$

The Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2} > 1$. *Equation 2* is the relation between time dilation and energy difference of the photon and electron wave functions of the TDSE in the Galilean frame.

4 CONCLUSION

This paper has derived a novel relation (*Equation 2*) between time dilation and the energy difference of quantum photon and electron wave functions in the Galilean frame. The time-dependent Schrödinger equation was used to construct wave functions of the photon (relativistic KE : $E_{n_1} = (\gamma - 1)mc^2$) and the electron (classical KE : $E_{n_2} = \frac{1}{2}m_e v^2$). By taking the ratio of these two wave functions in the Galilean frame and applying the Lorentz time dilation, the energy E_{n_2} is expressed in terms of E_{n_1} , the Lorentz factor γ , and the time variables t_1 and t_2 :

$$E_{n_2} = (\gamma - 1) mc^2 t_1 \left[1 + \frac{\gamma}{2} \frac{t_1^2 - t_2^2}{t_1 t_2} \right].$$

Python simulation confirms the derived relation numerically, showing that the energy ratio E_{n_2}/E_{n_1} approaches 0.5 in the non-relativistic limit and tends to zero as $v \rightarrow c$. The result represents a quantum mechanical description of time dilation and demonstrates how the two quantum systems i.e. light and matter are connected through relativistic effects within the TDSE framework.

Future work may extend this framework to include the full relativistic Dirac equation and explore the wave function phase ratio under curved spacetime metrics.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

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