



International Journal of Science and Research Archive

eISSN: 2582-8185

Cross Ref DOI: 10.30574/ijrsra

Journal homepage: <https://ijrsra.net/>

(RESEARCH ARTICLE)



AN HEWMA-BASED ESTIMATOR FOR POPULATION MEAN UNDER SIMULTANEOUS NON-RESPONSE AND MEASUREMENT ERRORS IN TIME-BASED SURVEYS

Sanjay Kumar * and Amit

Department of Statistics, Central University of Rajasthan, Kishangarh, Ajmer, Rajasthan, India.

* Corresponding Author

ORCID Details

Sanjay Kumar: <https://orcid.org/0000-0002-8527-9543>

International Journal of Science and Research Archive, 2026, 20(03), 321–335

Publication history: Received on 30 July 2026; revised on 05 September 2026; accepted on 07 September 2026

Article DOI: <https://doi.org/10.30574/ijrsra.2026.20.3.1729>

Copyright © 2026 Author(s) retain the copyright of this article. This article is published under the terms of the Creative Commons Attribution License 4.0

ABSTRACT

Estimating key unknown population parameters, such as population mean and the population total, has been a fundamental objective in sample survey theory. However, this task becomes challenging particularly in time-based surveys when the observed data are contaminated by non-response and measurement errors. The presence of non-response and measurement errors can substantially reduce the accuracy and reliability of conventional estimators. Motivated by these practical challenges, this study proposes a novel estimator for the population mean based on a hybrid exponentially weighted moving average (HEWMA) framework. The HEWMA framework effectively integrates information from both current and past surveys. The proposed estimator optimally combines time-based information while mitigating the adverse impact of missing and mis-specified observations. Analytical expressions for the bias and variance of the proposed estimator are derived, and explicit efficiency conditions are established to compare it with the existing classical estimator. The performance of the estimator is further evaluated through extensive Monte Carlo simulations across measurement error and varying levels of non-response. The practical applicability of the proposed estimator is also demonstrated using a real survey dataset. The results indicate that the proposed HEWMA-based estimator consistently achieves lower variance and higher efficiency than the existing estimators across a wide range of practical scenario. Therefore, the proposed HEWMA-based estimator offers a reliable and efficient alternative for estimating the population mean in time-based surveys affected by non-response and measurement errors.

Keywords: HEWMA; Non-Response; Measurement Error; Time-Based Survey; Simulation Study.

1 INTRODUCTION

Due to the impracticality of complete enumeration in some situations, sample surveys provide reliable information across several disciplines, including agricultural sciences, business management, demography, economics, education, engineering, medical sciences, and social sciences. It enables inference about unknown parameters in a population, such as totals, means, proportions, etc. The most fundamental inference about a population is estimating its mean with

greater precision of the proposed estimators. Several authors such as Cochran (1940), Murthy (1964), Sahai (1979), Prasad (1989), Singh (2003), Kadilar and Cingi (2004), Kadilar et al. (2009), Ozturk (2014), Hafeez and Shabbir (2015), Singh et al. (2016), Sharma and Kumar (2021), Kusum (2022), Yadav (2022), and others, have studied the estimation of the population mean.

In real-life situations, a survey most often encounters non-sampling errors in addition to sampling error. Among these, non-response and measurement error substantially influence the accuracy and reliability of statistical estimates.

Measurement error constitutes a major form of non-sampling error. It occurs when the recorded response deviates from the true value of the variable of interest. Such errors may arise due to recall bias, misunderstanding of questions, interviewer effects, ambiguous questionnaire design, or data processing inaccuracies. Measurement error can introduce bias and inflate variability, thereby reducing the overall accuracy and credibility of survey findings. For example, in online surveys, people may give higher ratings than they truly feel or misunderstand the scale. In rural health surveys, some respondents may not fully understand the questions, leading to wrong answers. In studies involving elderly or ill individuals, memory problems or weakness can lead to inaccurate responses. A large number of researchers have studied parameter estimation in the presence of measurement error, including works such as Cochran (1968, 1977), Fuller (2009), Biemer and Stokes (2004), Singh and Karpe (2009, 2010), Kumar et al. (2011), Shukla et al. (2012), Sharma and Singh (2013), Zahid and Shabbir (2019), Tiwari et al. (2022), and others.

Non-response is another important type of non-sampling error. It arises when information can not be obtained from all the sampled units. Broadly, the non-response is classified into item non-response and unit non-response. Item non-response refers to situations in which respondents participate in the survey but fail to provide complete information for certain variables. In contrast, unit non-response occurs when some sampled units do not provide any information, leaving those units with no observations. In this study, we consider the problem of unit non-response, which may lead to biased and insufficient survey outcomes. We usually encounter this issue more often in mail surveys involving human respondents rather than personal interviews. Several factors contribute to unit non-response, including lack of availability, limited interest, unwillingness to cooperate, low literacy levels, physical or mental constraints, and concerns. Such factors may systematically affect participation and consequently compromise the representativeness and validity of survey estimates. For example, consumers may ignore online product feedback surveys because they do not see a personal benefit, leading to biased estimates of customer satisfaction. In rural areas, respondents may not understand written questionnaires on health practices, potentially affecting the accuracy of public health estimates. Elderly or severely ill individuals may be unable to respond due to physical weakness or cognitive decline, which may lead to biased health indicators.

Hansen and Hurwitz (1946) first addressed the issue of non-response in mail surveys and suggested random sub-sampling from the initial non-respondents after the initial mail questionnaire, then contacting them with a more expensive method. Further, several authors, such as Rao (1986, 1987), Khare & Srivastava (1993, 1995), Singh (2003), Singh & Kumar (2008), Khare and Kumar (2009a, 2009b, 2010, 2011a, 2011b, 2015), Ahmed et al. (2016), Sharma & Kumar (2020), and others have examined the problem of non-response.

In practical survey applications, measurement error and non-response often occur simultaneously rather than separately. This combined occurrence can substantially affect the accuracy and reliability of survey estimates. For example, in online product feedback surveys, many consumers may not respond at all (non-response) and among those who do respond, some may give inaccurate or overly positive ratings (measurement error), in rural health surveys, some individuals may not participate (non-response) and even when they respond, they may misinterpret questions and provide incorrect answers (measurement error), in health and aging studies, elderly or severely ill individuals may fail to respond (non-response) and those who do respond may give imprecise or memory-based answers (measurement error). To address these situations in sample surveys, several authors including Jackman (1999), Dixon (2010), Azeem (2014), Kumar et al. (2018), Zahid and Shabbir (2018), Kumar and Choudhary (2021), Kumar et al. (2023), Audu et al. (2025), Kumar and Kour (2026), and others studied the effects of both measurement error and non-response on the estimation of population parameter. These studies highlight the importance of developing estimation methods that explicitly account for both sources of non-sampling error.

The studies discussed above are primarily designed for cross-sectional surveys, in which data are generally collected from a current survey only for estimating parameters. In many practical surveys, however, information is collected repeatedly over time such as weekly, monthly, or annually. For example, in online product surveys, customer satisfaction is monitored regularly to track changes in consumer preferences, in rural health surveys, health indicators are measured periodically to evaluate the impact of public health programs, in health and aging studies, physical and cognitive conditions are followed over several years, in agricultural surveys, crop yields are recorded across multiple growing seasons, in economic surveys, household income and expenditure are tracked over successive quarters. In such situations, conventional estimators may not fully utilize the historical information in the data. These applications highlight the need for estimators specifically designed for time-based surveys. For such time-based surveys, we can

improve the performance of estimators if we utilize information from both the current and the past surveys. To address this issue, several authors have proposed memory-type estimators that use current observations along with information from previous survey periods. Important contributions include those by Aslam et al. (2020), Noor-ul Amin (2020, 2021), Chhapparwal and Kumar (2022), Kumar and Chhapparwal (2024), Kumar et al. (2024), Jakhar and Kumar (2024), and Koçyiğit (2025). By incorporating historical information, these estimators often provide more accurate and efficient estimates. Their work has extended survey estimation methods to a wide range of real-world time-dependent applications.

Research on the combined effects of non-response and measurement error in time-based surveys remains very limited. Only a few studies have addressed this issue. Kumar and Chhapparwal (2025) developed an estimator for the population mean in the presence of nonresponse in time-based surveys under a simple random sampling design. Qureshi et al. (2024) developed estimators for population variance in the presence of measurement error in time-based surveys under simple random and stratified sampling designs. Despite these contributions, the available literature remains sparse. This reveals a significant research gap and highlights the need for further methodological development in this area.

In this study, we propose an estimator for the population mean under the simultaneous presence of non-response and measurement errors. The proposed estimator is constructed using the hybrid exponentially weighted moving average (HEWMA) statistic. It combines information from the current and previous survey occasions to improve estimation efficiency.

2 Hybrid Exponentially Weighted Moving Average (HEWMA)

Let $\{Z_t\}_{t \geq 1}$ denote a sequence of independent and identically distributed normal random variables with mean μ and variance σ^2 , that is, $Z_t \sim N(\mu, \sigma^2)$. The hybrid exponentially weighted moving average (HEWMA) statistic, denoted by H_t , is defined recursively as follows:

$$H_t = \lambda_1 E_t + (1 - \lambda_1)H_{t-1}, \quad 0 < \lambda_1 \leq 1 \quad (1)$$

and

$$E_t = \lambda_2 Z_t + (1 - \lambda_2)E_{t-1}; 0 < \lambda_2 \leq 1, t \geq 1, \quad (2)$$

where the subscripts t and $t - 1$ are the survey occasions consist of a sample size n , the smoothing constants λ_1 and λ_2 control the relative contributions of the current and past survey information, and the quantities H_{t-1} and E_{t-1} represent the HEWMA and EWMA statistics for the previous survey occasion, respectively.

Haq (2013) introduced the HEWMA statistic, denoted by (H_t) , which is constructed using the exponentially weighted moving average (EWMA) statistic, (E_t) . The initial values of both (H_t) and (E_t) are taken as the expected population mean. In practice, these initial values can be obtained from a pilot survey or historical data. Haq (2017) further showed that the HEWMA statistic is an unbiased estimator of the population mean, that is,

$$E(H_t) = \mu$$

and

$$V(H_t) = \frac{(\lambda_2 \lambda_1)^2}{(\lambda_2 - \lambda_1)^2} \left\{ \frac{(1 - \lambda_1)^2 (1 - (1 - \lambda_1)^{2t})}{1 - (1 - \lambda_1)^2} + \frac{(1 - \lambda_2)^2 (1 - (1 - \lambda_2)^{2t})}{1 - (1 - \lambda_2)^2} - \frac{2(1 - \lambda_1)(1 - \lambda_2)(1 - (1 - \lambda_1)^t (1 - \lambda_2)^t)}{1 - (1 - \lambda_1)(1 - \lambda_2)} \right\} \frac{\sigma^2}{n}$$

$$= \lambda_t^* \frac{\sigma^2}{n}, \quad (3)$$

$$\text{where } \lambda_t^* = \frac{(\lambda_2 \lambda_1)^2}{(\lambda_2 - \lambda_1)^2} \left\{ \frac{(1 - \lambda_1)^2 (1 - (1 - \lambda_1)^{2t})}{1 - (1 - \lambda_1)^2} + \frac{(1 - \lambda_2)^2 (1 - (1 - \lambda_2)^{2t})}{1 - (1 - \lambda_2)^2} - \frac{2(1 - \lambda_1)(1 - \lambda_2)(1 - (1 - \lambda_1)^t (1 - \lambda_2)^t)}{1 - (1 - \lambda_1)(1 - \lambda_2)} \right\}.$$

3 EXISTING ESTIMATORS

Considering a finite population $U: (U_1, U_2, \dots, U_N)$ of size N , it is supposed that the population U is composed of two separate and non-overlapping groups: responding with N_1 units and non-responding with N_2 units. Now, let Y refer to the study variable with corresponding values $y_1, y_2, \dots, y_N$. One may need to estimate the population mean $\mu_Y (= \sum_{i=1}^N y_i / N)$ of the study variable on the basis of a sample of size n drawn from the population U by the simple random sampling without replacement (SRSWOR). The sample mean estimator $\bar{y} (= \sum_{i=1}^n y_i / n)$ is considered an

unbiased estimator of the population mean μ_Y . However, in real-life situations, a survey most often encounters non-response. In this situation, Hansen and Hurwitz (1946) first addressed this issue of non-response. The summary of the technique suggested by Hansen and Hurwitz (1946) is given below:

(i) The prepared questionnaire is mailed to all the sampled respondents. (ii) The non-respondents are identified after the first attempt to respond. (iii) A random subsampling is made from the identified non-respondents. (iv) Data is collected from the subsampled non-respondents through conducting personal interviews, considering that all the subsampled units respond this time. (v) The data so obtained from the respondents at the first attempt and at the second attempt are combined to estimate the population parameters.

Let n_1 out of n units respond in the first attempt, and n_2 units do not, considered as non-respondents. From n_2 non-respondents, let a SRSWOR sub-sample of size $r = n_2/k$; $k > 1$, to be interviewed personally, and measurements are taken from all the r units. It is considered that all the r units respond this time.

To account for non-response in sample surveys, Hansen and Hurwitz (1946) developed the following unbiased estimator for the population mean (μ_Y) of the study variable:

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{2r}, \quad (4)$$

where, $w_j = n_j/n$, $j = 1, 2$, $\bar{y}_1 = \sum_{i=1}^{n_1} y_i/n_1$ denotes the sample mean of the respondent group based on the observed values obtained in the first survey attempt, and $\bar{y}_{2r} = \sum_{i=1}^r y_i/r$ denotes the sample mean of the subsampled non-respondent group based on the observed values interviewed in the second attempt.

and showed the variance of the estimator $\bar{y}^*$, given by

$$V(\bar{y}^*) = \delta S_Y^2 + \frac{W_2(k-1)}{n} S_{Y(2)}^2, \quad (5)$$

where, $\delta = (1/n - 1/N)$, $S_Y^2 = \sum_{i=1}^N (y_i - \mu_Y)^2 / (N - 1)$, $S_{Y(2)}^2 = \sum_{i=1}^{N_2} (y_i - \mu_{Y(2)})^2 / (N_2 - 1)$, $\mu_{Y(2)} = \sum_{i=1}^{N_2} y_i / N_2$, $W_2 = N_2/N$, and $\bar{y}_{2r} = \sum_{i=1}^r y_i/r$.

Following the pioneering work of Hansen and Hurwitz (1946), several researchers have further investigated estimation under non-response. Notable contributions include those of Rao (1986, 1987), Khare & Srivastava (1993, 1995), Singh (2003), Singh & Kumar (2008), Khare and Kumar (2009a, 2009b, 2010, 2011b, 2015), Ahmed et al. (2016), Sharma & Kumar (2020), and many others, who proposed improved estimators and methodologies to address non-response in sample surveys.

Measurement error is another important source of non-sampling error that may affect the accuracy of survey estimates. Let (y_i, y_i^m) denote the true and observed values, respectively, of the study variable corresponding to the i^{th} unit in the sample. Then the measurement error is defined as $U_i = y_i^m - y_i$. Assuming that the measurement errors are random with zero mean, the unbiased estimator for the population mean in the presence of the measurement error is given by

$$\bar{y}^m = \sum_{i=1}^n y_i^m / n \text{ with } V(\bar{y}^m) = \delta(S_Y^2 + S_U^2), \quad (6)$$

where $S_U^2 = \sum_{i=1}^N (y_i^m - \mu_U)^2 / (N - 1)$ is the population variance of the measurement errors with $\mu_U = \sum_{i=1}^N y_i^m / N$.

We now consider the situation in which non-response and measurement errors occur simultaneously. Let (y_i, y_i^{*m}) denote the true and observed values, respectively, of the study variable corresponding to the i^{th} unit in the sample. Then the measurement error is given by $U_i^* = (y_i^{*m} - y_i)$, which is random in nature with mean zero and their variances are denoted by S_U^2 for the population and $S_{U(2)}^2$ for the non-responding part of the population. Under these assumptions, the Hansen and Hurwitz's (1946) estimator in the simultaneous presence of non-response and measurement error is given by

$$\bar{y}^{*m} = w_1 \bar{y}_1^m + w_2 \bar{y}_{2r}^m, \quad (7)$$

where $w_j = n_j/n$, $j = 1, 2$, $\bar{y}_1^m = \sum_{i=1}^{n_1} y_i^{*m} / n_1$ denotes the sample mean of the respondent group based on the observed values obtained in the first survey attempt, and $\bar{y}_{2r}^m = \sum_{i=1}^r y_i^{*m} / r$ denotes the sample mean of the subsampled non-respondent group based on the observed values interviewed in the second attempt.

The variance of the estimator $\bar{y}^{*m}$ can be written as

$$V(\bar{y}^{*m}) = \delta(S_Y^2 + S_U^2) + \frac{W_2(k-1)}{n}(S_{Y(2)}^2 + S_{U(2)}^2), \quad (8)$$

where $S_{U(2)}^2 = \sum_{i=1}^{N_2} (y_i^{*m} - \mu_{U(2)})^2 / (N_2 - 1)$, and $\mu_{U(2)} = \sum_{i=1}^{N_2} y_i^m / N_2$.

The studies discussed above primarily utilize information from the current survey occasion for estimating population parameters. However, in many practical applications, data are collected repeatedly over time, such as on a weekly, monthly, quarterly, or annual basis. Incorporating information from previous survey occasions, one can improve the efficiency of the estimators. Motivated by the HEWMA statistic proposed by Haq (2013, 2017) and the non-response framework of Hansen and Hurwitz (1946), Kumar and Chhaparwal (2025) proposed an HEWMA-based estimator for the population mean of the study variable (Y) under non-response in time-based surveys, which is given by

$$H_t^* = \gamma_1 E_t^* + (1 - \gamma_1) H_{t-1}^*; 0 < \gamma_1 \leq 1, \quad (9)$$

and

$$E_t^* = \gamma_2 \bar{y}_t^* + (1 - \gamma_2) E_{t-1}^*; 0 < \gamma_2 \leq 1. \quad (10)$$

where $\bar{y}_t^*$ denotes the Hansen–Hurwitz (1946) estimator at the t -th survey occasion. The quantities H_{t-1}^* and E_{t-1}^* represent the HEWMA and EWMA statistics, respectively, obtained from the previous survey occasion. The smoothing constants γ_1 and γ_2 determine the relative contributions of the current and past survey information. The initial values of E_t^* and H_t^* are taken as the expected population mean and may be obtained from historical information or a pilot survey.

The authors further established that the proposed HEWMA estimator is unbiased for the population mean, that is,

$$E(H_t^*) = \mu_Y, \quad (11)$$

and its variance is given by

$$V(H_t^*) = \gamma_t^* \left[\delta S_Y^2 + \frac{W_2(k-1)}{n} S_{Y(2)}^2 \right], \quad (12)$$

$$\gamma_t^* = \left(\frac{\gamma_1 \gamma_2}{\gamma_1 - \gamma_2} \right)^2 \left[\frac{(1-\gamma_1)^2 \{1-(1-\gamma_1)^{2t}\}}{1-(1-\gamma_1)^2} + \frac{(1-\gamma_2)^2 \{1-(1-\gamma_2)^{2t}\}}{1-(1-\gamma_2)^2} - \frac{2(1-\gamma_1)(1-\gamma_2) \{1-(1-\gamma_1)^t(1-\gamma_2)^t\}}{1-(1-\gamma_1)(1-\gamma_2)} \right]$$

It is worth noting that when $\gamma_1 = 1$ and $\gamma_2 = 1$, the estimator H_t^* reduces to the estimator $\bar{y}^*$, which shows that the Kumar and Chhaparwal (2025) estimator (H_t^*) generalizes the classical Hansen–Hurwitz (1946) estimator $\bar{y}^*$ under non-response.

4 PROPOSED ESTIMATOR

Motivated by the preceding results, we now extend the HEWMA framework to account for the simultaneous presence of non-response and measurement errors in time-based surveys. We propose an HEWMA-type estimator for estimating the population mean, which is given by

$$H_t^{*m} = \alpha_1 E_t^{*m} + (1 - \alpha_1) H_{t-1}^{*m}; 0 < \alpha_1 \leq 1, \quad (13)$$

and

$$E_t^{*m} = \alpha_2 \bar{y}_t^{*m} + (1 - \alpha_2) E_{t-1}^{*m}; 0 < \alpha_2 \leq 1. \quad (14)$$

where $\bar{y}_t^{*m}$ denotes the Hansen–Hurwitz estimator at the t -th survey occasion in the presence of both non-response and measurement errors. The quantities H_{t-1}^{*m} and E_{t-1}^{*m} represent the HEWMA and EWMA statistics, respectively, obtained from the previous survey occasion. The smoothing parameters α_1 and α_2 determine the relative contributions of the current and historical survey information. The initial values of E_t^{*m} and H_t^{*m} are taken as the expected population mean and may be obtained from a pilot survey or reliable historical information.

After repeated substitution, Eqs. (13) and (14) can be written in the forms as

$$H_t^{*m} = \alpha_1 \sum_{i=1}^{t-1} (1 - \alpha_1)^i E_{t-i}^{*m} + (1 - \alpha_1)^t H_0^{*m} \quad (15)$$

and

$$E_t^{*m} = \alpha_2 \sum_{j=0}^{t-1} (1 - \alpha_2)^j \bar{y}_{t-j}^{*m} + (1 - \alpha_2)^t E_{0-1}^{*m}. \quad (16)$$

Now, putting Eq. (16) in Eq. (15), we obtain

$$\begin{aligned} H_t^{*m} &= \alpha_1 \alpha_2 \sum_{i=0}^{t-1} (1 - \alpha_1)^i \sum_{j=0}^{t-i-1} (1 - \alpha_2)^j \bar{y}_{t-i-j}^{*m} + \alpha_1 \sum_{i=0}^{t-1} (1 - \alpha_1)^i (1 - \alpha_2)^{t-i} \mu_Y + (1 - \alpha_1)^t \mu_Y \\ &= \alpha_1 \alpha_2 \sum_{i=1}^t \left\{ (1 - \alpha_2)^{t-i} \sum_{j=0}^{t-i} \left(\frac{1 - \alpha_1}{1 - \alpha_2} \right)^j \right\} \bar{y}_i^{*m} + \alpha_1 \sum_{i=0}^{t-1} (1 - \alpha_1)^i (1 - \alpha_2)^{t-i} \mu_Y + (1 - \alpha_1)^t \mu_Y \end{aligned} \quad (17)$$

Taking expectations from both sides of Eq. (17), we obtain,

$$\begin{aligned} E(H_t^{*m}) &= \alpha_1 \alpha_2 \sum_{i=1}^t \left\{ (1 - \alpha_2)^{t-i} \sum_{j=0}^{t-i} \left(\frac{1 - \alpha_1}{1 - \alpha_2} \right)^j \right\} E(\bar{y}_i^{*m}) + \alpha_1 \sum_{i=0}^{t-1} (1 - \alpha_1)^i (1 - \alpha_2)^{t-i} \mu_Y + (1 - \alpha_1)^t \mu_Y \\ &= \alpha_1 \alpha_2 \sum_{i=1}^t \left\{ (1 - \alpha_2)^{t-i} \sum_{j=0}^{t-i} \left(\frac{1 - \alpha_1}{1 - \alpha_2} \right)^j \right\} \mu_Y + \alpha_1 \sum_{i=0}^{t-1} (1 - \alpha_1)^i (1 - \alpha_2)^{t-i} \mu_Y + (1 - \alpha_1)^t \mu_Y \end{aligned} \quad (18)$$

On simplification of Eq. (18), we obtain,

$$E(H_t^{*m}) = \mu_Y \quad (19)$$

Further, bias and variance of the estimator (H_t^{*m}) are obtained as

$$\text{Bias}(H_t^{*m}) = E(H_t^{*m}) - \mu_Y = 0 \quad (20)$$

and

$$\begin{aligned} V(H_t^{*m}) &= \alpha_1 \alpha_2 \sum_{i=1}^t \left\{ (1 - \alpha_2)^{t-i} \sum_{j=0}^{t-i} \left(\frac{1 - \alpha_1}{1 - \alpha_2} \right)^j \right\} V(\bar{y}_i^{*m}) \\ &= \left(\frac{\alpha_1 \alpha_2}{\alpha_1 - \alpha_2} \right)^2 \left[\sum_{i=1}^2 \frac{(1 - \alpha_1)^2 \{1 - (1 - \alpha_1)^{2t}\}}{1 - (1 - \alpha_1)^2} - \frac{2(1 - \alpha_1)(1 - \alpha_2) \{1 - (1 - \alpha_1)^t (1 - \alpha_2)^t\}}{1 - (1 - \alpha_1)(1 - \alpha_2)} \right] V(\bar{y}_i^{*m}) \\ &= \alpha_t^* \left[\delta(S_Y^2 + S_U^2) + \frac{W_2(k-1)}{n} (S_{Y(2)}^2 + S_{U(2)}^2) \right] \quad (\text{also see Kumar and Chhapparwal (2025)}) \end{aligned} \quad (21)$$

The proposed estimator includes several existing estimators as special cases.

(i) When both non-response and measurement errors are present, setting $\alpha_1 = 1$ and $\alpha_2 = 1$ reduces the proposed estimator H_t^{*m} to the Hansen–Hurwitz estimator in the presence of both non-response and measurement errors, that is $H_t^{*m} = \bar{y}^{*m}$.

(ii) When only non-response is present, setting $\alpha_1 = 1$ and $\alpha_2 = 1$ reduces the proposed estimator H_t^{*m} to the Hansen–Hurwitz estimator in the presence of non-response, that is $H_t^{*m} = \bar{y}^*$.

(iii) When only measurement error is present, setting $\alpha_1 = 1$ and $\alpha_2 = 1$ reduces the proposed estimator H_t^{*m} to the sample mean based on the observed measurements, that is, $H_t^{*m} = \bar{y}^m$, and

(iv) In the absence of both non-response and measurement errors, setting $\alpha_1 = 1$ and $\alpha_2 = 1$ reduces the proposed estimator H_t^{*m} to the usual sample mean estimator, that is, $H_t^{*m} = \bar{y}$.

The above results show that the proposed estimator generalizes several existing estimators. Therefore, it offers a unified estimation framework for complete-response surveys and for surveys affected by non-response, measurement errors, or both.

5 MATHEMATICAL COMPARISON

In this section, we establish the condition under which the proposed HEWMA estimator performs better than the existing Hansen–Hurwitz estimator in the simultaneous presence of non-response and measurement errors. The performance of the proposed estimator H_t^{*m} is better than that of the existing estimator $\bar{y}^{*m}$, that is,

$$V(H_t^{*m}) < V(\bar{y}^{*m}) \text{ if } 0 < \alpha_t^* < 1, \quad (22)$$

which is always true as the values of α_1 and α_2 range from 0 and 1.

6 SIMULATION STUDY

This section evaluates the performance of the proposed estimator through a Monte Carlo simulation study implemented in R (version 4.5.0). For this purpose, a finite population of size ($N = 500$) is generated from the normal distribution with assumed parameters, that is $Y \sim N(0, 1)$. To estimate the variance of the proposed estimator in (13), $M = 51000$ simple random samples of size n ($= 50, 100$) are drawn without replacement from the population using the SRSWOR sampling scheme. Each selected sample represents one survey occasion. Thus, the value of t ranges from 1 to M . The proposed estimator H_t^{*m} is computed for each of the $M = 51000$ samples. The non-response rates are fixed at 20% and 15%. For simplicity, the last 20% and 15% units in each selected sample are treated as non-respondents. A subsample of the non-respondents is then selected by taking the subsampling rate $k = 2.0$. The performance of the proposed estimator is examined under different combinations of the smoothing parameters. First, α_1 is fixed at 0.10 while α_2 varied over 0.15, 0.50, and 0.9999. Next, α_2 is fixed at 0.9999 and α_1 varied over 0.10, 0.25, and 0.999.

We obtain the simulated values of variance and percent relative efficiency (PRE) as follows:

$$V(T) = \frac{1}{M} \sum_{j=1}^M (T_j - \mu_Y)^2 \text{ for } T = \bar{y}^*, H_t^*, \bar{y}^{*m}, \text{ and } H_t^{*m}, \quad (23)$$

$$PRE(T) = \frac{V(\bar{y}^*)}{V(T)} * 100\% \text{ for } T = \bar{y}^* \text{ and } H_t^*, \text{ (Case of non-response)}, \quad (24)$$

and

$$PRE(T) = \frac{V(\bar{y}^{*m})}{V(T)} * 100\% \text{ for } T = \bar{y}^{*m} \text{ and } H_t^{*m} \text{ (Case of non-response \& measurement error)} \quad (25)$$

Table 1: Simulated values of variances and PREs of the estimators at different values of α_2 and n for the fixed value of $\alpha_1 = 0.10$.

	$W_2 = 0.20$				$W_2 = 0.15$			
	$n = 50$		$n = 100$		$n = 50$		$n = 100$	
Est.	Var.	PRE	Var.	PRE	Var.	PRE	Var.	PRE
Case: non-response								
$\alpha_2 = 0.15$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.000801	2843.32	0.000741	3025.09	0.000755	2898.83	0.000290	3212.78
$\alpha_2 = 0.50$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.001120	2034.64	0.001059	2116.90	0.001068	2049.30	0.000420	2214.00
$\alpha_2 = 0.9999 \cong 1.0$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.001264	1802.75	0.001202	1865.50	0.001207	1812.19	0.000480	1938.92
Case: non-response and measurement error								
$\alpha_2 = 0.15$								

$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.001667	2680.10	0.002559	1777.86	0.001370	3107.78	0.001417	1398.64
$\alpha_2 = 0.50$								
$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.002287	1953.18	0.003189	1426.40	0.001966	2165.00	0.001684	1176.90
$\alpha_2 = 0.9999 \cong 1.0$								
$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.002568	1739.25	0.003470	1311.09	0.002237	1903.44	0.001804	1097.98

Table 2: Simulated values of variances and PREs of the estimators at different values of α_1 and n for the fixed value of $\alpha_2 = 0.9999 \cong 1.0$.

	$W_2 = 0.20$				$W_2 = 0.15$			
	$n = 50$		$n = 100$		$n = 50$		$n = 100$	
Est.	Var.	PRE	Var.	PRE	Var.	PRE	Var.	PRE
Case: non-response								
$\alpha_1 = 0.15$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.001903	1197.00	0.001842	1217.00	0.001831	1194.87	0.000743	1252.32
$\alpha_1 = 0.50$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.007603	299.61	0.007477	299.79	0.007356	297.46	0.003093	300.71
$\alpha_1 = 0.9999 \cong 1.0$								
$\bar{y}^*$	0.022778	100.00	0.022416	100.00	0.021880	100.00	0.009301	100.00
H_t^*	0.022728	100.22	0.022367	100.22	0.021833	100.22	0.009281	100.22
Case: non-response and measurement error								
$\alpha_1 = 0.15$								
$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.003815	1171.01	0.004730	961.89	0.003438	1238.16	0.002343	845.49
$\alpha_1 = 0.50$								
$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.014946	298.87	0.015815	287.67	0.014123	301.43	0.007126	278.03
$\alpha_1 = 0.9999 \cong 1.0$								
$\bar{y}^{*m}$	0.044669	100.00	0.045494	100.00	0.042570	100.00	0.019813	100.00
H_t^{*m}	0.044571	100.22	0.045396	100.22	0.042476	100.22	0.019771	100.21

From Tables 1 and 2, it is evident that the proposed estimator (H_t^{*m}) has a smaller variance than the existing Hansen–Hurwitz estimator ($\bar{y}^{*m}$). This indicates that the proposed estimator is more efficient when non-response and measurement errors are present simultaneously. Furthermore, the results indicate that the simultaneous presence of non-response and measurement errors leads to a higher variance than when only non-response is present. This highlights the additional variability introduced by measurement errors.

The results also show that the variance decreases as more historical information is incorporated through the HEWMA framework. The smoothing parameters α_1 and α_2 , which determine the contributions of the current and previous survey occasions, have a noticeable effect on the estimator's performance. As the values of α_1 and α_2 increase, the variance of the proposed estimator increases, leading to a corresponding decrease in its PRE.

In addition, the variance decreases as the non-response rate becomes smaller. Finally, Table 2 shows that, as $\alpha_1 \rightarrow 1$ and $\alpha_2 \rightarrow 1$, the proposed estimator H_t^{*m} converges to the Hansen–Hurwitz estimator $\bar{y}^{*m}$. Consequently, its variance approaches that of $\bar{y}^{*m}$ which agrees with the theoretical result established in Section 4. The results obtained from Tables 2 and 3 are graphically depicted in Figs. 1 and 2, respectively, for better visualization and comparison between the estimators.

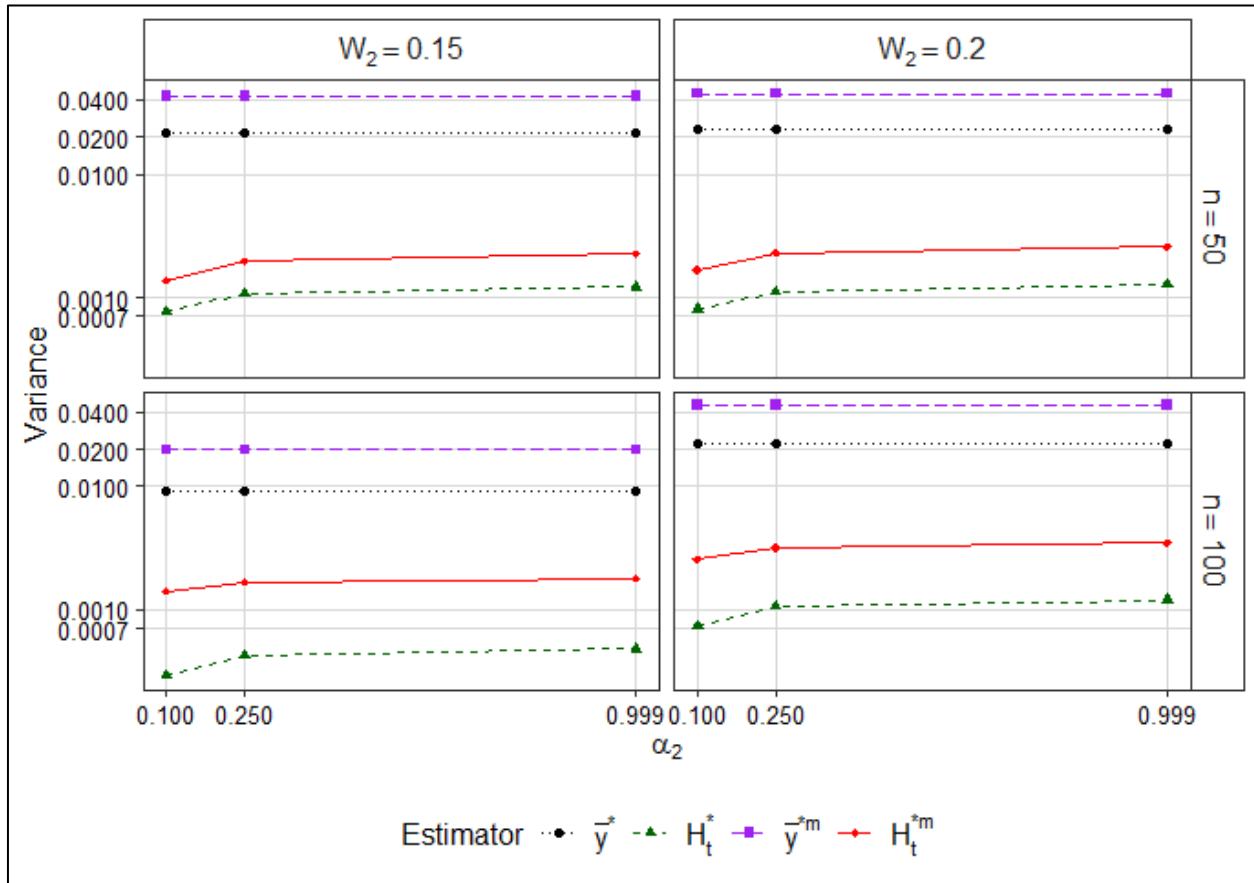


Figure 1: Simulated variances of the estimators corresponding to the results presented in Table 1.

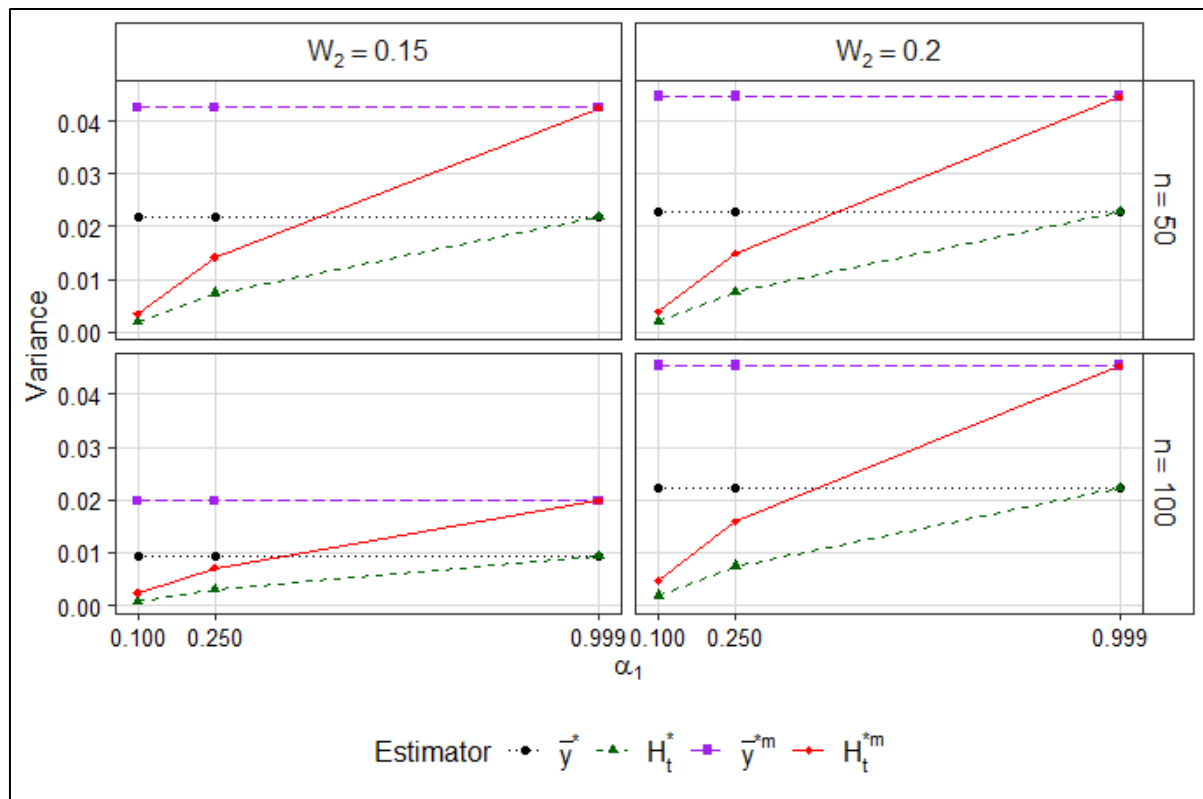


Figure 2: Simulated variances of the estimators corresponding to the results presented in Table 2.

7 REAL DATA APPLICATION

To illustrate the practical applicability of the proposed estimator, we consider a real birth weight dataset available at <https://online.stat.psu.edu/stat501/lesson/8/8.1>. The dataset was originally analyzed by Guha et al. (1973) and later by Kumar et al. (2024). In the present study, birth weight (in grams) of a baby was considered as the study variable (Y). The population consists of 32 birth weights with the population mean ($\mu_Y = 3090.88$ grams).

A simple random sample of 20 birth weights is drawn without replacement from the population. To represent repeated survey occasions, three independent samples ($t = 1, 2, 3$) are selected using the SRSWOR sampling scheme. The non-response rates are assumed to be 25% and 15%. For simplicity, the last 25% and 15% units in each sample are treated as non-respondents. A subsample of the non-respondents is then selected using the Hansen–Hurwitz procedure with a subsampling rate of $k = 2.0$.

The performance of the proposed estimator is evaluated under different combinations of the smoothing parameters. First, α_1 is fixed at 0.10 while α_2 varied over 0.15, 0.50, and 0.9999. Next, α_2 is fixed at 0.9999 and α_1 varied over 0.10, 0.25, and 0.999. Using Eqs. (7), (13), (8) and (21), we compute the values of $\bar{y}^m, H_t^{*m}, V(\bar{y}^m)$ and $V(H_t^{*m})$, respectively. The results are reported in Tables 3 and 4.

The results show that the proposed estimator (H_t^{*m}) has a smaller variance than the existing Hansen–Hurwitz estimator ($\bar{y}^m$). This indicates that the proposed estimator is more efficient when non-response and measurement errors are present simultaneously. Furthermore, the results indicate that the simultaneous presence of non-response and measurement errors leads to a higher variance than when only non-response is present. This highlights the additional variability introduced by measurement errors.

The results also show that the variance decreases as more historical information is incorporated through the HEWMA framework. The smoothing parameters α_1 and α_2 , which determine the contributions of the current and previous survey occasions, have a noticeable effect on the estimator's performance. As the values of α_1 and α_2 increase, the variance of the proposed estimator increases, leading to a corresponding decrease in its PRE.

In addition, the variance decreases as the non-response rate becomes smaller. Finally, Table 2 shows that, as $\alpha_1 \rightarrow 1$ and $\alpha_2 \rightarrow 1$, the proposed estimator H_t^{*m} converges to the Hansen–Hurwitz estimator $\bar{y}^m$. Consequently, its variance approaches that of $\bar{y}^m$ which agrees with the theoretical result established in Section 3.

Table 3: Computations of values of $\bar{y}^*, H_t^*, V(\bar{y}^*)$ and $V(H_t^*)$ under non-response and $\bar{y}^{*m}, H_t^{*m}, V(\bar{y}^{*m})$ and $V(H_t^{*m})$ under non-response and measurement errors for various values of α_2 at fixed value of $\alpha_1 = 0.10$.

	Under non-response				Under non-response and measurement error			
	$\bar{y}^*$	H_t^*	$V(\bar{y}^*)$	$V(H_t^*)$	$\bar{y}^{*m}$	H_t^{*m}	$V(\bar{y}^{*m})$	$V(H_t^{*m})$
$W_2 = 0.25$								
t	$\alpha_2 = 0.15$							
1	3171.18	3020.63	3475.88	0.08690	3171.05	3020.59	3477.54	0.08694
2	2989.08	3019.72	3475.88	0.38430	2989.25	3019.68	3477.54	0.38449
3	3068.45	3020.12	3475.88	0.95713	3068.48	3020.08	3477.54	0.95759
	$\alpha_2 = 0.50$							
1	3171.18	3027.44	3475.88	8.68971	3171.05	3027.40	3477.54	8.69386
2	2989.08	3018.34	3475.88	25.72155	2989.25	3018.31	3477.54	25.73382
3	3068.45	3022.30	3475.88	45.53496	3068.48	3022.27	3477.54	45.55669
	$\alpha_2 = 0.9999 \cong 1.0$							
1	3171.18	3034.99	3475.88	34.69	3171.05	3034.94	3477.54	34.71
2	2989.08	3016.80	3475.88	62.85	2989.25	3016.78	3477.54	62.88
3	3068.45	3024.73	3475.88	85.66	3068.48	3024.69	3477.54	85.70
$W_2 = 0.15$								
t	$\alpha_2 = 0.15$							
1	3141.73	3020.48	2958.58	0.07396	3141.74	3020.44	2961.10	0.07403
2	2947.00	3019.51	2958.58	0.32711	2947.06	3019.47	2961.10	0.32739
3	3023.08	3019.89	2958.58	0.81469	3023.02	3019.85	2961.10	0.81538
	$\alpha_2 = 0.50$							
1	3141.73	3025.97	2958.58	7.40	3141.74	3025.93	2961.10	7.40
2	2947.00	3016.23	2958.58	21.89	2947.06	3016.20	2961.10	21.91
3	3023.08	3020.04	2958.58	38.76	3023.02	3019.99	2961.10	38.79
	$\alpha_2 = 0.9999 \cong 1.0$							
1	3141.73	3032.05	2958.58	29.53	3141.74	3032.01	2961.10	29.55
2	2947.00	3012.59	2958.58	53.50	2947.06	3012.57	2961.10	53.54
3	3023.08	3020.19	2958.58	72.91	3023.02	3020.15	2961.10	72.97

Table 4: Computations of values of $\bar{y}^*, H_t^*, V(\bar{y}^*)$ and $V(H_t^*)$ under non-response and $\bar{y}^{*m}, H_t^{*m}, V(\bar{y}^{*m})$ and $V(H_t^{*m})$ under non-response and measurement errors for various values of α_2 at fixed value of $\alpha_1 = 0.9999 \cong 1.0$.

	Under non-response				Under non-response and measurement error			
	$\bar{y}^*$	H_t^*	$V(\bar{y}^*)$	$V(H_t^*)$	$\bar{y}^{*m}$	H_t^{*m}	$V(\bar{y}^{*m})$	$V(H_t^{*m})$
$W_2 = 0.25$								
t	$\alpha_2 = 0.15$							
1	3171.18	3027.44	3475.88	8.68797	3171.05	3027.40	3477.54	8.69212
2	2989.08	3018.34	3475.88	16.53052	2989.25	3018.31	3477.54	16.53841

3	3068.45	3022.30	3475.88	23.60842	3068.48	3022.27	3477.54	23.61969
$\alpha_2 = 0.50$								
4	3171.18	3095.52	3475.88	868.80	3171.05	3095.44	3477.54	869.21
5	2989.08	3004.48	3475.88	1086.08	2989.25	3004.55	3477.54	1086.60
6	3068.45	3044.16	3475.88	1140.41	3068.48	3044.16	3477.54	1140.95
$\alpha_2 = 0.9999 \cong 1.0$								
1	3171.18	3171.01	3475.88	3468.24	3171.05	3170.89	3477.54	3469.90
2	2989.08	2989.11	3475.88	3468.25	2989.25	2989.29	3477.54	3469.90
3	3068.45	3068.40	3475.88	3468.25	3068.48	3068.43	3477.54	3469.90
$W_2 = 0.15$								
t	$\alpha_2 = 0.15$							
1	3141.73	3025.97	2958.58	7.39497	3141.74	3025.93	2961.10	7.40
2	2947.00	3016.23	2958.58	14.07033	2947.06	3016.20	2961.10	14.08
3	3023.08	3020.03	2958.58	20.09484	3023.02	3019.99	2961.10	20.11
$\alpha_2 = 0.50$								
4	3141.73	3080.79	2958.58	739.50	3141.74	3080.78	2961.10	740.13
5	2947.00	2983.44	2958.58	924.44	2947.06	2983.45	2961.10	925.23
6	3023.08	3021.47	2958.58	970.68	3023.02	3021.43	2961.10	971.51
$\alpha_2 = 0.9999 \cong 1.0$								
1	3141.73	3141.59	2958.58	2952.07	3141.74	3141.61	2961.10	2954.59
2	2947.00	2947.08	2958.58	2952.08	2947.06	2947.14	2961.10	2954.59
3	3023.08	3023.07	2958.58	2952.08	3023.02	3023.02	2961.10	2954.59

8 CONCLUSIONS

In many practical surveys, non-response and measurement errors occur simultaneously, particularly when data are collected repeatedly over time. Such errors can substantially reduce the precision of conventional estimators. To address this problem, this study proposes a hybrid exponentially weighted moving average (HEWMA)-based estimator for estimating the population mean in time-based surveys. The proposed estimator effectively combines information from the current and previous survey occasions, thereby improving estimation efficiency.

Theoretical investigations establish that the proposed estimator is more efficient than the existing Hansen–Hurwitz estimator under appropriate conditions. The simulation study and the real data application further support these findings. In both cases, the proposed estimator consistently produces a smaller variance than the existing estimator. The results also indicate that incorporating historical survey information improves estimation efficiency. Furthermore, the simultaneous presence of non-response and measurement errors leads to a higher variance than when only non-response is present. This highlights the additional variability introduced by measurement errors and emphasizes the importance of accounting for both sources of error. Moreover, the variance decreases as the non-response rate becomes smaller. When the smoothing parameters satisfy ($\alpha_1 = 1$) and ($\alpha_2 = 1$), the proposed estimator reduces to the Hansen–Hurwitz estimator, confirming it as a generalized form of the classical estimator. Therefore, it offers a unified estimation framework for complete-response surveys and for surveys affected by non-response, measurement errors, or both.

The proposed estimator is developed under simple random sampling without replacement for homogeneous populations. Future research may extend this framework to other probability sampling designs, including stratified, cluster, and systematic sampling. The proposed approach may also be adapted for estimating other finite population parameters in the presence of non-response and measurement errors in time-based surveys.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Acknowledgments

The authors are deeply grateful to the Editors and the referees.

Funding Source

There is no funding.

Disclosure of conflict of interest

The authors declare no conflicts of interest.

Data Availability Statement

The corresponding author can provide the data generated and/or analyzed during the current study upon a reasonable request.

REFERENCES

- [1] Ahmed S, Shabbir J, Hafeez W. Hansen and Hurwitz Estimator with scrambled response on second call in stratified random sampling. *J. Stat. Appl. Pro.* 2016; 5(2): 1-13.
- [2] Aslam I, Noor-ul Amin M, Yasmeen U, Hanif M. Memory type ratio and product estimators in stratified sampling. *J Reliab Stat Stud.* 2020;13(1):1–20.
- [3] Audu A, Aphane M, Ishaq OO, et al. New estimators of population variance based on logarithmic transformation in the presence of random non-response and measurement errors under successive sampling. *Afr Mat.* 2025;36:104. <https://doi.org/10.1007/s13370-025-01324-7>
- [4] Azeem M. On estimation of population mean in the presence of measurement error and non-response [PhD thesis]. Lahore: National College of Business Administration and Economics; 2014.
- [5] Biemer P, Stokes SL. Approaches to the modeling of measurement errors. In: Biemer PP, Groves RM, Lyberg LE, Mathiowetz NA, Sudman S, editors. *Measurement errors in surveys*. New York: Wiley; 1991. p. 487–516.
- [6] Chhaparwal P, Kumar S. Improving efficiencies of ratio- and product-type estimators for estimating population mean for time based survey. *J Reliab Stat Stud.* 2022; 15(1):325–340.
- [7] Cochran WG. Errors of measurement in statistics. *Technometrics.* 1968;10(4):637–666.
- [8] Cochran WG. *Sampling techniques*. 3rd ed. New York: Wiley; 1977.
- [9] Cochran WG. The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce. *J Agric Sci.* 1940;30(2):262–275. <https://doi.org/10.1017/S0021859600048012>
- [10] Dixon J. Assessing non-response bias and measurement error using statistical matching. In: *Proceedings of the Section on Survey Research Methods, American Statistical Association*; 2010. p. 3388–3396.
- [11] Fuller WA. *Measurement error models*. New York: Wiley; 2009.
- [12] Guha DK, Rashmi A, Kochar M. Relationship between length of gestation, birth weight and certain other factors. *Indian J Pediatr.* 1973;40: 44–53.
- [13] Hafeez W, Shabbir J. Estimation of the Finite Population Mean, using Median based Estimators in Stratified Random Sampling. *J. Stat. Appl. Pro.* 2015; 4(3): 367-374.
- [14] Hansen MH, Hurwitz WN. The problem of non-response in sample surveys, *J. Amer. Stat. Assoc.* 1946; 41, 517-529.
- [15] Haq A. A new hybrid exponentially weighted moving average control chart for monitoring process mean, *Qual. Reliab. Eng. Int.* 2013; 29(7), 1015–1025.
- [16] Haq A. A new hybrid exponentially weighted moving average control chart for monitoring process mean: discussion, *Qual. Reliab. Eng. Int.* 2017; 33(7), 1629–1631.
- [17] Jackman S. Correcting surveys for non-response and measurement error using auxiliary information. *Elect Stud.* 1999;18:7–27.

- [18] Jakhar A, Kumar S. Robust HEWMA-type estimators for population mean under non-normality. *Life Cycle Reliab Saf Eng*. 2024;13:33-49 <https://doi.org/10.1007/s41872-024-00244-y>
- [19] Kadilar C, Cingi H. Ratio estimators in simple random sampling. *Appl Math Comput*. 2004;151(3):893–902. [https://doi.org/10.1016/S0096-3003\(03\)00803-8](https://doi.org/10.1016/S0096-3003(03)00803-8)
- [20] Kadilar C, Unyazici Y, Cingi H. Ratio estimator for the population mean using ranked set sampling. *Stat Pap*. 2009;50(2):301–309. <https://doi.org/10.1007/s00362-007-0079-y>
- [21] Khare BB, Kumar S. A generalised chain ratio type estimator for population mean using coefficient of variation of the study variable. *Nat Acad Sci Lett India*. 2011;34(9–10):353–358.
- [22] Khare BB, Kumar S. Chain regression type estimators using additional auxiliary variable in two-phase sampling in the presence of non-response. *Nat Acad Sci Lett India*. 2010;33(11–12):369–375.
- [23] Khare BB, Kumar S. Estimation of population mean using known coefficient of variation of the study character in the presence of non-response. *Commun Stat Theory Methods*. 2011;40(11):2044–2058.
- [24] Khare BB, Kumar S. Generalised chain ratio-in-regression estimators for population mean using two-phase sampling in the presence of non-response. *J Inf Optim Sci*. 2015;36(4):317–338.
- [25] Khare BB, Kumar S. Transformed two-phase sampling ratio and product type estimators for population mean in the presence of non-response. *Aligarh J Stat*. 2009;29:91–106.
- [26] Khare BB, Kumar S. Utilization of coefficient of variation in the estimation of population mean using auxiliary character in the presence of non-response. *Nat Acad Sci Lett India*. 2009;32(7–8):235–241.
- [27] Khare BB, Srivastava S. Estimation of population mean using auxiliary character in presence of non-response. *Nat Acad Sci Lett India*. 1993;16(3):111–114.
- [28] Khare BB, Srivastava S. Study of conventional and alternative two-phase sampling ratio, product and regression estimators in presence of non-response. *Proc Nat Acad Sci India Sect A*. 1995;65:195–203.
- [29] Koçyiğit EG. Using past sample means in exponential ratio and regression type estimators under simple random sampling. *Soft Comput*. 2025;29:1389–1406.
- [30] Kumar M, Singh R, Sawan N, Chauhan P. Exponential ratio method of estimation in the presence of measurement errors. *Int J Agric Stat Sci*. 2011;7(2):457–461.
- [31] Kumar S, Chhapparwal P, Kumar K et al. Generalized memory type estimators for time-based surveys: simulation experience and empirical results with birth weight dataset. *Life Cycle Reliab Saf Eng*. 2024; 13:15-23. <https://doi.org/10.1007/s41872-023-00239-1>
- [32] Kumar S, Chhapparwal P. Estimation of population mean in the presence of non-response for time-based surveys. *Thail Statist* 2025;23(2):438–446.
- [33] Kumar S, Chhapparwal P. Utilization of priori information in the estimation of population mean for time-based surveys. *Ann Data Sci*. 2024;11(5):1675–1685. <https://doi.org/10.1007/s40745-023-00472-6>
- [34] Kumar S, Choudhary. Estimation of population product in the presence of non-response and measurement error in successive sampling. *Math Sci Lett*. 2021;10(3):71–83.
- [35] Kumar S, Kour S. ORRT models in the presence of non-response and measurement error under stratified random sampling. *Afr Mat*. 2026;37:38. <https://doi.org/10.1007/s13370-026-01412-2>
- [36] Kumar S, Kour SP, Gupta R, et al. A class of logarithmic type estimator under non-response and measurement error using ORRT models. *J Indian Soc Probab Stat*. 2023;24:333–356. <https://doi.org/10.1007/s41096-023-00156-7>
- [37] Kumar S, Trehan M, Joorel JPS. A simulation study: estimation of population mean using two auxiliary variables in stratified random sampling. *J Stat Comput Simul*. 2018;88(1):1–14.
- [38] Kusum SM. New log type estimator in simple random sampling. *Math Stat Eng Appl*. 2022;71(4):992–998. <https://doi.org/10.17762/msea.v71i4.587>
- [39] Murthy MN. Product method of estimation. *Sankhya*. 1964;26:69–74.
- [40] Noor-ul Amin M. Memory type estimators of population mean using exponentially weighted moving averages for time-scaled surveys. *Commun Stat Theory Methods*. 2021;50(12):2747–2758.
- [41] Noor-ul-Amin M. Memory type ratio and product estimators for population mean for time-based surveys. *J Stat Comput Simul*. 2020;90(17):3080–3092. <https://doi.org/10.1080/00949655.2020.1795660>

- [42] Ozturk O. Estimation of population mean and total in a finite population setting using multiple auxiliary variables. *J Agric Biol Environ Stat.* 2014;19(2):161–184. <https://doi.org/10.1007/s13253-013-0163-9>
- [43] Prasad B. Some improved ratio type estimators of population mean and ratio in finite population sample surveys. *Commun Stat Theory Methods.* 1989;18(1):379–392. <https://doi.org/10.1080/03610928908829905>
- [44] Qureshi MN, Alamri OA, Riaz N, Iftikhar A, Tariq MU, Hanif M. Memory-type variance estimators using exponentially weighted moving average statistic in the presence of measurement error for time-scaled surveys. *PLoS One.* 2024;19(9):e0310870.
- [45] Rao PSRS. Ratio and regression estimates with subsampling the non-respondents. *Proceedings of the International Statistical Association Meetings;* 1987; Tokyo, Japan.
- [46] Rao PSRS. Ratio estimation with subsampling the non-respondents. *Survey Methodol.* 1986;12(2):217–230.
- [47] Sahai A. An efficient variant of the product and ratio estimators. *Stat Neerl.* 1979;32:27–35. <https://doi.org/10.1111/j.1467-9574.1979.tb00659.x>
- [48] Sharma P, Singh R. A generalized class of estimators for finite population variance in presence of measurement errors. *J Mod Appl Stat Methods.* 2013;12(2):13.
- [49] Sharma V, Kumar S. Class of ratio-cum-product type estimator under double sampling: A simulation study. *Thailand Stat.* 2021;19(4):734–742.
- [50] Sharma V, Kumar S. Estimation of population mean using transformed auxiliary variable and non-response. *Rev Investig Oper.* 2020;41(3):438–444.
- [51] Shukla D, Pathak S, Thakur NS. An estimator for mean estimation in presence of measurement error. *Res Rev J Stat.* 2012;1(1):1–8.
- [52] Singh HP, Karpe N. Estimation of mean, ratio and product using auxiliary information in the presence of measurement errors in sample surveys. *J Stat Theory Pract.* 2010;4:111–136.
- [53] Singh HP, Karpe N. On the estimation of ratio and product of two population means using supplementary information in the presence of measurement errors. *Statistica.* 2009;69(1):27–47.
- [54] Singh HP, Kumar S. A regression approach to the estimation of the finite population mean in the presence of non-response. *Aust N Z J Stat.* 2008;50(4):395–408.
- [55] Singh HP, Lashkari P, Pal SK. New product-type and ratio-type exponential estimators of the population mean using auxiliary information in sample surveys. *J Stat.* 2016;23:1–18.
- [56] Singh S. *Advanced Sampling Theory with Applications (Vol.1).* Dordrecht, The Netherlands: Kluwer Academic Publishers; 2003.
- [57] Tiwari KK, Bhogal S, Kumar S, Rather KUI. Using randomized response to estimate the population mean of a sensitive variable under the influence of measurement error. *J Stat Theory Pract.* 2022;16(2):28.
- [58] Yadav SK. Modified Searls Predictive Estimation of Population Mean Using Known Auxiliary Character. *Thailand Statistician.* 2022; 20(1): 53-65.
- [59] Zahid E, Shabbir J. Estimation of finite population mean for a sensitive variable using dual auxiliary information in the presence of measurement errors. *PLoS One.* 2019;14(2):e0212111. <https://doi.org/10.1371/journal.pone.0212111>
- [60] Zahid E, Shabbir J. Estimation of population mean in the presence of measurement error and non-response under stratified random sampling. *PLoS One.* 2018;13(2):e0191572.