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DOUBLY CONSTRAINED GRAVITY MODEL FOR ORIGIN-DESTINATION TRIP DISTRIBUTION: A PYTHON-BASED FOUR-ZONE CASE STUDY

Lima N Fernandes Bonfim Marcal *

College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China.
* Corresponding Author

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ABSTRACT

This study presents a reproducible implementation of a doubly constrained gravity model for origin-destination trip distribution. The formulation preserves prescribed trip productions and attractions while representing the effect of generalized travel cost through an exponential deterrence function. A four-zone numerical case is implemented in Python 3.12.7 using iterative balancing of origin and destination factors. For the baseline case, with β = 0.10, the model converges in 16 balancing cycles to a maximum marginal relative error below 10^-10, producing an OD matrix whose row and column totals match the target trip ends simultaneously. A sensitivity analysis for β values from 0.05 to 0.20 shows that stronger cost sensitivity reduces the flow-weighted mean travel cost from 10.25 to 6.79 cost units and increases the intrazonal share from 37.54% to 61.82%. The results demonstrate the practical importance of explicit convergence checks and parameter calibration in gravity-based demand modelling. The paper provides a compact computational framework suitable for model verification, teaching, and as a transparent baseline for larger empirical transport-planning applications.

Keywords*:* Gravity Model, Origin-Destination Matrix, Trip Distribution, Furness Balancing, Deterrence Function, Python

1 INTRODUCTION

Trip distribution links zonal trip-generation estimates to the pattern of movements between origins and destinations. In conventional travel-demand analysis, this step converts origin productions and destination attractions into an OD matrix that can subsequently support mode choice, network assignment, accessibility assessment, and infrastructure planning. Among the available spatial-interaction approaches, the gravity model remains attractive because it combines a simple behavioral interpretation with explicit control of travel impedance and zonal trip totals [1-3].

The gravity concept was introduced into traffic analysis by adapting the idea that interaction grows with the size of two locations and decreases as separation becomes more difficult [1]. Later developments replaced the direct physical analogy with constrained spatial-interaction formulations in which trip productions, trip attractions, and a calibrated deterrence function define the flow between zones [2,3]. This distinction is important: in practical transportation modelling, the objective is not to reproduce Newtonian mechanics, but to estimate a feasible matrix that respects observed or forecast trip ends while reflecting the friction of travel.

A doubly constrained model must satisfy two sets of marginal conditions at the same time. Every origin row must equal its prescribed production total, and every destination column must equal its prescribed attraction total. Because the balancing factors are mutually dependent, the solution is obtained iteratively. A computation that checks only one margin can therefore appear plausible while remaining inconsistent with the stated demand constraints. This issue is especially relevant in instructional implementations, where stopping rules are sometimes based on visual inspection or on changes between successive matrices rather than on direct marginal errors.

This paper develops a verified Python implementation of a doubly constrained gravity model using a four-zone case study. The study has three objectives: first, to formulate the model and its balancing procedure in a reproducible form; second, to demonstrate strict simultaneous closure of origin and destination totals; and third, to examine how the deterrence parameter β changes the spatial concentration of predicted flows. The contribution is methodological rather than empirical: the numerical data are illustrative, and no claim of city-specific forecasting validity is made.

2 RELATED WORK

Gravity-type spatial interaction models have a long history in transport geography and demand modelling. Casey [1] provided an early traffic-engineering application, while Wilson [2] gave a statistical foundation for constrained spatial-interaction models. Modern transport texts continue to treat gravity models as a core trip-distribution technique because they can reconcile marginal trip totals with a measurable representation of separation between zones [3].

The calibration of deterrence functions is a central part of applied gravity modelling. Abdel-Aal [4] demonstrated that gravity parameters can differ across trip purposes, emphasizing that a single impedance relationship should not be assumed to describe all travel behavior. Iterative proportional fitting and Furness-type procedures are widely used to determine balancing factors, including extensions designed for more complex multidimensional constraints [5]. Comparative work has also shown that gravity models remain competitive as interpretable benchmarks when evaluated against alternative trip-distribution laws [6].

Recent mobility research has broadened the modelling landscape. Radiation models and other parameter-light formulations have been proposed for mobility and migration [7], while large-scale reviews describe a spectrum ranging from classical spatial-interaction laws to data-driven and deep-learning methods [8,12]. Gravity-based formulations nevertheless remain useful for accessibility analysis [9], practical transport decision support [10], and studies where transparent assumptions and constrained totals are more important than algorithmic complexity. Social-media and other digital traces can also provide auxiliary demand information, although they do not remove the need for careful model validation [11].

3 GRAVITY MODEL FORMULATION AND COMPUTATIONAL METHOD

3.1 Doubly constrained formulation

Consider origins i = 1, ..., m and destinations j = 1, ..., n. Let O_i denote the prescribed production at origin i, D_j the prescribed attraction at destination j, and c_{ij} the generalized travel cost between the two zones. The doubly constrained gravity model used in this study is written as

$$T_{ij} = A_i O_i B_j D_j f(c_{ij}) \qquad (1)$$

where T_{ij} is the estimated number of trips from i to j, $f(c_{ij})$ is the deterrence function, and A_i and B_j are balancing factors. When the total production equals the total attraction, the final matrix must satisfy $\sum_j T_{ij} = Oi$ for every origin and $\sum_i T_{ij} = D_j$ for every destination.

The balancing factors are updated alternately according to

$$A_i = [\sum_j B_j D_j f(c_{ij})]^{-1} \qquad (2)$$

$$B_j = [\sum_i A_i O_i f(c_{ij})]^{-1} \qquad (3)$$

3.2 Deterrence function

Travel impedance is represented by an exponential function because it produces a smooth monotonic decline in interaction as generalized cost increases. The adopted form is

$$f(c_{ij}) = \exp(-\beta c_{ij}) \tag{4}$$

where $\beta > 0$ controls sensitivity to travel cost. Small β values produce a relatively flat impedance surface, whereas larger values shift more demand toward low-cost OD pairs. In applied studies β should be estimated against observed travel behavior; here it is varied systematically to illustrate its mathematical effect.

3.3 Iterative balancing and convergence criterion

The calculation was implemented in Python 3.12.7. Destination factors were initialized to one, the origin factors were computed from Eq. (2), and destination factors were then updated from Eq. (3). After each complete cycle, the OD matrix was reconstructed and both sets of marginal totals were checked. Iteration continued until the maximum relative marginal error E satisfied the selected tolerance:

$$E = \max\{\max_i \left|\frac{\Sigma_j T_{ij} - O_i}{O_i}\right|, \max_j \left|\frac{\Sigma_i T_{ij} - D_j}{D_j}\right|\} \tag{5}$$

A tolerance of 10^-10 was used for the reported calculations. This stringent numerical condition is not intended as a behavioral calibration criterion; it is used only to verify that the implemented balancing procedure solves the stated constrained matrix problem without residual marginal inconsistency.

3.4 Four-zone case-study data

The illustrative system contains four zones with production vector O = [400, 460, 400, 702] and attraction vector D = [260, 400, 500, 802]. Both sets sum to 1962 trips. The generalized cost matrix is summarized in Table 1. The baseline analysis uses $\beta = 0.10$.

Table 1: Travel-Cost Matrix for the Four-Zone Case Study

Origin	Z1	Z2	Z3	Z4
Z1 (400)	3.0	11.0	18.0	22.0
Z2 (460)	12.0	3.0	12.0	19.0
Z3 (400)	15.5	13.0	5.0	7.0
Z4 (702)	24.0	18.0	8.0	5.0

Destination totals: Z1 = 260, Z2 = 400, Z3 = 500, Z4 = 802 trips.

3.5 Sensitivity design

To isolate the effect of the impedance parameter, all zonal totals and travel costs were held fixed while β was set to 0.05, 0.10, 0.15, and 0.20. For each case, three outputs were recorded: the number of balancing cycles required to reach the strict convergence tolerance, the flow-weighted mean travel cost, and the share of intrazonal trips. The latter is calculated as Σ_i T_ii divided by the total number of trips.

4 RESULTS AND DISCUSSION

4.1 Baseline origin-destination matrix

For $\beta = 0.10$, the iterative procedure reached E < 10^-10 after 16 complete balancing cycles. The resulting OD matrix is shown in Table 2. Row totals reproduce the four prescribed productions and the column totals reproduce all four attractions, confirming simultaneous satisfaction of both constraint sets.

The largest single movement is the Zone 4-to-Zone 4 intrazonal flow, 440.24 trips. This result follows from three interacting conditions rather than from cost alone: Zone 4 has the largest production total, it is also the largest destination, and its intrazonal cost is low. Conversely, high-cost pairs receive smaller flows unless the marginal totals require sufficient demand to be allocated to them. The matrix therefore illustrates the defining characteristic of a constrained gravity model: impedance influences the distribution, while the balancing factors enforce the overall demand structure.

Table 2: Balanced OD Matrix for β = 0.10

Origin	Z1 (260)	Z2 (400)	Z3 (500)	Z4 (802)
Z1 (400)	157.04	100.36	66.14	76.46
Z2 (460)	57.48	201.09	108.50	92.92
Z3 (400)	25.26	46.13	136.24	192.37
Z4 (702)	20.23	52.42	189.11	440.24

The exact marginal closure is also a useful implementation check. If only destination totals are normalized after each iteration, the columns may appear correct while origin totals remain inconsistent. Evaluating both margins through Eq. (5) avoids that error and makes the numerical stopping rule independent of the visual appearance of the matrix.

4.2 Effect of the deterrence parameter

Table 3: Sensitivity of Model Outputs to β

β	Cycles	Mean cost	Intrazonal (%)
0.05	8	10.25	37.54
0.10	16	8.65	47.64
0.15	28	7.50	55.89
0.20	41	6.79	61.82

Table 3 shows a monotonic response to β. Raising β from 0.05 to 0.20 lowers the mean predicted travel cost from 10.25 to 6.79 cost units, while the intrazonal share rises from 37.54% to 61.82%. These changes are consistent with the exponential deterrence function: larger β values penalize expensive OD pairs more severely, so more of the fixed total demand is concentrated in relatively low-cost cells.

The stronger deterrence also increases the number of balancing cycles. The strict tolerance is reached after 8 cycles at β = 0.05 but requires 41 cycles at β = 0.20. This does not imply that the higher-β model is intrinsically worse; rather, stronger cost contrasts produce a more uneven preliminary flow pattern, and additional factor updates are required to reconcile that pattern with the same row and column constraints.

4.3 Interpretation, strengths, and limitations

The case study demonstrates two distinct aspects of gravity modelling. Numerical correctness concerns whether the algorithm solves the constrained matrix problem, whereas empirical validity concerns whether the chosen costs, zonal totals, and deterrence parameter represent real travel behavior. The first aspect is verified here through exact marginal checks. The second cannot be established because the case-study inputs are illustrative and no observed trip-length distribution or survey OD matrix is available for calibration.

The principal strength of the implementation is transparency. Each model component can be inspected directly: zonal totals define the amount of demand, the cost matrix defines separation, β governs the strength of deterrence, and the balancing factors enforce conservation. This makes the method useful for teaching, debugging, and as a benchmark against more complex OD-flow models. It also facilitates sensitivity analysis because changes in predicted flows can be traced to an explicit model parameter rather than to a black-box learning process.

The limitations are equally important. A four-zone synthetic system cannot reproduce the behavioral heterogeneity, congestion effects, modal competition, temporal variation, or socioeconomic segmentation present in a real urban network. In addition, a single exponential function may not be appropriate for all trip purposes. Previous calibration studies show that impedance parameters can vary substantially by travel purpose and context [4]. Future work should therefore estimate β using observed trip-length distributions, compare alternative deterrence functions, and test the model on a larger empirically derived zoning system.

Contemporary OD research increasingly combines theory-driven models with large mobility datasets and machine-learning methods [8,11,12]. In that setting, the gravity model remains valuable not because it is universally superior, but because it offers a highly interpretable baseline. A complex model that improves prediction should ideally be compared against a correctly calibrated and correctly balanced gravity benchmark rather than against an implementation with unresolved marginal errors.

5 CONCLUSION

A doubly constrained gravity model was implemented in Python and verified using a four-zone trip-distribution case. The baseline solution at $\beta = 0.10$ converged in 16 balancing cycles and reproduced all prescribed origin and destination totals to a maximum relative marginal error below 10^-10. The balanced matrix therefore provides a consistent numerical solution to the stated OD problem.

The sensitivity analysis confirms that β materially changes the spatial pattern of modeled demand. Increasing β from 0.05 to 0.20 reduced the flow-weighted mean cost and substantially increased the proportion of intrazonal trips, while also increasing the number of iterations required under the strict convergence criterion. These results reinforce the need to distinguish computational convergence from behavioral calibration.

The study is intended as a reproducible methodological example rather than a real-city forecast. Its main contribution is a transparent workflow for formulating, balancing, checking, and interpreting a constrained gravity model. The same workflow can be extended to empirical applications by replacing the illustrative inputs with surveyed or otherwise validated trip productions, attractions, travel costs, and calibrated impedance parameters.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The author declares no conflict of interest regarding this manuscript.

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