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(Research Article)

Teaching the addition and subtraction of integers in Lower Secondary School: An Inclusive and Inquiry-Based Approach

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Abstract

This paper presents a proposed teaching scenario for the instruction of addition and subtraction of integers in the first grade of lower secondary school, aimed at students with or without learning difficulties. The scenario applies principles of discovery learning and emphasizes the use of both manipulative and digital tools, such as the number line and GeoGebra. Through mathematical projects, students actively construct knowledge by discovering sign rules and transformation of operations. The goal of the teaching plan is to enhance conceptual understanding and to foster student engagement.

Keywords: Integers; Learning Difficulties; Inclusion; Discovery Learning

1. Introduction

Learning the addition and subtraction of integers presents conceptual and procedural challenges for many students, both with and without learning difficulties. Misconceptions related to the meaning of negative numbers, the dual role of mathematical symbols, and the abstract nature of operations often hinder understanding. These difficulties are exacerbated in students with mathematical learning disabilities due to cognitive and linguistic factors, as well as insufficiently adapted instructional materials. Addressing these issues through inclusive, well-designed teaching strategies that promote multiple representations, and conceptual understanding can significantly support all learners.

The aim of the present study is to enhance the understanding of operations with integers through two inquiry-based projects. Through Project 1, students are encouraged to discover patterns and relationships related to integer addition, which in turn will lead them to formulate the rules governing this operation. Project 2 aims to provide meaning to the transformation of the subtraction operation into addition.

2. Teaching Integers in Inclusive Classrooms

The process of learning addition and subtraction of integers can be particularly challenging for students with or without learning difficulties, as it involves different rules and concepts from those used with natural numbers. The concept of negative numbers can be abstract, and students often struggle to visualize or represent them [1]. They also tend to confuse the symbols "+" and "−", which carry a dual meaning either as signs or as operations with the minus symbol "−" also being interpreted as indicating the opposite.

Students also face difficulties in understanding how to perform addition and subtraction with negative numbers, since the addition of numbers with different signs is reduced to subtraction of natural numbers, and subtraction of integers is transformed into addition [2]. Difficulties may arise with sign rules, which often seem contradictory compared to

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those used with natural numbers. For example, when adding a negative number to a positive number, the negative sign is maintained, whereas when subtracting a negative number, the sign becomes positive [3].

Furthermore, when working with negative numbers, students may hold misconceptions about the role of zero. Often, they believe that adding a negative number to a positive number always results in zero [1].

Additional difficulties may arise in understanding the use of the number line and the relationship between positive and negative numbers on it [4]. Furthermore, students may struggle to perform mental calculations with negative numbers; for example, when subtracting a larger negative number from a smaller one, resulting in a positive number, which causes confusion and contradiction [2]. The idea that operations with negative numbers can yield negative answers contradicts their prior knowledge, as they are accustomed to working only with positive numbers. This can lead to errors and frustration, especially when working on more complex problems [5].

In the case of students with learning difficulties in mathematics, these challenges are linked to deficits in working memory, spatial skills, and the linguistic comprehension of mathematical terms [6]. Difficulties in understanding abstract concepts, such as negativity, zero, or the properties of operations, are intensified by the lack of empirical contact with these concepts. Such difficulties may also arise from insufficient instructional design or from educational materials that have not been appropriately adapted to meet students' needs [7]. Although students with learning difficulties follow a different pace in developing mathematical thinking, they can be effectively supported through appropriately designed teaching scenarios that focus on the use of multiple representations and conceptual approaches [8]. All the above take place within an inclusive framework aimed at ensuring all students have access to quality education [9].

3. Proposed teaching plan

- **Grade:** First Grade of Lower Secondary School
- **Teaching Objectives:** Students will be able to add and subtract integers, initially using visual models and later mathematical equalities to describe additions and subtractions.
- **Duration:** 3 teaching hours, one hour for Project 1 and two hours for Project 2.

3.1. Material and Technical Requirements

Each group requires a computer or tablet with internet access, or an interactive whiteboard. If these are not available, a computer with internet access and a projector should be provided. Coins, red, blue, and white laminated cards, markers, a board with corresponding markers or chalk, and a printed and laminated number line for each student are also required.

3.2. Prerequisite Student Knowledge

- **Cognitive:** Students can plot points on an axis and associate these points with their coordinates on the axis. They recognize the need for negative numbers. They distinguish between positive and negative integers, as well as between like-signed and unlike-signed numbers. They understand the absolute value of integers as their distance from zero on the number line. They can describe the characteristics of two opposite numbers.
- **Socio-cultural:** Students cooperate in groups and communicate their thoughts and ideas effectively.

3.3. Teaching Actions / Practices of the Teacher and Students

The teaching scenario is situated within an environment of inquiry-based and discovery learning. In such a setting, the teacher takes on the role of orchestrator, facilitator, and mentor [10]. Specifically, the teacher selects appropriate tasks that effectively support the learning process and designs the way these tasks will be explored in the classroom [11]. They encourage dialogue and guide discussion by providing scaffolding, without revealing the final conclusions that students are expected to reach on their own. The teacher offers feedback and helps summarize the conclusions drawn by the students. Students are given space to act autonomously, take initiative, and assume ownership of the task they are working on [12]. Furthermore, the teacher fosters interaction and communication.

In this scenario, the teacher initially allows students a set amount of time to read through each project to address any questions or confusion. Then, students are divided into groups and first work individually for a predetermined period. This helps them become familiar with the task and better prepared to collaborate with their group, sharing their individual insights and thoughts [13]. After this phase, students are asked to work in groups to discuss their conclusions and arrive at a shared outcome. Whenever necessary, the teacher brings the class back into plenary discussion to clarify misunderstandings or to summarize key ideas that emerge. Throughout the process, the teacher adheres to the general principles of discovery and inquiry-based learning described above [10].

On the other hand, through their individual engagement with the mathematical task, students take responsibility for their own learning and, through interaction with others, are encouraged to reconsider their existing knowledge and experiences. Throughout the learning process, they take on an active role—posing questions, seeking mathematical ways to justify their conclusions, and exploring appropriate means to communicate them [14].

More specifically, students are given time to read the task and the opportunity to clarify any questions that may arise. Then, they work individually in order to take ownership of the problem and make use of their prior knowledge, skills, and abilities to discover mathematical laws and rules. Finally, they are expected to communicate their conclusions effectively within their group and, through evidence and reasoning, reach a shared conclusion.

3.4. Classroom Dynamics Management

The management of classroom dynamics is primarily based on applying the principles of inclusive and differentiated teaching. The tasks have been selected in a way that takes into account each student's different starting point on social, cultural, and emotional levels, as well as the individual learning style and pace of each student. Given the diverse entry points of students into mathematical tasks, active engagement of all students is achieved. At every stage of the process, the teacher can clarify any misunderstandings or errors that may arise in students' reasoning by posing appropriate questions that help them overcome these obstacles independently. The teacher can also utilize students' responses to highlight important aspects of the tasks and summarize the conclusions that emerge at each step.

The use of manipulatives (colored cards), visual tools (number line), and digital tools (GeoGebra software), as proposed in the teaching scenario, enhances the learning process, which takes on a multimodal form to accommodate all types of learners. Working in groups, students communicate their ideas, reflect, revise, and arrive at conclusions, which the teacher then validates or challenges by asking appropriate questions designed to promote further reflection.

3.4.1. Projects

Project 1: On the number line below, each time we move steps to the right, we denote this by the symbol "+" followed by the number of steps, while each time we move steps to the left, we denote this by the symbol "−" followed by the number of steps (that is, +2 means 2 steps to the right, while −2 means 2 steps to the left). Use two numbers a and b on the number line and represent with an arrow the sum of these two numbers. That is, the arrow will start at one of the two numbers you chose and end at the number you reach each time by moving right or left according to the second number you chose. The coordinate of the arrow's endpoint is the result of the addition a+b.

- Repeat the above process for various values of a and b and note the conclusions you reach about the addition of a and b.
- Can you calculate sums of integers without using the number line in Figure 1 [15]?

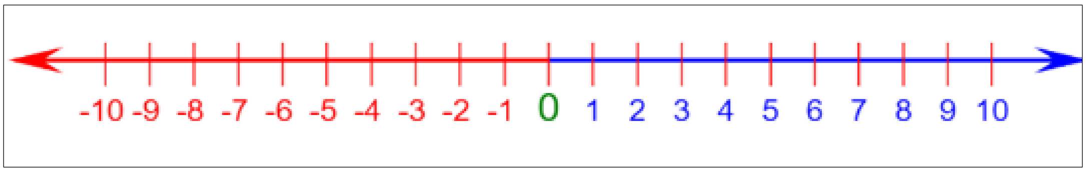

Figure 1 Number Line (Source: MathsisFun [15])

Project 2: Each group of students flips a coin 10 times, where "heads" corresponds to receiving a red card with the symbol "−", which we call negative, and "tails" corresponds to receiving a blue card with the symbol "+", which we call positive. On a white card, each group records the result. If the group loses points, they write the number of points lost preceded by the "−" sign. If the group wins, they write the number of points won preceded by the "+" sign. Finally, they write 0 if they have zero points.

- What is the maximum and minimum gain that a group can achieve?
  - If you belong to a group that lost, how many points does your group need to "make up" for the loss (i.e., regain the points lost)? More generally, how many points does a group that lost need to break even?

- If you belong to a group that won, how can you reduce your points to zero? More generally, how many points does a group that won need to reduce their score to zero?
- How many points does your group need to have the maximum gain? To have the minimum gain? More generally, how many points does a group need for their gain to be maximum? To be minimum?

- Which operation can represent the case where a group lost three points and then 2 points are subtracted from them? How many points does the group have now?
- A group with +8 cards (reminder: this corresponds to 8 positive cards and 2 negative cards) in what ways can they maximize their points? In what ways can they reduce their points to +3?
- How could 3 positive points be subtracted from a group with zero points? How about 2 negative points?
- Using the card game, what do the following operations mean and what are their results? $(-2)-(-5)$, $(-3)-(+6)$, $(+3)-(-4)$, $(-3)-(+2)$, $(+6)-(-3)$
- Can the above subtractions be performed using another operation?
- In a repeat of the game three times, the results recorded were −2, +3, and −5. What is the result?

3.5. Characteristics of the Mathematical Activity to be Highlighted during Students' Engagement with Each of the Specific Projects

In Project 1, students are expected to discover patterns and relationships related to the addition of integers, which will lead them to formulate the rules governing this operation. Initially, through experimentation by assigning values to a and b, they will visualize the addition process and develop a meaningful understanding of its extension to the set of integers. Next, they are expected to distinguish the existence of different cases depending on whether the numbers a and b are both positive (same sign), both negative (same sign), or have opposite signs (different signs). Then, they should discover that the sum of two negative numbers is always negative, while the sum of two numbers with opposite signs can sometimes be positive and sometimes negative. In the latter case, they are expected to observe that if they add a negative number with a greater absolute value to a positive number, the result is negative; whereas if they add a negative number with a smaller absolute value to a positive number, the result is positive. This observation can also occur in the case where a positive number is added to a negative number: if the positive number has greater absolute value, the result is positive; if smaller, the result is negative. Students may reach these conclusions without strictly using the concept of absolute value but rather describing it in a simpler way. At this point, a whole-class discussion is recommended to introduce the need for the use of the terminology "absolute value" so that conclusions are clearly and precisely expressed in mathematical language. Finally, through this mathematical project, students will realize that in the addition of integers with the same sign, the numbers are added by their absolute values, while in the addition of integers with opposite signs, the absolute value of the smaller is subtracted from the absolute value of the larger. Helpful for this mathematical project could be the use of digital tools such as GeoGebra in Figure 2 [16] or Mathsisfun in Figure 3 [15] to provide variety and a multisensory use of visual aids.

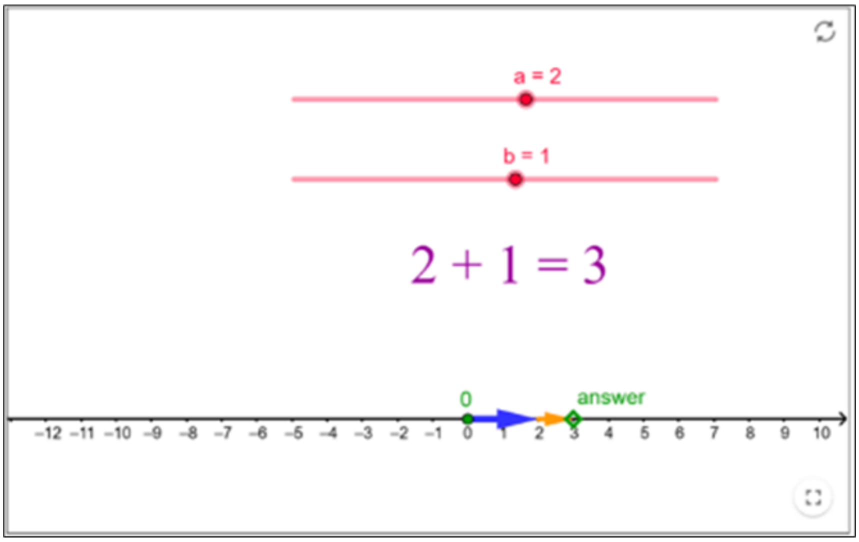

Figure 2 Adding Numbers with a Number Line (Source: GeoGebra [16])

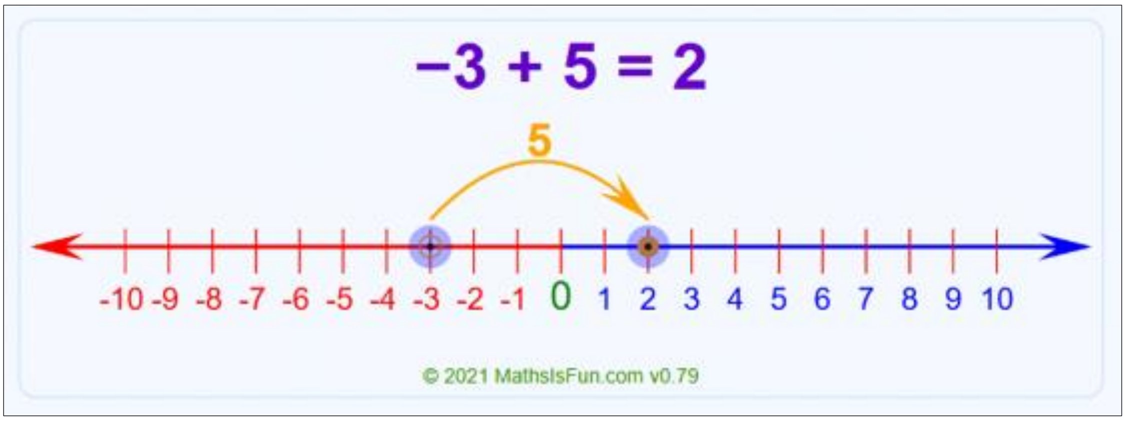

Figure 3 Using the Number Line (Source: MathisFun [15])

Project 2 helps in the conceptual understanding of the transformation of subtraction into addition. The questions in part A aim to familiarize students with the project, engage their interest as it has a game format, and introduce the concept of subtraction. Various answers are expected regarding the procedures followed in the individual questions, where students experiment with additions and subtractions to reach their conclusions. In part B, the use of the subtraction operation is targeted more specifically, where students discover that instead of subtracting one integer from another, they can add the opposite of the subtracted number. The use of the cards is very helpful for reaching this conclusion. For example, the operation (+6) −(−3) means that from six positive cards corresponding to +6 points, three negative cards must be subtracted. Since there are no negative cards, they need to "borrow" three positive and three negative cards so that the points remain six but there are also negative cards to subtract. After subtracting the three negative cards, the result turns out to be nine positive cards. The idea of "borrowing" cards, which does not alter the number of points, is clarified in question (c). In question (d), students continue familiarizing themselves with the transformation of subtraction into addition through the card game, leading them to generalize their conclusions in question (e), where symbolic expression of subtraction was already introduced in part (a). In question (f), the process extends to addition and subtraction of more numbers. Questions (b), (c), and (e) need to be explored at multiple levels in class: individually, in groups, and in plenary, with substantial discussion each time. Especially question (b) requires thorough investigation so that students conclude that adding (subtracting) blue cards is equivalent to subtracting (adding) red cards. The above concepts can be further reinforced using the digital tool GeoGebra, specifically via the activity Addition and Subtraction of Integers [17].

3.6. Assessment

The assessment of learning and teaching is ongoing and multifaceted, aiming to monitor students' progress both cognitively and socially/emotionally. The achievement of learning goals related to the addition and subtraction of integers is checked through various activities (individual, group, digital), observation, and conceptual maps, with feedback also provided via the e-class platform. Simultaneously, skills such as collaboration, respect, creativity, and metacognition are assessed. The evaluation of teaching practice is based on observing students' mathematical and social behavior and reflecting on teaching effectiveness through targeted questions [18].

3.7. Reflection

The process of reflection invites the educator to evaluate their practice through four main axes: lesson planning, the learning process, the teaching approach, and personal feedback. It examines the alignment of activities with objectives, inquiry, use of tools, as well as student participation in the communication and development of mathematical ideas. Emphasis is placed on conceptual understanding, personalization of teaching, and classroom collaboration, while highlighting the need for professional development through collaboration and systematic reflection [11].

4. Conclusion

The scenario offers an effective approach for teaching integers in heterogeneous learning groups. The use of both simple and digital tools facilitates the understanding of highly abstract concepts. According to international literature on the use of multiple representation tools and the enhancement of mathematical understanding through playful activities, the approach proposed in this work highlights the value of experiential and inquiry-based learning in mathematics. The use of physical and digital representations aids the comprehension of abstract concepts, while the game element increases engagement and interest. Students grasp the rules of signs through an empirical approach, without relying on rote memorization.

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