

eISSN: 2582-8185 Cross Ref DOI: 10.30574/ijsra Journal homepage: https://ijsra.net/



(RESEARCH ARTICLE)

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# Analysis of the charging process for a phase change thermal energy storage system using the heat balance integration method

Stanley A. Omenai \*

Department of Mechanical Engineering, Georgia Institute of Technology, Atlanta, USA.

International Journal of Science and Research Archive, 2024, 13(02), 2476–2488

Publication history: Received on 31 October 2024; revised on 05 December 2024; accepted on 07 December 2024

Article DOI: https://doi.org/10.30574/ijsra.2024.13.2.2443

# Abstract

Thermal Energy Storage (TES) is a significant topic of interest as it offers a low-cost method to store and deliver energy when needed. There are several forms of thermal energy storage ranging from sensible, latent, and thermochemical.

In this paper, the various forms of thermal energy storage have been reviewed, and the mathematical modelling of the charging process for phase change thermal energy storage system has been formulated for use in a typical fast-food outfit where significant amounts of heat energy are readily available for capture during the day and re-use at night.

The behavior of the Phase Change Material (PCM) during melting in a cylindrical encapsulation under fixed boundary conditions: location of the solid-liquid interface across the phase change material, transient temperature distribution of the liquid phase and latent heat absorbed are also analyzed.

**Keywords:** Thermal Energy Storage; Phase Change Material; Heat Balance Integration Method; Thermal Diffusivity; Stefan Number

# 1. Introduction

In the fast-food industry, it is typical to burn large quantities of gas to provide heat for cooking and processing food during the day. Large amounts of heat energy, with temperatures up to 600K, are also lost with the flue gases during this combustion process.

During the nights, there is a reduction in the workforce and the focus for night work is packaging, material preparation and planning for the next day.

The nights are usually colder than the day and require additional heat energy to keep personnel warm. Electric heaters are typically used in such instances with the attendant cost of electric energy.

This paper considers capturing the heat energy from the flue gases during the day by channeling the flow through a heat storage system where most of the heat is absorbed by a Phase Change Material (PCM). During the nights, heating air flows through the heat exchanger to provide warmth for night activities, thus saving costs on heating.

The storage system consists of cylindrical pipe heat exchangers with the Phase Change Material (PCM) encapsulated within the annulus. The hot flue gases are channeled to flow through the tubes during the day, while heating air flows through them during the night.

<sup>\*</sup> Corresponding author: Stanley A. Omenai

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In this study, a mathematical model for a characteristic pipe heat exchanger is developed. The thermal energy storage process for the Phase Change Material (PCM) is then reviewed and analyzed.

# 2. Literature review

## 2.1. Thermal energy storage

Thermal Energy Storage (TES) is a technology that stocks thermal energy by heating or cooling a storage medium so that the stored energy can be used at a later time for heating and cooling applications and power generation [1]. TES systems are used particularly in buildings and in industrial processes.

Advantages of using TES in an energy system include an increase in overall efficiency and better reliability, as well as improved economics, reductions in investment and running costs, and less pollution of the environment, i.e., lower carbon dioxide (CO<sub>2</sub>) emissions.

Solar thermal systems, unlike photovoltaic systems with striving efficiencies, are industrially mature and utilize a major part of the Sun's thermal energy during the day. However, they suffer the disadvantage of not having enough (thermal) backup to continue operating during the low or no solar radiation hours.

TES is becoming particularly important for electricity storage in combination with concentrating solar power (CSP) plants where solar heat can be stored for electricity production when sunlight is not available. New materials are selected, characterized, and enhanced in their thermo-physical properties to serve the purpose of a 24-hour operation in an efficient TES system.

## 2.2. Classification of thermal energy storage systems



Thermal Energy Storage technologies can be divided into three categories: sensible, latent, and thermochemical heat storage [1].

Figure 1 Classification of TES Systems

Sensible heat storage systems include Tank (TTES), Pit (PTES), Borehole (BTES) and Aquifer (ATES) Thermal Energy Storage.

Latent Heat Storage systems use different types of Phase Change Materials (PCM), while Thermochemical Heat Storage (THS) refers to the use of reversible chemical reactions to store large quantities of heat in a compact volume.

The table below summarizes the most common types of Thermal Energy Storage systems and their energy sources.

Table 1	l Types	of Thermal	Energy	Storage	Systems
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Type of TES	Description	Energy Input
TTES	Tank systems usually storing hot water, but molten salts and heat transfer oils have also been used extensively (depending on temperature required).	All conventional and renewable heating systems (boilers, CHP, heat pumps, biomass, solar thermal).
PTES	Shallow pits dug in the ground, which are then lined and filled with gravel and / or water for energy storage.	Larger solar thermal installations, as PTES is most beneficial at scale (plus interaction with other heat inputs for district heating).
BTES	Regularly spaced vertical holes are drilled into the ground, with heat exchangers inserted to transfer heat to and from the ground.	Solar thermal, ground source heat pump for extraction, potentially CHP, gas turbines, waste heat.
ATES	Open-loop system utilising natural underground water- bearing permeable layers from which groundwater is extracted.	Ground source heat pump, waste heat, CHP.
РСМ	Using organic or inorganic compounds to store energy in the form of heat in the material's change of phase (usually from solid to liquid, but also from liquid to gas).	All conventional and renewable heating systems (boilers, CHP, heat pumps, biomass, solar thermal), solar PV.
THS	Reversible chemical reactions to store large quantities of heat in a compact volume.	Most likely industrial heat, but theoretically variety of heat sources.

## 2.3. Sensible heat storage systems

Sensible heat storage systems store thermal energy by varying the temperature of the storage material, without undergoing any form of phase change within the working temperature range. The amount of thermal energy stored or released is proportional to the density  $\rho$ , volume V, specific heat  $c_p$ , and temperature variation of the storage materials [2]:

$$Q_s = \int_{T_i}^{T_f} mc_p dT = \rho V c_p (T_f - T_i)$$

where  $Q_s$  is the amount of sensible heat stored, dT is the temperature interval,  $T_i$  is the initial temperature and  $T_f$  the final temperature of the storage medium during the storage process. Basically, specific heat  $c_p$ , density  $\rho$  and thermal conductivity k are the key thermal properties of sensible heat storage materials. According to the materials' phase state, sensible heat storage materials can be divided into two main categories: solid and liquid heat storage.

#### 2.4. Latent heat storage systems

Latent Heat storage utilizing Phase Change Materials (PCMs) is an alternative TES technique compared to sensible heat storage. PCMs are substances which can absorb or release large amounts of energy when they experience phase transitions between solid, liquid and gas states. Although the highest latent heat of phase change is the liquid-gas phase change, it is hard to utilize this due to the enormous volume change associated with material evaporation.

Solid-liquid PCMs are commonly applied for latent heat thermal energy storage. Solid-liquid PCMs should have a melting point near the required operation temperature range of the TES system, melt congruently with minimum subcooling and also be chemically stable, cost competitive, non-toxic, and non-corrosive. The amount of energy storage of the latent heat system with PCMs is calculated as [3]:

$$Q_{L} = m[c_{p,s}(T_{m} - T_{i}) + \gamma L + c_{p,l}(T_{f} - T_{m})]$$

where  $Q_L$  is the amount of heat stored,  $c_{p,s}$  and  $c_{p,l}$  are the specific heat of the PCM in solid and liquid states respectively, L is the latent heat of fusion,  $\gamma$  is the melting fraction,  $T_i$  and  $T_f$  are the initial and final temperatures of the storage materials, and  $T_m$  is the melting temperature.

Phase Change Materials usually have high latent heat of fusion and can store more amount of heat than sensible heat storage systems. This large difference gives PCMs the advantage of a high energy storage density, which reduces the volume of the Thermal Energy Storage (TES) vessel and the outer wall surface area, thus minimizing heat losses.

Compared to the sensible heat storage, the discharging process of the latent heat storage can maintain the constant temperature for both the PCM and contacting medium (Heat Transfer Fluid or adjacent space).

The main drawback of latent heat storage is the low thermal conductivity of PCMs. Moreover, the heat transfer design and media selection are more difficult, and the experience with low temperature PCMs like inorganic salts has shown that the performance of the materials can degrade after moderate number of freezing-melting cycles.



Phase change materials can be classified as follows [1]:

Figure 2 Classification of Phase Change Materials

#### 2.5. Considerations for selection of phase change storage systems

The following factors are key in specifying a phase change Thermal Storage system:

#### 2.5.1. Phase Change Material

The choice of PCM is very important in selecting a phase change Thermal Storage System. The Latent heat capacity of the PCM determines how much heat the system is able to store.

Also, the temperature range of application determines the type of PCM to be used as the melting temperature of the PCM should be lower than the temperature of the heat transfer medium.

#### 2.5.2. Geometry of PCM Container

Once the PCM has been selected the next most important factors to consider are the geometry of the PCM container and the thermal and geometric parameters of the container required for a given amount of PCM. Each of these factors has a direct influence on the heat transfer characteristics in the PCM and ultimately affects the melt time and the performance of the PCM storage unit.

The most commonly used PCM container geometries are the rectangular and cylindrical types.

#### 2.5.3. Configurations of Cylindrical PCM Containers

Three modes of cylindrical PCM container configurations are available. The first is where the PCM fills the shell and the heat transfer fluid flows through a single tube designated as the pipe model. In the second model the PCM fills the tube and the HTF flows parallel to the tube. The third cylinder model is the shell and tube system commonly used to improve heat transfer in PCM.



Figure 3 Cylindrical PCM Containers (a) Pipe Model (b) Cylinder Model (c) Shell and Tube Model

Various studies have recommended the pipe model because it affords a shorter melt time [4]. This is because the thicker the PCM mass, the lower the surface area of contact and thus, the longer the melt time.

An added advantage for the pipe model is the fact that the pipe model has a lower heat loss rate to the environment because most heat supplied from the center ends up heating the PCM [4].

## 2.6. System modelling and analysis

Phase change materials store thermal energy due to their large heat storage capacity at constant temperature. The choice of PCM is based on their melting temperature and the application.

For this study, we shall consider a cylindrical pipe heat exchanger consisting of a solid phase change material (organic paraffin) encapsulated in a shell. The heat transfer fluid (hot gas) enters the tube at temperature  $T_s = 600$ K. Heat is transferred across the walls of the tube to the phase change material while melting takes place. The heat is then retained in the material for re-use.

This process of storing heat is called "charging". Our study shall focus on the charging process.

The properties of the heat storage medium (organic paraffin) are listed below [2]:

Table 2 Thermophysical Properties of Organic Paraffin

Thermophysical Property	Magnitude
Melting Temperature, T <sub>m</sub>	300 K
Latent Heat of Fusion, L	200 kJ/kg
Thermal conductivity, k	0.20 W/mK
Specific Heat Capacity, <i>C</i> <sub>p</sub>	2.4 kJ/kgK
Density, ρ	750 kg/m <sup>3</sup>

Consider the following cylindrical geometry for our study:



#### Figure 4 Geometry of Phase Change Storage System

The dimensions of the heat exchanger are:

Tube radius  $(R_i) = 0.1m$ 

Shell radius  $(R_o) = 0.3m$ 

Total length of heat exchanger (z) = 10m

The following simplifying assumptions are made:

The phase change material is homogenous and isotropic throughout

The thermophysical properties of the phase change material are constant with temperature.

Natural convection in the liquid phase of the phase change material is negligible.

The flow of the heat transfer fluid is fully developed and the temperature at the walls of the tube is uniform throughout

Azimuthal and axial symmetry are assumed for the melting of the phase change material; hence, the problem reduces to a 1-D cylindrical problem.

At the initial time (ambient conditions), the phase change material is at its melting temperature,  $T_m$ .

The temperature of the solid phase of the material remains constant throughout,  $T_m$ .

The temperature of the hot gas remains the same throughout and is same as the temperature at the geometry boundary,  $T_s$ .

The position of the moving boundary *s*(*t*) is a function of time, hence, as time elapses, the solid PCM will melt due to the boundary temperature applied at the tube wall [2].

Since the temperature of the heat transfer fluid and the solid phase of the phase change material are constant and defined, our analysis shall focus on the temperature distribution in the liquid phase of the material.



Figure 5 Temperature Distribution of PCM during Charging

The governing equation for the liquid phase is given by [2]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The boundary conditions are given by:

At  $r = R_i$ , t > 0:  $T = T_s$ 

At r = s, t > 0:  $T = T_m$ 

The initial condition is given by:

At 
$$t = 0$$
,  $R_i \le r \le R_o$ :  $T = T_s$ 

The energy balance at the solid-liquid interface is given by:

At 
$$r = s$$
,  $t > 0$ :  
 $k \frac{\partial T}{\partial r} = -\rho L \frac{\partial s}{\partial t}$ 

To simplify the governing equation, let us introduce non-dimensional variables as follows [5]:

$$\bar{r} = \frac{r}{R_i}; \quad \tau = \frac{\alpha t}{{R_i}^2}; \quad \eta = \frac{s}{R_i}; \quad \theta = \frac{T - T_m}{T_s - T_m}; \quad \mu = \frac{C_p(T_s - T_m)}{L}$$

Where:

$$\alpha = \frac{k}{\rho c_p}$$

## The dimensionless number, $\mu$ , is known as the Stefan number.

The governing equation transforms as follows:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}} \left( \bar{r}\frac{\partial \theta}{\partial \bar{r}} \right) = \frac{\partial \theta}{\partial \tau}$$

The boundary conditions transform as follows:

At  $\bar{r} = 1$ ,  $\tau > 0$ :  $\theta = 1$ 

At 
$$\bar{r} = \eta$$
,  $\tau > 0$ :  $\theta = 0$ 

The initial condition transform as follows:

At 
$$\tau = 0$$
,  $\bar{r} \ge 1$ :  $\theta = 0$ 

The energy balance at the solid-liquid interface transform as follows:

At 
$$\bar{r} = \eta$$
,  $\tau > 0$ :  
 $\frac{\partial \theta}{\partial \bar{r}} = -\frac{1}{\mu} \frac{\partial \eta}{\partial \tau}$ 

Using the heat balance integral method [6], we integrate the governing equation over the domain defined by the liquid phase.

$$\frac{\partial}{\partial \bar{\mathbf{r}}} \left( \bar{\mathbf{r}} \frac{\partial \theta}{\partial \bar{\mathbf{r}}} \right) = \bar{\mathbf{r}} \frac{\partial \theta}{\partial \tau}$$
$$\int_{1}^{\eta} \frac{\partial}{\partial \bar{\mathbf{r}}} \left( \bar{\mathbf{r}} \frac{\partial \theta}{\partial \bar{\mathbf{r}}} \right) d\bar{\mathbf{r}} = \int_{1}^{\eta} \left[ \bar{\mathbf{r}} \frac{\partial \theta}{\partial \tau} \right] d\bar{\mathbf{r}}$$

Integrating and expanding the above expression, we obtain:

$$\frac{\partial}{\partial \tau} \left[ \int_{1}^{\eta} (\bar{\mathbf{r}}\theta) d\bar{\mathbf{r}} \right] - \left[ \bar{\mathbf{r}}\theta \frac{\partial \eta}{\partial \tau} \right]_{\bar{\mathbf{r}}=\eta} = \left[ \bar{\mathbf{r}} \frac{\partial \theta}{\partial \bar{\mathbf{r}}} \right]_{\bar{\mathbf{r}}=\eta} - \left[ \bar{\mathbf{r}} \frac{\partial \theta}{\partial \bar{\mathbf{r}}} \right]_{\bar{\mathbf{r}}=1}$$

Substituting the interface energy balance equation, we obtain the energy integral equation as follows:

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$$\frac{\partial}{\partial \tau} \left[ \int_{1}^{\eta} (\bar{\mathbf{r}}\theta) d\bar{\mathbf{r}} \right] - \left[ \bar{\mathbf{r}}\theta \frac{\partial \eta}{\partial \tau} \right]_{\bar{\mathbf{r}}=\eta} = - \left[ \frac{\bar{\mathbf{r}}}{\mu} \frac{\partial \eta}{\partial \tau} \right]_{\bar{\mathbf{r}}=\eta} - \left[ \bar{\mathbf{r}} \frac{\partial \theta}{\partial \bar{\mathbf{r}}} \right]_{\bar{\mathbf{r}}=1}$$

Let us assume a temperature profile for the liquid phase as follows:

$$\theta = A + \frac{B}{\bar{r}}(\bar{r} - \eta)$$

The above equation must satisfy the boundary conditions:

Applying BC-2,

$$0 = A + \frac{B}{\eta}(\eta - \eta)$$
$$A = 0$$

Applying BC-1,

$$1 = \frac{B}{1}(1-\eta)$$

$$B = \frac{1}{(1-\eta)}$$

Thus, the temperature profile is given by:

$$\theta = \frac{(\bar{r} - \eta)}{\bar{r}(1 - \eta)}$$

Substituting  $\theta$  into the energy integral equation, we solve for the interface location as follows:

$$\int_{1}^{\eta} (\bar{r}\theta) d\bar{r} = \int_{1}^{\eta} \frac{(\bar{r}-\eta)}{(1-\eta)} = \frac{\eta-1}{2}$$
$$\left[ \bar{r}\theta \frac{\partial \eta}{\partial \tau} \right]_{\bar{r}=\eta} = 0$$
$$- \left[ \frac{\bar{r}}{\mu} \frac{\partial \eta}{\partial \tau} \right]_{\bar{r}=\eta} = -\frac{\eta}{\mu} \frac{\partial \eta}{\partial \tau}$$
$$- \left[ \bar{r} \frac{\partial \theta}{\partial \bar{r}} \right]_{\bar{r}=1} = \frac{\eta}{\eta-1}$$

The expression becomes:

$$\frac{\partial}{\partial \tau} \left[ \frac{\eta - 1}{2} \right] = -\frac{\eta}{\mu} \frac{\partial \eta}{\partial \tau} + \frac{\eta}{\eta - 1}$$

Simplifying further:

$$\Big(\frac{1}{2} + \frac{\eta}{\mu}\Big)\frac{\partial\eta}{\partial\tau} \,=\, \frac{\eta}{\eta-1}$$

Collecting terms for  $\eta$  and  $\tau$  and integrating the above expression, we obtain:

$$\int d\tau = \int \frac{\eta - 1}{\eta} \left(\frac{1}{2} + \frac{\eta}{\mu}\right) d\eta$$
$$\tau = \frac{\eta}{2} - \frac{1}{2} \ln(\eta) + \frac{\eta^2}{2\mu} - \frac{\eta}{\mu} + C$$

Where *C* is a constant of integration.

Applying the initial condition ( $\tau = 0, \eta = 1$ ), we obtain the constant of integration:

$$0 = \frac{1}{2} - 0 + \frac{1}{2\mu} - \frac{1}{\mu} + C$$
$$C = \frac{1}{2\mu} - \frac{1}{2} \text{ Hence:}$$
$$\tau = \frac{\eta}{2} - \frac{1}{2} \ln(\eta) + \frac{\eta^2}{2\mu} - \frac{\eta}{\mu} + \frac{1}{2\mu} - \frac{1}{2}$$

Thus, the variation of interface location with time is given by the expression:

$$\tau = \frac{1}{2}(\eta - \ln(\eta) - 1) + \frac{1}{2\mu}(\eta^2 - \eta + 1)$$

The amount of Latent heat absorbed by the phase change material is given by:

$$Q_L = mL$$

Where:

 $m = \rho V$ 

The volume of melted liquid PCM is a function of interface location, which is a function of time, and is given by the relation:

$$V = \pi (s^2 - R_i^2) z$$

Thus:

 $Q_L = \rho \pi L z \left( s^2 - R_i^2 \right)$ 

Expressing in terms of non-dimensional interface variable, we obtain:

$$Q_L = \rho \pi L z R_i^2 (\eta^2 - 1)$$

#### 3. Results and discussions

From the analytical solution obtained using the heat balance integral method (HBIM), we can determine the location of the solid-liquid interface at any time during melting.

Using MATLAB, the non-dimensional variation of the interface location ( $\eta$ ) was plotted with non-dimensional time ( $\tau$ ).



We can infer from the plot that the rate of change of the interface location is higher at the beginning of melting and decreases with time. This is because during the initial time of melting of the phase change material, the heat transfer rate increases more rapidly because of the large temperature difference available between the two phases. However, with time, the heat transfer rate becomes linear because the temperature difference between the solid and liquid phases decreases.



A very important factor in thermal energy storage is the dimensionless number,  $\mu$ , known as the Stefan number. This depends on the Specific heat capacity and Latent heat of the phase change material as well as the temperature gradient.

A material with high Latent heat and low Specific heat capacity will have a lower value of  $\mu$  and will require a longer time to melt than a different material with lower Latent heat and higher Specific heat capacity subject to the same temperature gradient.

At lower Stefan numbers, the rate of interface change (or interface velocity) is lower.

Let us consider two additional materials with different values of Latent heat and Specific heat capacities as shown below [2]:

MATERIAL	C <sub>p</sub> (kJ/kg-K)	L (kJ/kg)	μ
Organic Paraffin	2.4	200	3.6
L-30	1.23	270	1.4
PEG-600	2.5	127	6.0

A plot of the rate of interface change with time for the additional materials was made and compared with our initial phase change material. It can be seen that the rate of interface change depends on the value of  $\mu$ .



**Figure 7** Comparison of interface location ( $\eta$ ) variation with time ( $\tau$ ) for different materials

The total amount of Latent heat absorbed by the phase change material during melting was also plotted as a function of interface location and time, respectively.

We also observe that the heat transfer rate is higher at the initial time and gradually decreases to a linear profile with time.

The amount of Latent heat stored, however, increases exponentially as the interface location approaches the shell diameter. This is because the volume of the liquid phase of the material, and thus the mass, increases exponentially with change in radius.



Figure 8 Variation of latent heat stored (QL) with interface location  $(\eta)$ 



Figure 9 Variation of latent heat stored (QL) with time  $(\tau)$ 

From our results, we observe that it takes 1 unit of dimensionless time (corresponding to approximately 25 hrs.) for the entire phase change material to melt and absorb maximum Latent heat corresponding to  $Q = 3.8 \times 10^8$  J (380 MJ).

For a typical 12-hour day ( $\tau$  = 0.48), approximately 200 MJ of heat energy is stored for re-use during the 12-hour night period, which is equivalent to 56 kWh of electric energy.

# 4. Conclusion

Thermal Energy Storage (TES) using Phase Change Materials (PCM) is an effective means of storing heat which would have otherwise been dissipated for re-use.

The amount of heat stored largely depends on the Latent heat capacity of the phase change material. The rate at which it is stored depends on other factors including the thermal diffusivity and the Stefan number.

#### **Compliance with ethical standards**

#### Disclosure of conflict of interest

No conflict of interest to be disclosed.

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