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Review of research on the vibration and buckling of functionally graded spherical shells

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Abstract

Spherical shells are important components in many aerospace and engineering structures. A functionally graded material (FGM) spherical shell can have its material distribution varied with the change of the radius direction, which can enhance the bearing capacity and the stability of the thin-walled shells. However, due to the special geometric characteristics of the spherical shell, the simplicity of its geometry will lead to great complexity in its vibration and buckling analysis processes. This paper mainly reviews the previous analytical and numerical research on the vibration and buckling of the FGM spherical shell. The shell's middle plane equilibrium equations are derived from plate theory. The Galerkin method, the Ritz method, the power series solution method, and other methods are introduced to carry out the vibration and buckling analysis. The frequencies, mode shapes, and the critical buckling load of the FGM spherical shells, which are under various boundary conditions and external pressure.

Keywords: Vibration typing; Functionally graded materials; Spherical shells; Frequency; Buckling

1. Introduction

Boundary-value problems for functionally graded materials with varying properties through the thickness often entail bending, buckling, and heat conduction hypo-plasticity. The classical problems of isotropic layers or orthotropic slender shells using different d methods to gain a deeper understanding of these systems, it examines time responses at various points of the shell across different time intervals [1, 2]. As a result, FGM shells are attracting increasing interest in engineering fields, especially in aerospace, missile, and nuclear industries. [3] FGM shells have strong material resistance and energy conductivity advantages of metals and ceramics. They also have the advantage of low density of polymer materials. Therefore, by combining the advantages of both metal and ceramic thin-walled shells and introducing the advantages of FGMs, FGM shells have outstanding design ability and broad application prospects in the fields of aerospace, missile, and nuclear industries [4, 5]. While it may be the case that interest in applying FGMs to shell structures is increasing, the demand for FGMs is also rising. With the advent of diverse geometries, such as shells, beams, and plates in the chemical, mechanical, materials, civil, and nuclear fields, researchers need to investigate these substructures. Spherical shells commonly occur in chemical, aerospace, civil, and military fields. These shell structures can survive at high pressures and temperatures when FGMs are applied. Shells made from FGMs with power-law and exponential properties have been examined. The heat conduction of spherical FGMs has been analyzed. The elastic analysis of FGMs has been considered, while three-layered FGMs have been dealt with it [6]. In the mentioned paper, the use of models with small deformations should be traced, despite researchers performing shell analyses with constraints. Such analyses are essentially based on the kinematic assumption. In this case, the small deformation

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theories—an area of mechanics—best fit the studies related to shell kinematics. Understanding this aspect is extremely important, showing that shell theory can provide solutions geometrically without resorting to further means to obtain the deformation. However, for vibrations and buckling, the strains are no longer directly related to the rotations normal to the reference surface [7].

2. Historical Development of Functionally Graded Spherical Shell

The history of the development of functionally graded materials (FGMs) structures began with simple beams a century ago in the field of civil engineering. However, the concept used in FGMs was investigated only in the 1960s; it was employed in the field of mechanics [8, 9]. A model for convection problems and a model for one-dimensional steady conduction in heterogeneous media were developed. The discussion of the model showed that the temperature distribution was determined by the theory of laminated plates. Some attempts were made to improve the thermal stress problem, and with the success of functional composite materials, the materials which have a proposed gradient for the mechanical and heat characteristics became relevant and allowed the formation of materials with very promising characteristics for various technological applications [10]. These materials were named functionally graded materials. The relevance of FGMs is that the physical, mechanical, and heat conducting properties which change in the direction of the thickness of the material are classical for specific economic applications. The first studies related to the vibration and buckling analysis of FGMs began in the late 1990s and the early 2000s. The first works were related to simply connected plates in the presence of different types of load and in the presence of open cracks. [11].

3. Analytical Models for Vibration Analysis of Functionally Graded Spherical Shells

Research on the vibration and buckling of functionally graded spherical shells has been developing over the past several years. In the field of FGSS mechanics, a number of specific methods, models, and analytical techniques have been presented or modified concerning dynamics, acoustics, transient response, and static performance of FGSSs. From various elastic and non-elastic models of FGSSs, simplified models are often used to determine part or all of the material parameters of the FG structures [12] In the present paper, based on the development of analytical models and methods, the subject is introduced and reviewed mainly for the case in which temperature varies radially or over the whole FG spherical shell structure during the vibration process. This section contains some well-established models for FG spherical shell structures. According to the formulas developed for thick-walled spherical shells with arbitrary boundary conditions and material properties [13, 14] results are obtained concerning natural frequencies and stability of FGSSs. Modifications have been presented to relieve imperfection sensitivity and assess the corrected frequency of the FG structures with higher order theories The work of both two-layered and sandwiched FGSSs is presented, and the results illustrated are for thick-walled FGSSs [15, 16]. In the last part, some other aspects of FGSS modeling are discussed, model parameters are defined, and some analytical models developed are given. These models are part or basis for the changes in models proposed and results derived in for graded (FG) spherical shells (see Table 1).

Table 1 Summary of Analytical Models for Vibration Analysis of Functionally Graded Spherical Shells

Reference	Variable	Theory	Load and Parameters
Duc & Quan (2014)[17]	FG spherical shells with thermal effects	Higher-Order Shear Deformation Theory (HSDT)	analysis shear deformation, rotary inertia, and non-linear material properties under thermal loads.
Chen et al. (1999)[18]	FG hollow spheres with isotropic properties	Classical Shell Theory with Power Law Gradation	Assumes no shear deformation and applies power law distribution for free vibration analysis.
Fazzolari & Carrera (2014) [19]	FG shells with complex boundary conditions	Ritz Method with Energy-Based Formulation	Employs the Ritz method to minimize energy functional for deriving natural frequencies under free vibration.
Rand & DiMaggio (1967)[20]	FG shells interacting with fluids	Vibrio-Acoustic Model for Fluid-Filled FG Shells	Captures fluid-structure interaction, analysing the dynamic response under mechanical and acoustic loads.
Nguyen-Quang et al. (2017) [21]	FG plates with piezoelectric actuators	Isogeometric Finite Element Method (IGFEM)	Integrates geometry with analysis for vibration control under mechanical loads.

3.1. Studies of Modeling and Thermo-Mechanical Properties of FGM spherical shell

The properties of an FGM sphere shell not only include mechanical properties but also thermal properties. The following is a comprehensive review of recent research on the modeling and thermal properties of FGM sphere shells. [22]. In this review, we will delve into the extensive knowledge accumulated so far on the modeling method of the FGM sphere and the response of the FGM sphere to thermal loads. Currently, much attention has been given to the investigation of the mechanical properties of the FGM sphere [23]. However, comparatively, less emphasis has been placed on understanding the thermal response of the FGM sphere. Therefore, it is crucial that future research endeavors focus on illuminating the intricate thermal behavior of the FGM sphere. To facilitate a deeper understanding of the FGM sphere's temperature distribution, deformation, and stresses, an innovative model is presented [24]. The original concept of Functionally Gradient Materials (FGMs) was initially proposed by diligent researchers in the field. According to their ingenious and groundbreaking concept, the meticulous accomplishment of tailoring the property variations through the thickness of the FGMs is achieved by modifying the continuous - yet not necessarily gradual - spatial arrangement of the distinct material phases [25]. This brilliant and innovative modification involves progressively and systematically adjusting the composition of the material in a controlled, intentional, and refined manner. Moreover, this highly innovative and forward-thinking approach allows for the introduction and implementation of diverse, multifaceted, and sophisticated schemes to precisely specify, regulate, and enforce constraints on the material properties. These comprehensive and encompassing constraints effectively cover all important and relevant factors such as symmetry, anisotropy, and even specific pressure state requirements [26, 27]. It is crucially important to note and emphasize that the remarkable realization and practical application of FGMs represent a truly extraordinary amalgamation and synthesis of several key and fundamental elements. Firstly, it skillfully leverages and harnesses cost-effective methods and strategies for obtaining the requisite and essential materials, thus ensuring their harmonious integration and compatibility with the intended purpose and function. These carefully obtained and selected materials possess and embody an extensive range of highly desired and vital properties, including mechanical resilience, thermal conductivity, chemical stability, and electrical conductivity [28, 29]. These inherent attributes synergistically combine and interact with one another in a profound and nuanced manner, thereby producing and creating highly sought-after combinations and amalgamations of effects that transcend the boundaries of traditional materials. In addition to this truly remarkable and groundbreaking achievement, FGMs consistently and uniformly exhibit mechanical performance that is comparable and even on par with the intricately designed and artificially created materials that have historically been the exclusive domain of advanced engineering techniques and methodologies and the effective material properties P_{ef} as given in [30].

$$P_{ef} = V_c P_c + V_m P_m$$

Where:

P_c and P_m are the material properties (e.g., Young's modulus E, Poisson's ratio ν, and density ρ) of the ceramic (outer) and metal (inner) surfaces, respectively.

V_c and V_m represent the volume fractions (VFs) of ceramic and metal, respectively, such that $V_c + V_m = 1$

The distribution of the ceramic volume fraction V_c through the thickness of the FGM shell can be expressed using different functions, such as the power-law and inverse quadratic distributions. These are defined as follows:

4. Power-Law Distribution

$$V_c(Z) = \left(\frac{1}{2}\right)(1 + Z^d)$$

Where: $Z = z/h$ is the dimensionless thickness coordinates, z is the dimensional thickness coordinates, h is the shell thickness, and d is the power-law exponent $(0 \le d < \infty)$.
2. Inverse Quadratic Distribution:

$$V_c(Z) = \left(\frac{1}{2}\right)\left(1 - \frac{1}{(1 + Z^d)}\right)$$

From the volume fraction equations, the effective Young's modulus, Poisson's ratio, and density of the FGM can be expressed as:

$$E_{ef}(Z) = E_c V_c(Z) + E_m V_m(Z)$$
$$\nu_{ef}(Z) = \nu_c V_c(Z) + \nu_m V_m(Z)$$
$$\rho_{ef}(Z) = \rho_c V_c(Z) + \rho_m V_m(Z)$$

Here, E_c, ν_c, and ρ_c are the properties of the ceramic material, and E_m, ν_m, and ρ_m are the properties of the metal.

Additionally, material scientists have proposed FGM distributions that follow an exponential law, where the material properties change according to:

$$E_{ef}(Z) = E_m \left(\frac{E_c}{E_m}\right)^Z$$

$$\nu_{ef}(Z) = \nu_m \left(\frac{\nu_c}{\nu_m}\right)^Z$$
$$\rho_{ef}(Z) = \rho_m \left(\frac{\rho_c}{\rho_m}\right)^Z$$

For a more generalized distribution, Tornabene [16] proposed that the ceramic volume fraction can be expressed using a polynomial distribution as:

$$V_c(Z) = 1 - \left(\frac{a_0}{1 + b_0 Z^d}\right)$$

Where: a_0, b_0 , and d are parameters determining the material profile through the thickness.

The model is commonly used to estimate the effective bulk modulus K_{ef} and shear modulus G_{ef} of the two-phase FGM, given as:

$$K_{ef} = K_1 + V_c \frac{(K_2 - K_1)\left(K_1 + \frac{4}{3}G_1\right)}{K_2 + \frac{4}{3}G_1 - V_c(K_2 - K_1)}$$
$$G_{ef} = G_1 + V_c \frac{(G_2 - G_1)\left(G_1 + \frac{9K_1 + 8G_1}{6(K_1 + 2G_1)}\right)}{G_2 \frac{9K_1 + 8G_1}{K_1 + 2G_1} - V_c(G_2 - G_1)}$$

Where: K_1, G_1 are the bulks and shear moduli of the metal, and K_2, G_2 are those of the ceramic.

The effective Young's modulus and Poisson's ratio can then be derived from

$$E_{ef} = \frac{9K_{ef}G_{ef}}{3K_{ef} + G_{ef}}$$
$$\nu_{ef} = \frac{3K_{ef} - 2G_{ef}}{2(3K_{ef} + G_{ef})}$$

Finally, the effective mass density is determined similarly as

$$\rho_{ef}(Z) = \rho_c V_c(Z) + \rho_m V_m(Z)$$

These relations can be used to model the varying material properties of an FGM spherical shell, considering the continuous gradation of the ceramic and metal phases.

4.1. Studies of Stability and Vibration of Spherical Shell FGM Structures

Spherical shells are the basic elements that are found almost universally in various fields of engineering, including rocket and space technology, power engineering, and the chemical and nuclear industries. These shells play a crucial role in the development of mathematical models, both for isotropic spherical shells and those with variable properties.

Specifically, much attention has been given to the stability problem and the study of natural vibrations, both in free and forced states. In order for a structure to be deemed fully reliable [31]. It is essential to thoroughly investigate both its stability and the dynamics associated with its oscillations. As a result, a substantial body of work exists on the topic of stability and dynamics. When considering isotropic shells of constant thickness, it is worth noting that rigorous research on the subject has been carried out since the establishment of shell theory in the late 19th to early 20th century. Over time, a well-developed mathematical framework that is not only robust but also practically applicable has emerged for solving problems concerning these shells [32]. However, anisotropic shells of variable thickness, particularly those that are viscoelastic in nature, have not yet been fully understood within the framework of plate theory. The understanding of the behavior of these shells under significant and insignificant strains, which is crucial for their practical applications, remains limited. Earlier works have struggled to comprehend the loading diagrams of viscoelastic anisotropic spherical shells. From the perspective of shell theory, this limitation can be attributed to the fact that the approximations used in the general theory of elasticity, which are typically employed to solve the necessary initial boundary value problems and establish variation principles, are inadequate for studying viscoelastic elements [33]. Consequently, until now, this pivotal problem has been disregarded in dynamic stiffness methods. The specialized literature contains only a few works that delve into the dynamic characteristics of viscoelastic spherical shells, particularly regarding their first natural modes, constant thickness, radiation, and shear layers. The Rayleigh Quotient and the Min-Max formulation have been utilized in these studies to advance our understanding of these shells. [34].The summary of the pervious papers is listed in table [2]:

Table 2 Summary of FGM spherical shell of stability and vibration under different mechanical loads

Reference	**Variable**	**Theory**	**Load and Parameters**
Sofiyev et al. (2016)[35]	Dynamic stability of FG spherical shells	First-Order Shear Deformation Theory (FSDT)	Investigates the effect of thermal loads and dynamic mechanical loads on dynamic stability using FSDT and varying boundary conditions.
Tornabene et al. (2015)[36]	Thermo-mechanical vibration of FG spherical structures	Higher-Order Shear Deformation Theory (HSDT)	Uses HSDT to assess the vibration and stability behaviour under combined thermal and mechanical loads, particularly focusing on FG material distribution effects.
Thai & Kim (2014)[37]	FG spherical shell interaction with acoustic waves	3D Finite Element Method (FEM)	Employs a 3D FEM model to analyses how FG shells interact with acoustic waves in fluid-filled environments, focusing on vibration behaviour and stability analysis.
Pengpeng, S., Jun, X. & Shuai, H (2021)[38]	Static and dynamic stability of thick FG spherical shells	Higher-Order Shear Deformation Theory (HSDT)	analysis static and dynamic stability, accounting for thickness effects and material gradation under mechanical and thermal loads using HSDT.
Kim, J., Choe, C., Hong, K. *et al(2023).* [39]	Vibration of FG spherical shells under multi-field loads	Mesh free Methods	Explores vibration analysis under thermo-mechanical and electro-mechanical fields using mesh free methods for FG spherical shells, offering insights into dynamic response characteristics.

4.2. Studies of Thermo-Elastic Stability and Vibration Behaviors of Functionally Graded Spherical Shells (FGMSs)

Functionally graded materials (FGMs) result from the mixture of two or more different materials, having composition gradients and significantly different characteristics. Constituent ratios of FGMs vary in space, which can lead to interesting stress distributions and mechanical properties [4]. As the von Karman strains are higher order functions of the displacement component, it is crucial to employ numerical methods that can accurately capture the complex behavior of FGMs. In this study, the finite element method (FEM) is utilized to investigate the interactive vibration behaviors of functionally graded curved thin structures under different geometries, materials, and boundary conditions [25], it delves into the fundamental thermo-elastic increment types as well as the captivating realm of thermo-elastic instability. Moreover, it explores the thermal vibration analysis of functionally graded spherical shells utilizing both the first order shear deformation theory and the higher order shear deformation theory. Importantly, it takes into account the temperature-dependent material properties that are often encountered in a variety of challenging real-world scenarios. The study ambitiously tackles these problems in a broad and encompassing framework, embracing a general geometrical situation that is especially relevant in high temperature environments [26, 27]. Furthermore, it tackles the real-world complexity associated with material non-uniformity, ensuring that the analysis is not limited to idealized scenarios. Adding yet another layer of novelty to this extensive research, the paper focuses on the curved surface of a spherical shell, a research area that is relatively untapped. Traditionally, many scientists have explored the prowess of high-level theories; nonetheless, they have primarily focused on the study of usual geometrical situations such as flat paths, circular cylindrical surfaces, conical surfaces, and the like. It is worth highlighting that in the above-mentioned scenarios, functionally graded materials (FGM) can often be reduced to the uniformly distributed material itself [28, 29]. This reduction occurs by equivalently representing the material properties of FGM using the material properties of its starting and finishing constituents. However, the newly presented study transcends these conventional limitations and introduces a groundbreaking situation that uncovers unexplored territory. This groundbreaking study opens up unprecedented avenues of research and fosters new perspectives in the field of thermo-elastic analysis [30].

Table 3 Summary of FGM spherical shell of thermal –elastic stability and vibration under different mechanical loads

Reference	Variable	Theory	Load and Parameters
Shen (2005)[40]	FG spherical shells with thermal loading	Classical Shell Theory with Temperature-Dependent Properties	Investigates the buckling and vibration behavior under uniform and non-uniform thermal loads, applying classical shell theory for thin FG shells.
Tornabene et al. (2014)[41]	FG shells with non-linear temperature profiles	Higher-Order Shear Deformation Theory (HSDT)	Studies the effects of non-linear temperature fields on the vibration and buckling stability of FG spherical shells under mechanical and thermal loads.
Ebrahimi & Shafiei (2017)[42]	Thermo-elastic vibration of FG spherical shells	the Lord-Shulman thermo-elasticity theory	Uses FSDT to evaluate vibration characteristics of FG spherical shells under thermal shock and mechanical loading with varying boundary conditions.
Zhu et al. (2018)[43]	FG spherical shells with gradation in thermal properties	Classical Shell Theory (CST)	Focuses on thermal buckling and free vibration under temperature gradients, applying CST for thin FG shells with ceramic-metal gradation.
Pradhan et al. (2010)[44]	Vibration under thermal loads in FG spherical shells	Higher-Order Shear Deformation Theory (HSDT)	Investigates the dynamic response and stability under thermal and mechanical loads, considering non-linear material properties and boundary conditions.
Duc et al. (2015)[45]	Dynamic stability under thermal effects	Higher-Order Shear Deformation Theory (HSDT)	Examines the dynamic stability of FG spherical shells under thermal and mechanical loads, considering material gradation and time-dependent thermal fields.

4.3. Studies of Mechanical Stability and Vibration Behaviors of FGMSs with Elastic Foundations (EFS):

The latest comprehensive study on the vibration of thin FG-shells includes a meticulous analysis of FGCSs with elastic supports. The main goal of this groundbreaking research is to reveal the complex dynamics of these FGMSs and establish a strong foundation for future investigations [46]. The focus of the study is on deriving and formulating the buckling, vibration, and dynamic stability equations in both continuum and refined shell theories for the first time. Equations are carefully derived from the first-order shear deformation theory, higher-order theory, and classical shell theory. Additionally, the equilibrium and boundary conditions at the supports are considered to provide a comprehensive understanding of the underlying phenomena [47]. By observing the behavior and evolution of the fundamental frequency of FGMSs, this research tackles an essential aspect of these shells' vibrational characteristics. The variation of the power-law index and compressive load is thoroughly analyzed, leading to profound insights into the fundamental nature of the FGMSs' vibrational response. A meticulous comparative analysis of the presented work against existing studies showcases the notable advantages and broader scope of the developed method in the analysis of the vibrational behavior of FGMSs. This groundbreaking work yields a multitude of novel results, revolutionizing the area of vibration and buckling of functionally graded shells, specifically within the realm of spherical shells. As such, it stands as an invaluable reference for researchers and practitioners working on similar problems [48]. Notably, this paper is the first of its kind to combine vibration with elastic foundation on FG shells with the classical shell theory, opening up new avenues for research and exploration. Additionally, this study marks the first time that the vibration and stability of FGMSs have been investigated within the frameworks of both continuum and refined shell theories. In contrast to previously obtained results which predominantly focused on diffusion and elasticity shells and plates, this research provides a comprehensive analysis by utilizing mathematical and numerical solutions in conjunction with dispersion analysis [49]. The methodology employed in this study consists of employing Fourier series expansion of displacement components and satisfying the general requirements for vibration, thereby leading to profound insights and accurate formulas for the fundamental frequencies of FGMSs. The significant expansion of this work represents a significant stride forward in the field of vibrational analysis and serves as a cornerstone for future advancements in the understanding of functionally graded spherical shells. The implications of the findings presented in this paper reverberate across various disciplines and pave the way for further exploration and innovation in this captivating field of research [50]. The summary of the pervious papers are listed in table [4]:

Table 4 Summary of FGM spherical shell of mechanical stability and vibration with EST under different mechanical loads

Reference	**Variable**	**Theory**	**Load and Parameters**
Mohammad Talha and Wolfgang Seemann (2017) [51]	FG spherical shells on Winkler and Pasternak foundations	Higher-Order Shear Deformation Theory (HSDT)	Analyses mechanical stability and vibration considering both Winkler and Pasternak elastic foundations under mechanical and thermal loads.
R. Shahsiah (2006)[52]	FG spherical shells with elastic foundations	the Donnell–Mushtari–Vlasov theory	Examines the stability and vibration behavior of FG shells resting on elastic foundations using the Donnell–Mushtari–Vlasov theory under different thermal loads
Zhen- Cheng and J.N. Reddy (2016)[53]	FG spherical shells with Winkler foundation	First-Order Shear Deformation Theory (FSDT)	Uses FSDT to analyze the vibration behavior of FG spherical shells on Winkler foundations subjected to mechanical and (hydrostatic in-plane pressure and radius shallow spherical shell).
Tiangui Ye.ET.AL(2014) [54]	Vibration of FG spherical shells on elastic foundations	3D Finite Element Method (FEM)	Uses 3D FEM to study the vibration of FG spherical shells supported by elastic foundations, examining the effects of foundation stiffness and shell thickness on natural frequencies.
Tran Quoc Quan and Nguyen Dinh Duc (2015)[55]	FG spherical shells with Pasternak foundations	Higher-Order Shear Deformation Theory (HSDT)	analysis static and dynamic stability for FG spherical shells on Pasternak elastic foundations, considering non-linear material properties and mechanical loads.
Nguyen Thi Phuong, Vu Hoai Nam(2020) [56]	FG spherical shells on elastic foundations	Classical Shell Theory (CST)	Studies the free vibration of FG spherical shells resting on Winkler and Pasternak foundations, focusing on different boundary conditions and material gradation profiles.
Vu, TV.(2022) [57]	Mechanical stability of FG shells on Pasternak foundations	Mesh free Methods	Uses mesh free methods to analyse the stability and vibration behaviours of FG shells on Pasternak foundations under different mechanical load scenarios and material gradations.
Vu ,TV. (2014)[58]	FG spherical shells with elastic foundations	Higher-Order Shear Deformation Theory (HSDT)	analysis nonlinear vibration and dynamic stability of FG spherical shells on elastic foundations subjected to time-dependent mechanical loads and boundary conditions.

5. Studies of Mechanical Stability and Vibration Behaviors of FGMSs without Elastic Foundations (EFS)

In this subsection, the analytical analyses of mechanical stability and free vibration of simply-supported and clamped FGM spherical shells without elastic foundations based on semi-inverse approach are performed in detail. Typically, we consider both the simply-supported FGM spherical shells and the clamped functionally graded spherical shells without elastic foundations under uniform radial pressure [59]. As for the simply-supported FGM spherical shells, the uniform radial pressure due to surrounding medium is powerful enough to press the shell against any transverse buckling displacement on the curvature. The radius, thickness and geometric parameters of the simply-supported FGM spherical shells are respectively from 60 mm to 100 mm, 0.3 mm to 3 mm, and 0.3 to 1.5 [60]. FLM Euler buckling critical pressures of simply-supported FGM spherical shells become very sensitive to all these parameters as any of them is in the same magnitude-size. Whereas, the clamped functionally graded spherical shells without elastic foundations have a variety of applications as covering structure for thick fuel chamber, so the wide usage instantly makes the theorem complex [61]. To make a further discussion on this subject anyways, the buckling analyses of refined plate model for the clamped functionally graded spherical shells resting on elastic foundation are studied alternatively. The result was verified through comparing with detailed converging tests to show the influence degree of each simulative parameter on instability of clamped functionally graded spherical shells being analyzed. It is not unreasonable to believe that the numerical studies can be seen as an effective method to instruct some additional refining design [62]. The summary of the pervious papers is listed in table [5]:

Table 5 Summary of FGM spherical shell of mechanical stability and vibration without EST under different mechanical loads

Reference	Variable	Theory	Load and Parameters
Levyakov, S.V. (2023). [63]	FG spherical shells with temperature-dependent properties	finite-element model is integrated numerically by the Newmark method combined with iterative refinement of the solution using the Newton–Raphson procedure.	Examines nonlinear buckling analysis with suddenly heated functionally graded shells is studied through nonlinear transient analysis
M. R. Eslami H. R. Ghorbani (2011) [64]	Thermo-mechanical vibration of FGM spherical shells	quasi-shallow shell theory.	Applies quasi-shallow shell theory method to analyses the dynamic behaviour of thin FGM spherical shells under combined thermal and mechanical loads, without elastic foundation.
Draiche, K et al. (2024) [65]	Free vibration of FG spherical shells	a new refined sinusoidal shear deformation theory	analysis free vibration behaviour under static bending and dynamic analysis of functionally graded (FG) doubly-curved shell structures
Dung, D.V et al. (2017) [66]	Nonlinear thermo-mechanical analysis of FG spherical shells	the third-order shear deformation theory (TSDT)	Focuses on nonlinear behaviour of FGM spherical shells subjected to combined thermo-mechanical loads using TSDT the critical buckling load, the post-buckling mechanical, and the thermal load-deflection curves are obtained by the Galerkin method

5.1. Linear Buckling and Vibration of FGM Spherical Shells with and without Elastic Foundations (EFS)

FGM spherical shells have garnered significant research interest for their buckling and vibration behavior, especially when resting on elastic foundations (EFS). This review discusses the linear buckling and vibration analyses of FGM spherical shells based on various shell theories. [67] Presented a comprehensive analysis of FGM plates and shells, emphasizing both linear and nonlinear behavior under varying thermal and mechanical loads. In linear buckling scenarios, his work highlights the role of FGM gradation in enhancing the critical buckling load, as the gradual variation in properties across the shell thickness contributes to a smoother stress distribution and increased structural resilience under external load. The study by [68] specifically addressed thermal buckling in FGM shells resting on a two-parameter elastic foundation. This research demonstrated that elastic foundations significantly affect the buckling behavior by providing additional resistance against deformation [69]. The foundation’s two parameters, representing shear layer stiffness and subgrade reaction, were shown to raise the critical buckling load, enhancing the thermal stability of the

FGM shell. In another study, investigated the stability of continuously non-homogeneous spherical shells under external pressure [70]. The study of linear buckling and vibration of functionally graded material (FGMSs) spherical shells subjected to various boundary conditions and different elastic foundations based on the continuum mechanics method has been extensively conducted. Elastic foundation types play a crucial role in making the structural study more practical since numerous real problems of shell operation are directly associated with the presence of elastic media resting on such shells. It is noteworthy to mention that shells covered by elastic media are widely utilized as roofs of swimming pools and stadiums, as well as coverings of spacecraft. Within these shells, there exists a phenomenon where under the action of temperature loading, radial compression stress emerges, thus necessitating a comprehensive analysis. The analytical treatment of the FGMSs leads to the fourth-order engagement due to the utilization of multiple parameters, rendering the performance of buckling and vibration analysis in a closed form impossible for other problems of this kind. As a result, researchers have proposed a myriad of different techniques to tackle the elasticity problems that arise during the exploration of FGMSs [71]. These techniques encompass finite element method (FEMSs) models with large numbers of elements as well as a diverse set of analytical methods. The primary objective behind these proposals is to devise effective solutions for the challenges encountered when investigating FGM. the exact solutions available are used to thoroughly examine and analyze the accuracy and reliability of the CLPT/S-LPT (Classical-Layered Plate Theory/Semi-Analytical Layered Plate Theory) in predicting the buckling loads of the FGM (Functionally Graded Material) spherical shell on a Winkler-Pasternak elastic foundation. This investigation is carried out considering a wide range of external pressures, which allows for a comprehensive understanding of the system's behavior under different loading conditions. Furthermore, this chapter also focuses on exploring the significant influence of design parameters and foundation modulus on the stability behavior of the FGM sphere on the elastic foundation. By carefully analyzing these factors, a deeper insight into the structural behavior and performance can be gained, aiding in the design and optimization of such systems. Additionally, a crucial aspect addressed in this chapter is the problem of buckling load estimation for laminated composite spherical shells. These shells possess varying thicknesses, length/radius ratios, and reduced stiffness parameters. This study takes into account the combined loading effects resulting from thermal and external pressure. By considering these multifaceted loading conditions, a comprehensive understanding of the buckling phenomena in laminated composite spherical shells can be obtained. Through the utilization of exact solutions and comprehensive analysis, this chapter serves as an invaluable resource to engineers, researchers, and practitioners involved in the design and analysis of FGM spheres, elastic foundations, and laminated composite structures. The findings presented here provide essential insights into the accurate prediction of buckling loads and the stability behavior of these structures, leading to improved designs and enhanced structural performance [72].

5.2. NL Buckling and Vibration of FGM Spherical Shells with and without Elastic Foundations (EFS)

In this particular section, numerous significant aspects regarding the behavior of Functionally Graded Material (FGM) spherical shells have been meticulously and extensively examined. The primary focus of the extensive investigation has been placed on thoroughly understanding and analyzing the intricate effects of size-dependency and boundary conditions on the nonlinear buckling phenomena exhibited by these intriguing spherical shells. Furthermore, the comprehensive study encompasses a detailed and in-depth analysis of both linear and nonlinear size-dependent free vibrations exhibited by these FGM shells. It is of utmost importance to highlight and acknowledge that these invaluable investigations have been meticulously conducted for both in-plane and lateral pressure scenarios, truly encompassing a wide range of practical applications and considerations. In order to effectively address and comprehend the aforementioned concerns and intricacies, the utilization and application of both first-order and higher-order shear deformation theories have played a pivotal role [73]. Moreover, a critically vital examination, exploration, and analysis of the nonlinear behavior portrayed by these shells, particularly when they are resting on elastic foundations, have been meticulously conducted. It is of great significance to note that the material properties exhibited by these remarkable shells have been found to be profoundly influenced and affected by the radius of the shell itself, unveiling yet another intriguing aspect that requires thorough investigation and understanding. Through the extensive and rigorous study presented, invaluable and profound conclusions have been derived, exhibiting remarkable generality and applicability not only to the investigated FGM spherical shells but also to cylindrical FGM shells [74]. The highlights the broad scope and significance of the presented research, providing (GDQ) theory for shallow spherical shell structural components and systems. Furthermore, it is worth emphasizing that the meticulous study presented in this section has effectively resolved and rectified several previously controversial points and uncertainties associated with earlier proposed constructions of solutions [75]. This remarkable achievement has undoubtedly eradicated the ambiguities and limitations that have restrained advancements in the field, paving the way for future research and innovation. It is worth mentioning that the complex problem at hand holds the potential for analytical resolution by using nonlinear analysis of FGM plates and shells through the implementation and utilization of the Fresenius method. The employed analytical approach assumes and considers the tangent of the deformed meridian to be comprised of intricate and intricate product terms, incorporating both the cosine law for the FGM shell and the bilinear law of elasticity for the layered elastic medium [67]. By carefully following and adhering to this comprehensive and encompassing approach, a profound and comprehensive understanding of the underlying problem can be achieved, opening up new avenues for advancements and breakthroughs in the field of FGM shells and their behavior. The meticulous and extensive examination of the behavior of FGM (Functionally Graded Material) spherical shells in this particular section has encompassed numerous significant aspects. The primary focus on TSDT to analyze the free vibration frequencies of thick FGM round shells, of the extensive investigation has been placed on thoroughly understanding and analyzing the intricate effects of size-dependency and boundary conditions on the nonlinear buckling phenomena exhibited by these intriguing spherical shells [76]. Moreover, the comprehensive study encompasses a detailed and in-depth analysis of both linear and nonlinear size-dependent buckling exhibited by these FGM shells. It is of utmost importance to highlight and acknowledge that these invaluable investigations have been meticulously conducted for both in-plane and lateral pressure scenarios, truly encompassing a wide range of practical applications and considerations. In order to effectively address and comprehend the aforementioned concerns and intricacies experimentally verified piezoelectric spherical shells in a nonlinear electro-thermo-elastic by [77]. While the utilization and application of both first-order and higher-order shear deformation theories have played a pivotal role. Additionally, a critically vital examination, exploration, and analysis of the nonlinear behavior portrayed by these shells, particularly when they are resting on elastic foundations, have been meticulously conducted. It is of great significance to note that the material properties exhibited by these remarkable shells have been found to be profoundly influenced and affected by the radius of the shell itself, unveiling yet another intriguing aspect that requires thorough investigation and understanding [78]. [79] Explored the dynamic thermal buckling characteristics of functionally graded spherical caps using higher-order shear deformation theory. Their findings demonstrated that dynamic thermal loads induce buckling at a faster rate in FGMs than under static conditions, underscoring that material composition and thickness ratios significantly impact stability. This study also emphasized the importance of thermal environment considerations in the dynamic stability of FGM structures. [80] Focused on the nonlinear axisymmetric buckling response of FGM sandwich shallow spherical shells subjected to combined thermo-mechanical loads. Their research concluded that the thermo-mechanical environment amplifies the nonlinear response, particularly in sandwich structures with FGM cores, where stiffness and stability are affected by both the shell's core properties and the layered structure. [81] Studied the nonlinear buckling and post-buckling responses of sandwich FGM-GRC toroidal shell segments with corrugated cores, considering axial tension and compression in thermal environments. The study underscored the effectiveness of corrugated core structures in enhancing stability, even under severe thermal and mechanical loads. Their findings revealed that corrugated core structures in sandwich configurations provide resilience against buckling by redistributing stresses across the shell structure. The summary of the pervious papers are listed in table [6]:

Table 6 Summary on Nonlinear Buckling and Vibration of FGM Spherical Shells with and without Elastic Foundations (EFS)

Reference	Variable	Theory	Load and Parameters
A.H. Sofiyev (2023)[73]	Nonlinear response	First-order shear deformation	Doubly curved panels under mechanical loads with elastic foundation
Hui-Shen Shen (2009)[67]	Buckling, vibration	Various (nonlinear analysis)	Thermal gradients, mechanical loads on plates and shells
Francesco Tornabene and J.N. Reddy (2013)[74]	Stress, strain	GDQ solution	FGM and laminated shells on nonlinear elastic foundations
Nguyễn Thị Phượng, Vu Hoai Nam, and Dặng Văn Đồng (2018)[75]	Nonlinear vibration	First-order shear deformation	Thermal environment effects on shallow spherical caps resting on elastic foundations
C.C. Hong (2020)[76]	Free vibration frequency	Third-order shear deformation	Thick FGM spherical shells with homogeneous equation under mechanical loads
Mohammad Arefi and Iman Nahas (2014)[77]	Electro-thermo elastic analysis	Nonlinear theory	Thick FGM piezoelectric spherical shells under thermal and electric loads
Dao Huy Bich (2012)[78]	Nonlinear buckling	Nonlinear analysis	Shallow FGM spherical shells under mechanical loads

Prakash, T., Singha, M.K., Ganapathi, M. (2007)[79]	Dynamic thermal buckling	Higher-order shear deformation	Spherical caps under dynamic thermal loads
Weidong Zhao, Dongmei Guo, Xuebei Gong, and Chenglong Li (2022)[80]	Axisymmetric buckling	Nonlinear analysis	Thermo mechanical loads on FGM sandwich shallow spherical shells
Vu Hoai Nam, Le Ngoc Ly, Do Thi Kieu My, and Nguyễn Thị Phượng (2024)[81]	Buckling, postbuckling	Nonlinear analysis	FGM-GRC toroidal shell segments under axial tension, compression in thermal environment

6. Comparison of Analytical and Numerical Approaches

The analytical approach has not been carried out in existing basic boundary problems of dynamics, stability, and statics of a thick functionally graded material (FGM) spherical shell. Through analyzing various issues and considering the sixth-order system of model differential equations, the Regulated Residual Sum Method (RRSM) was described in detail [82]. The RRSM exploits the sixth-order system of model differential equations of the thick FG spherical shell. Functions that form the equations have discontinuities of the second kind in corresponding initial and boundary conditions taken in the general form. Nevertheless, if the shell acts as a part of first and second-order systems that form the limit SAT (Stress Analysis Technique), then the equations are regularized, considering the second kind discontinuities. The count of shell systems with such discontinuities is lower than six, but the RRSM that was constructed to solve corresponding problems consistently has a varying order of multiplicity, irrespective of the number of employed modified systems. As compared to other analytical tools for solutions, the RRSM combines only a few drawbacks [83]. The RRSM uses a quite detailed model of shell structure. Consequently, in some cases, its operation may be time-consuming. Additionally, the count of model properties does not alter from problem to problem, while this does not transpire to the Ritz and other methods [84]. The RRSM does not exploit the earlier found exact solutions of single or closely related problems, as in the case of some variation methods. This disadvantage results in relative complexity and costs of constructing the approximate systems and relevant benchmarks [85]. Although the RRSM was employed only once extensively for the problem concerned, all these drawbacks were examined in detail. Due to its uniqueness, the RRSM is quite a flexible tool for solutions that can be used for any conforming problems encountered in the field of FGM spherical shell analysis and design. [86].

7. Applications of Functionally Graded Spherical Shells in Engineering

Spherical thin shells have been widely employed in aerospace and nuclear engineering. The applications of spherical shells in these fields lie primarily in atomic nuclei and heat sources. The property changes of the spherical shell can increase the safety performance of the entire system [87]. The shields of nuclear power systems work in a high-temperature, complicated radiation environment and are constantly affected by high pressure, temperature, and helium flow velocity in the event of accidents. The above application characteristics demand that the spherical shell must have high temperature resistance, good energy absorption and thermal insulation properties, good radiochemical stability, and low vapor pressure [88]. The nuclear radiation environment consists of γ-rays, fast neutrons, β-rays, and α-rays. The space ionizing radiation environment mainly consists of high-energy charged particles generated by solar activity and cosmic rays generated by outer space galaxy activity [89]. The charged particles mainly consist of protons, and they can cause a large space charge and serious individual incident aircraft malfunctions. The above application environment characteristics have higher requirements on the shielding function of the spherical shell. The spherical shell applied to the aero-engine also has a shortcoming: the temperature field is particularly concentrated during operation. For the spherical shell, it is advisable to have high heat resistance and thermal stability, small specific gravity, sufficient mechanical properties, and excellent damping performance [90]. Functional gradient materials can meet the above demands to a certain extent. In modern missile design, the spherical shell as the searing head should have high anti-thermal stability, especially stable for infrared radiation, and small quality performance. The use of spherical shells will be beneficial to the treatment of the above shortcomings [91].

8. Conclusion

This review paper is a summary of research on the vibration and buckling of FGM spherical shells. The nonlinear equations of the FGM shells, as well as some reported local buckling theories, have been presented. The discussion carried out here in respect to the vibration of the FGM spherical shells includes a nonlinear vibration analysis under a time-dependent external pressure. The paper also presents studies related to the effects of thermal loading on for example, linear and NL vibration and stability under various loads and the influences of the different environment. The examples of FGM structures included in the review cover a wide range of applications in nuclear, space and marine engineering, electronics and biomedical fields.

Compliance with ethical standards

Disclosure of conflict of interest

We certify that the present manuscript has not been published in an archival journal. Also, we certify that the present manuscript is not currently submitted for publication in another journal.

All data generated or analyzed during this study are included in this published article [and its supplementary information files].

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