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# Modelling selected stock prices at the Nairobi securities exchange using Markov chains

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## Abstract

This paper modelled four stock prices traded at the Nairobi stock exchange using Markov Chains. Markov chain is a stochastic process that has a Markovian property. The study focused primarily on the application of Markov Chain in analysing Nairobi Securities Market daily returns to describe the distribution of the market returns, cross-checking if a current market return depends on its preceding market returns and using the analysis to predict the future returns. Discrete Markov Chains were fitted to determine the probability of change of states in the returns of stock prices and to determine the extent to which the Markov model can be used to forecast returns. The idea of using Markov chains to predict the behaviour of market prices is widely used since prospective stock investors are concerned with stock price movements which might lead to an optimum investment strategy. The Markovian test carried out in the study on the daily returns showed that the returns had a Markov property. The model predicted that the share prices would either decrease to a range of minimum and lower average, decrease to a range of lower average and zero, remain unchanged, increase to a range of zero and upper average and increase to a range of upper average. The steady-state was found in the 21st, 23rd, 12th and 21st trading days for Centum, Equity Bank, East African Breweries Limited (EABL) and Kenya Airways respectively.

**Keywords:** Hidden Markov Model (HMM); Initial Price Offer (IPO); Markov Chain Model (MCM); Nairobi Securities Exchange (NSE); Transition Probability Matrix (TPM).

## 1. Introduction

Climate In investment, a time series tracks the movement of the chosen data points such as security's price over a specified time with data points recorded at regular intervals. A time series is a sequence of numerical data points in successive order. The two main goals of time series analysis are to summarize the time series data and to make predictions of the future values of time series variables. Once the pattern of financial time series data is identified, an interpretation of the data can be made. Applications of time series cover all areas in statistics most importantly the economic and financial time series.

Financial time series analysis deals with the analysis of data collected in financial markets. Its main goal is to obtain reliable information to make good decisions about the future. Predicting the future behaviour of financial markets is difficult, as they are determined by a variety of interdependent factors whose evaluation can hardly be measured and quantified. These factors include political disorders, economic crises and natural disasters. It forms the foundation of making inferences, a key feature that distinguishes financial time series analysis from other time series analyses. Both financial theory and its empirical time series contain an element of uncertainty. As a result of the added uncertainty, statistical theory and methods play an important role in financial time series analysis.

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Stock prices act as one of the major factors which enable one to get a whole idea about the performance of the market generally. Stock traders always wish to buy a stock at a low price and sell at a higher price. However, the best time to buy or sell a stock is a challenging question. Stock investments can have a huge return or a big loss due to the high volatility of stock prices.

In the early 1920's, trading shares in Kenya was under the London Stock Market. It was in 1954 that the Nairobi Securities Exchange was founded. The Nairobi Stock Exchange is the leading stock market and the fastest-growing economy in the East and Central African region with a market capitalization of about Ksh.1.62 trillion. The Nairobi Securities Exchange has a six-decade heritage in listing equity and debt securities. It offers a world-class trading facility for local and international investors seeking to gain exposure to Kenya's and Africa's economic growth. Our study is going to be based on data from the Nairobi Securities Exchange market. Interest in investing in equities is fast gaining momentum at the Nairobi Securities Exchange.

The introduction of Initial Public offers in Kenya signed a considerable shift to more risky and yet more profitable investment options for local and international investors. Due to the favourable investment climate in Kenya since the end of post-election violence in 2008 and the peaceful general election in August 2013, investors' confidence has greatly improved both direct foreign investment and active participation in the stock exchange market. Daily stock prices in the Kenyan market are greatly influenced by a variety of factors including strong political forces, fuel prices, exchange rate of major international currencies, inflation rates, dividend announcements and introduction of new products.

Investors always keep track of the movement of share prices since this directs them in their decision-making process. Due to globalization, investors have expanded their investment capacity making investments in more than one market which deals with different goods and services. This has enabled them to enjoy diversification of markets Vasanthi et al., (2011). For investors to base their decisions on investments, they must have done a study of the current market price movements. The ultimate aim is to earn high profit. Therefore, investors have shown a keen interest in predicting stock values. However, forecasting index prices may be hard due to market volatility that needs an accurate forecast model. The stock prices fluctuate and rise at high levels.

According to Zhang et al., (2009b), the stock market is uniquely characterized by characteristics such as non-linearity, non-parametric, essentially dynamic and exhibits no correlation. The movement also has a relationship with political temperature, commodity price index, bank rate and so on. This makes it hard to predict stock prices with certainty. He divided market researchers into three groups; The group convinced that investors do not get profit higher than average trading advantages just from owning historical and present information, the second group has some fundamental analysis from studying various macro-economic factors and information such as financial conditions thus they could find correlation between this information and stock prices and the third group tries to predict the stock prices by finding a good model.

Several approaches have been used to model stock prices. They include neural networks, data mining, moving averages, regression analysis, ARIMA models and Markov Chains. Markov Chains, used in this study is a prediction method based on the probability forecasting approach which is used to predict immediate probabilities of stock prices under the above market mechanisms (Zhang et al., 2009b). Markov model takes advantage of the stochastic nature of the share prices and helps researchers determine the probability of movement of the share prices from one state to another.

### 1.1. Statement of the problem

Price volatilities make stock investments risky, leaving investors in a critical position when an uncertain decision is made. Investors need market information to decide which stocks to invest in, when to buy, when to sell and when to wait. However, due to uncertainties in the stock market trends, investors can only maximize returns by studying the history of listed companies, performance and development prospects of such fundamentals and being familiar with a variety of technical analyses. The purpose of this study was to analyse the discontinuous jumps in stock prices which will be of help to the investors in making investment decisions. To improve investor evaluation confidence in securities exchange markets, the share prices were specified as a stochastic process assumed to possess Markov dependency with respective state transition probabilities matrices.

### 1.2. Empirical literature review

In the earlier studies, other forecasting methods have been proposed for the analysis of the stock market, for instance, Hassan et al., (2005) used the Hidden Markov Model for Securities Market forecasting. He used HMM with four observations, close, open, high and low, of airline stock and predicted their future close price using four states they used a day in the past similar to the recent day and used the change in that day's price and price of the recent day to predict a future close price.

Previous studies have shown that stock markets have a Markov property and hence they can be modelled as random walk processes. Markov chains have been widely used in the modelling of many practical systems such as telecommunications that is in speech recognition, inventory, queuing and manufacturing systems (Ching et al., 2008).

Markov chains have been proposed as a reasonably acceptable generator of synthetic wind speed data. Authors have used various transition matrix sizes, various time steps, and various orders. Shamshad et al., (2005) used a first-order transition matrix for hourly time steps with eight states. He stated that second and third-order autocorrelation coefficients are significant, and suggests higher-order transition matrices for future work.

Doubleday et al., (2011) used the Markov chain in the modelling of the Dow Jones Industrial Average. His research aimed to determine the relationship between a diverse portfolio of stocks and the market as a whole. Two models were highlighted, where the Dow Jones Industrial Average was considered as being in a state of (1) gain or loss and (2) small, moderate, or large gain or loss. Results indicated that the portfolio behaved similarly to the entire Dow Jones Industrial Average, both in the simple model and the partitioned model. Conclusions were made that when treated as a Markov process, a diverse portfolio of stocks will mirror the movements of the entire market. He recommended that future work may include different classifications of states to refine the transition matrices.

Markov chain model has been used to analyse and make predictions on the three states that exist in stock price change which are share prices increase, decrease or remain unchanged. Choji et al., (2013) modelled share price movement in two top banks, Guarantee Trust Bank of Nigeria and First Bank of Nigeria, using six years of data obtained from 2005 to 2010. From the matrices derived using the three states, he was able to predict the probability of moving from a given state to another state for a transition. He noticed that the probability of the share price of the two top banks in Nigerian stock continued to increase until equilibrium was reached which is after 20 years and then became constant. On the other hand, for the sake of investors or future investors to be well informed, the probability of the bank share price depreciating was approximately 0.4 which implies that an investor who bought shares during that year had an equal chance of the share price appreciating or depreciating by the year 2025.

Mitra et al., (2011) sought to predict the immediate future price for a company using Markov chains. He found the moving averages for the data and then grouped them into four different states of results. Markov Chain calculations were then applied to the data to create a 4x4 transitional probability matrix. Using this transition matrix, he solved a system of equations and found four steady states which were variables that represented the probability that a stock price for a given day would fall into one of the four states. He was able to successfully predict the next few days of stock prices using this method.

Markov chain was applied in modelling and forecasting Safaricom shares in NSE (Otieno et al., 2015). The study was conducted through a period covering 2008 to 2012 forming a 784 days trading data panel. A Markov chain model was determined based on the probability transition matrix and initial state vector and in the long run, the model predicted that the Safaricom share prices would depreciate, maintain value or appreciate with a probability of 0.3, 0.1 and 0.5 respectively. He noticed that the Markov model was able to predict trends due to its memoryless property and random walk capability, in that each state can be reached directly by every other state in the transition matrix, consequently giving good results. Otieno suggested that further studies could be conducted on several companies listed in the Nairobi Securities Exchange, using higher-order Markov chains to gain better insight into the behaviour of the stock market.

Agwuegbo et al., (2010) analysed Nigerian stock market price trends by determining probabilities of the market transitions between various states. He used the Markov chains method to analyse the behaviour of daily return of the stock market prices of all securities listed in the Nigeria Stock Exchange. His study showed that the stock market follows a random walk model and that the stock prices are but martingale and that all that the investors can do is to narrow differences between fairness and otherwise in a way that high chances of small gains may be exchanged with low chances of large gains.

Svoboda et al., (2012) applied the Markov chain to model the Prague stock exchange using time series of day closing prices. Their study compared models with different state sets. They used two states in the first model and eight states in the other model. They recommended further research be done by constructing models with different state spaces within MCA and the development and implementation of a non-homogeneous Markov chain.

A study carried out by Zhang et al., (2009a) on forecasting the stock market trend based on the stochastic analysis method, observed that an increase in trading days under stable conditions resulted in the convergence of the state probability to a value that is independent of the initial state and more or less stabilized. He obtained the closed state transition matrix of the Shanghai Composite using a tree states model which he named up, down and constant. He further used the past 24 trading day's closing prices to calculate a forecast of the subsequent day's closing price using a vector formula. After the calculation, they were able to find out that the closing price state interval after each day predicted was consistent with the actual situation. He explains the main difference between the Markov model and other statistical methods like regression and time series analysis in that the former does not need to consider mutual laws among the factors from the complex predictor, only to consider the characteristics of the evolution of the history situation of the event itself and to predict changes of the internal state by calculating the state transition probability. The article shows that the Markov model has a broader applicability in stock prediction.

Ghezzi et al., (2003) proposed a novel dividend valuation model by using a Markov chain. A general treatment is provided for the valuation problem in which the dividend growth rate is a discrete variable. To do so, a state of a stationary Markov chain is attached to each feasible value of the dividend growth rate. On deriving an existing condition, the valuation problem is turned into a system of linear equations, with the unknowns being price–dividend ratios, each corresponding to a different state of the Markov chains. This procedure also holds when the above-mentioned assumption of independent, identically distributed, random variables is relaxed. This paper showed different ways of predicting an uncertain future and enhanced the precise amount of predicted information.

Simonato, (2011) developed a numerical approach for computing American option prices in the lognormal jump–diffusion context. That approach uses the known transition density of the process to build discrete time-homogeneous Markov chains to approximate the target jump-diffusion process. He examines how a Markov chain approach, which uses the known transition density of the stock price to build an approximating Markov chain, can be used to compute prices for American options in the jump-diffusion context with log-normal jumps. The proposed approach is shown numerically to converge smoothly to benchmark values as the number of states of the Markov chains is increased. He recommends that further research examine and compare the recursive integration approach, which also uses the transition density of the stock price to compute derivatives prices.

Hoek et al., (2012) stated that the main contribution of their paper is to value a general finite expiration American option in the framework of Markov chain dynamics; the framework of dynamics is driven by a finite state Markov chain. He developed a model of a financial market and considered where the uncertainty is using a finite state Markov chain.

Mettle et al., (2014) specified equity price change of selected equities from Ghana Stock Exchange weekly trading data as a stochastic process assumed to possess Markov dependency with respective state transition probabilities matrices following the identified state, decrease, stable or increase. He established that identified states communicate and that the chains are aperiodic and ergodic thus possessing limiting distributions. He developed a methodology for determining expected mean return time for stock price increases and also established criteria for improving investment decisions based on the highest transition probabilities, lowest mean return time and highest limiting distributions.

Zhou, (2014) studied the stock price of China's Sports Industry and the theory of Weighted Markov Chain was applied to forecast the stock price. He used the stock price changes in China's Sports Industry in seventy trading weeks from February 6, 2012, to June 11, 2013, and used the Weighted Markov Chain model for prediction. The historical data of the stock's closing price was tested about Markov Property. A state transition matrix was constructed using the data and the weight values of every state were calculated with the method of Weighted Markov Chain theory and prediction intervals of the industry's future stock prices were obtained. The weighted Markov model was used to predict the stock prices. The six states used were plunge, flat plunge, downward flat, upward flat, rise and soar.

Based on previous literature reviews, researchers have been able to use the definite states Markov model to forecast financial time series data. This paper applied the previous methods used but used a recent data set, developed a Markov chain model for more than one company and used different states to monitor the transition in a Markov model. In addition, the extent to which the Markov model can be used to forecast share price was determined.

## 2. Methodology

### 2.1. Stochastic Process

A stochastic process is a family or set of ordered random variables. It is a collection of random variables $X_t$, one for each time $t$ in some set $J$. The order is indicated by indexing each random variable in the family by a subscript. Usually, the ordering is a result of the random variables being observed over time. The random variable $X_t$ denotes the price of a stock at the time $t$ and observations of the stock price for the last 5 trading days. These data were used to describe the process and to analyse the nature of its past behaviour over time. It was also used to estimate the parameters of our stochastic process model. Prediction of the future behaviour of the stock price was then done by the estimated stochastic process. It is the dependence between the random variables in the set that allowed us to make predictions by extrapolating past patterns into the future. The set of values that the random variables $X_t$ are capable of taking what is called the state space of the process. A possible model might say that the value of $X_t$ depends on the values at the end of the two previous trading days $X_{t-1}$ and $X_{t-2}$. In this study, our stochastic process was in the form of a set of state spaces which can be counted hence the state spaces are discrete that is $S = 1,2,3, \dots, n$.

### 2.2. Markov Chains

A Markov Chain is a stochastic process that has the Markovian property. A Markovian property is when the present state determines the future state but the past state has no significance in future predictions. That is a stochastic process $X_t$ is said to have the Markovian property if;

$P(X_{t+1} = j | X_0 = K_0, X_1 = K_1, \dots, X_{t-1} = K_{t-1}, X_t = i) = P(X_{t=1} = j | X_0 = K_0)$ for $t = 0,1,2, \dots, n$

The conditional probabilities below are called transition probabilities given by;

$$P(X_{t+1} = j | X_t = i) = P_{ij}$$

The current status of the system can fall into any one of a set of $M + 1$ mutually exclusive categories called states. The random variable $X_t$ represents the state of the system at time t, so it's only possible values are 0 to $M$. The system is observed at particular points in time, labelled $t = 0,1,2, \dots, m$. Thus, the stochastic process $X_t = (x_0, x_1, x_2, \dots, x_m)$ provides a mathematical representation of how the status of the physical system evolves. Markov Chains provide conditional transition probability distributions of state $S = s_1, s_2, \dots, s_k$ and transition probability matrix $p$ which consist of conditional probabilities $p_{ij} = P(X_{n+1} = S_j | X_n = S_i)$ for $i, j = 1,2,3, \dots, k$ where $p_{ij}$ does not depend on time thus called homogeneous Markov chains. The vector of conditional probabilities was $P(X_{n+1} = x_{n+1} | X_1 = x_1, X_2 = x_2, \dots, X_n = x_n)$.

The Markov model was built in each of the stock returns. Markov Chain models are useful in studying the evolution of systems over repeated trials. Repeated trials are often successive periods where the state of the system in any particular period cannot be determined with certainty. Rather, transition probabilities are used to describe how the system makes transitions from one period to the next. The TPM is used to determine the probability of the system being in a particular state at a given time.

### 2.3. Chapman-Kolmogorov Equations

The Chapman-Kolmogrov equations are used to point out that when one goes from one steady state to another in n steps, the process will be in some other state after exactly m states. Chapman-Kolmogorov equations allow us to obtain the n-step probabilities from the one-step transition probabilities recursively.

Given that $p_{ij}$ is the 1-step Transition Probability Matrix, $p_{ij}^n$ is the n-step Transition Probability Matrix, $p_{ij}^m$ is the m-step Transition Probability Matrix and $p_{ij}^{n-m}$ is the $(n-m)$-step Transition Probability Matrix.

$$p_{ij}^n = \sum_{k=0}^{m} p_{ik}^m p_{kj}^{n-m}$$

for all $i = 0,1, \dots, m; j = 0,1,2, \dots, m; m = 1,2, \dots, (n-1)$ and $n = (m+1), (m+2), \dots$

Thus, the summation is the conditional probability that, given a starting point in one state, the process goes to the other state after m-steps and then to the next state in $(n-m)$-steps. Therefore, by summing up these conditional probabilities over all the possible steady states must yield,

$p_{ij}^{n} = \sum_{k=0}^{M} p_{ik} p_{kj}^{n-1}$ and $p_{ij}^{n} = \sum_{k=0}^{M} p_{ik}^{(n-1)} p_{kj}$.

These expressions allowed the computation of the n-step probabilities from the one-step transition probabilities recursively.

### 2.4. Steady State Probabilities

Given that $p$ is the steady state in a 1-step transition of states and $\pi^n$ the steady state in the n-step transition of states, if at any step $n, \pi^n = \pi$, it is then said that the chain has reached the steady state or equilibrium and $\pi$ is called the steady-state distribution. After the n-step transition probabilities for a Markov chain have been calculated, the Markov chain will display the characteristic of a steady state. Meaning, that if the value of $n$ is large enough, every row of the matrix will be the same and as such, the probability that the process is in each state does not depend on the initial state of the process. Therefore, the probability that the process will be in each state $n$ after a certain number of transitions is a limiting probability that exists independently of the initial state. This can be defined as follows;

For any irreducible ergodic Markov chain $\lim_{n\to\infty} p_{ij}^{n}$ exists and is independent of $i$. Furthermore, $\lim_{n\to\infty} p_{ij}^{n} = \pi_j \geq 0$, where the $\pi_j$ uniquely satisfies the following steady-state equations; $\pi_j = \sum_{i=0}^{M} \pi_j p_{ij}$ and $\sum_{j=0}^{M} \pi_j = 1$, for $j = 0,1,2,\dots,M$.

The steady-state probabilities of the Markov chain are $\pi_j$. These values indicate that after a large number of transitions, the probability of finding the process in a particular state such as $j$ tends to the value of $\pi_j$ which is independent of the initial state. To solve for the steady state probabilities discussed above, the aforementioned formulas must be applied to the transition matrix, and the linear system needs to be solved. Suppose, for a given Markov chain, the one-step transition matrix is the following:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{bmatrix}$$

Then the corresponding $\pi_j = 1$ the following linear equation was then solved,

$$\begin{gathered} \pi_0 p_{01} + \pi_1 p_{02} + \pi_2 p_{03} + \cdots + \pi_n p_{0n} = \pi_0 \\ \pi_0 p_{01} + \pi_1 p_{02} + \pi_2 p_{03} + \cdots + \pi_n p_{0n} = \pi_1 \\ \vdots \\ \pi_0 p_{n1} + \pi_1 p_{n2} + \pi_2 p_{n3} + \cdots + \pi_n p_{nn} = \pi_n \\ \pi_0 + \pi_1 + \pi_2 + \cdots + \pi_n = 0 \end{gathered}$$

In testing Markovian property, a Markovian property means that the future state is determined by the current state and not the past states. This test is carried out to check if the current week's return depends on the preceding week's return and the hypothesis is as follows, $H_0: p_{ij} = p_j$ (Markov chain is of a zero order) versus $H_1: p_{ij} \neq p_j$ (Markov chain is of a first order). The test statistic is given by, $Q = 2\sum_{i=1}^{m}\sum_{j=1}^{m} n_{ij} \log_e\left(\frac{n_{ij}}{n_i n_j}\right)$. This follows a chi-square distribution with degrees of freedom $= (m-1)^2$, where $m$ is the number states. The decision rule is to reject $H_0$ for larger values of $Q$ otherwise do not reject.

## 3. Results and Discussion

Returns of data were computed and a Markov test was performed to check whether these returns had a Markov property. Descriptive statistics were obtained and frequencies were plotted against returns. The state transition diagram was drawn from the TPM. Steady states were generated to determine how much the Markov model can be used to forecast stock prices. Table 1 summarises descriptive statistics of the daily returns for Kenya Airways, Equity Bank, EABL and Centum investment for the period under review.

**Table 1** Summary statistics for returns

| Statistics | Centum | Equity bank | EABL | Kenya Airways |
|---|---|---|---|---|
| No. of observations | 1358 | 1358 | 1358 | 1358 |
| Mean | 0.0005 | 0.0004 | 0.0000 | -0.0015 |
| Minimum | -0.1646 | -0.1139 | -0.2564 | -0.2426 |
| Median | 0.1646 | 0.0000 | 0.0000 | 0.0000 |
| Variance | 0.0005 | 0.0003 | 0.0009 | 0.001 |
| Std. dev | 0.0220 | 0.0166 | 0.0295 | 0.0309 |
| Skewness | 0.0143 | -0.1328 | 0.8747 | -0.1625 |
| Kurtosis | 12.9266 | 6.7913 | 16.2386 | 10.5382 |

The analysis from table 1 showed that the mean of the returns approximately equal to zero and with excess kurtosis in the four companies. This shows that the distribution of the daily returns data has heavy tails meaning the daily returns has a leptokurtic distribution.

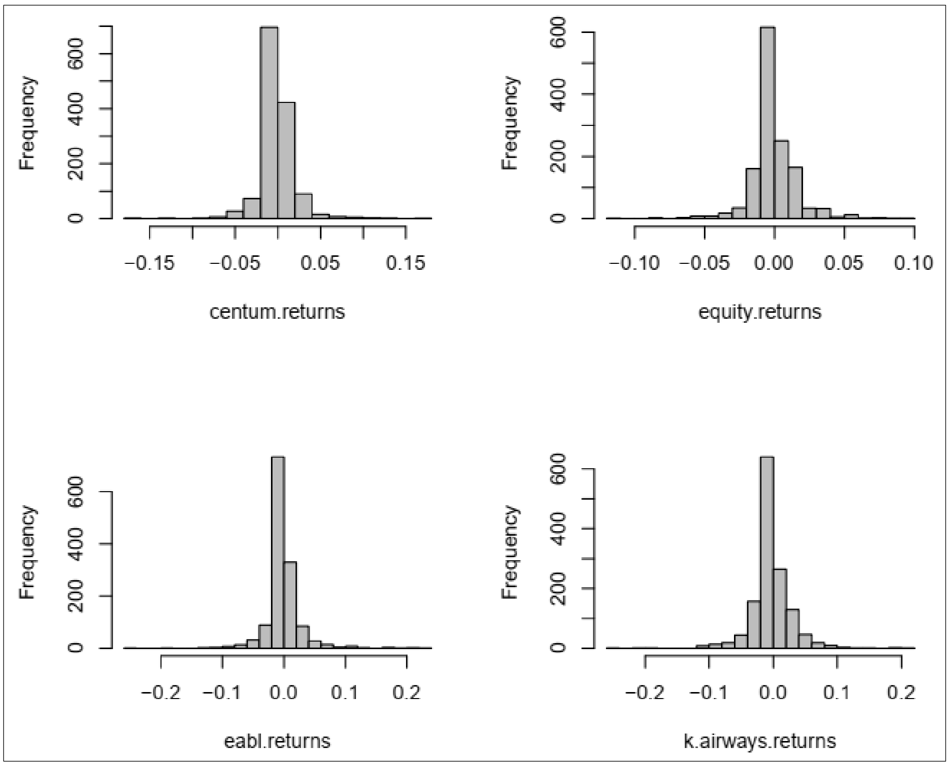


**Figure 1** Histogram of returns

Figure 1 shows the frequencies of the daily returns plotted in a histogram together with a line graph to describe the distribution as shown above. This implies that the distribution of the daily returns of the Nairobi Securities market is leptokurtic.


**Figure 2** Plots for the four selected stock prices

The figure 2 shows the general direction of the prices of shares for the four companies considered for analysis. The figure shows that the stock prices of the companies can fluctuate wildly over time due to the frequent change in market sentiment, sector or industries in play and profit taking.

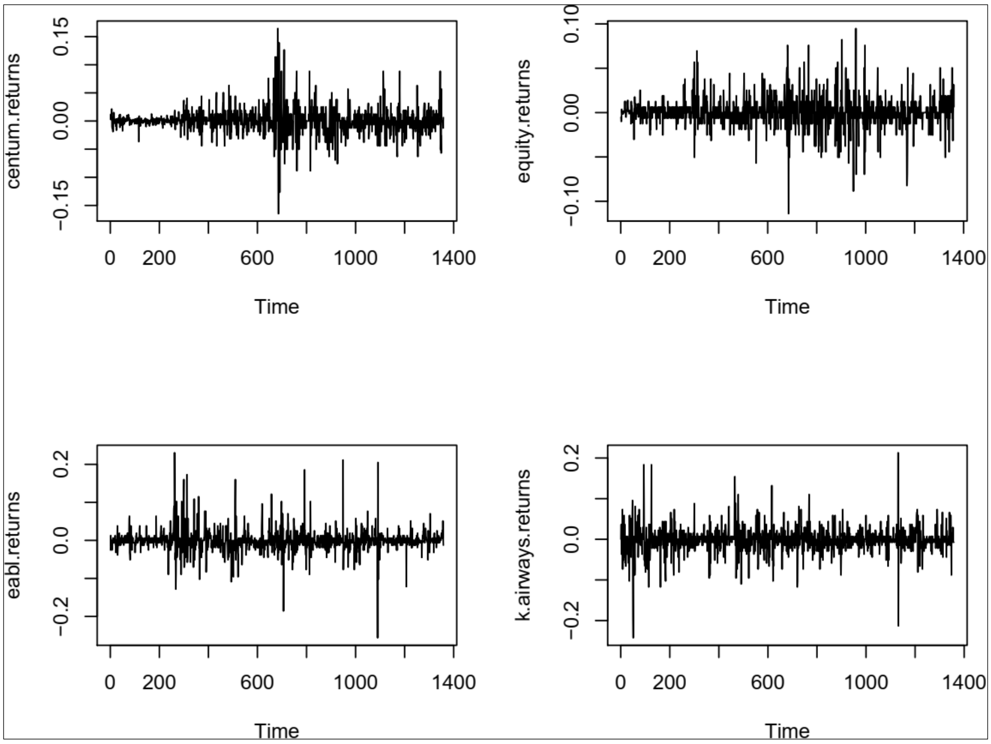


**Figure 3** Plots of the time series for the four company's daily returns

### 3.1. Transition Probability Matrix of the Returns

The three major states were defined as follows; $U$ for unchanged, $D$ for decrease and $I$ for increase. Then $D$ and $I$ states were classified further into two more states; $D_1$ and $D_2$ for decrease and $I_1$and $I_2$ for increase respectively to give a total of five states. These states are;

$Min \leq D_2 \leq Lower\ average$, $Lower\ average \leq D_1 \leq 0$, $U = 0$, $0 \leq I_1 \leq Upper\ average$, and $Upper\ average \leq I_2 \leq Max$.

R statistical software was used to obtain the states above for the four companies. The transition probability matrix of the daily returns of the Nairobi stock market index for the entire period for the four companies was obtained respectively.

$$Centum_{TPM} = \begin{bmatrix} 0.3333 & 0.0000 & 0.1667 & 0.3333 & 0.1667 \\ 0.0039 & 0.4650 & 0.1946 & 0.3327 & 0.0039 \\ 0.0000 & 0.3183 & 0.2942 & 0.3806 & 0.0069 \\ 0.0019 & 0.3358 & 0.1866 & 0.4646 & 0.0112 \\ 0.0833 & 0.2500 & 0.2500 & 0.3333 & 0.0833 \end{bmatrix}$$

In Centum investments, for instance the probability of being in state $D_2, D_1, U, I_1$ and $I_2$, is 0.3333, 0.4650, 0.2942, 0.4646 and 0.0833 respectively. This is also the probability of remaining in the same state.

$$Equity\ Bank_{TPM} = \begin{bmatrix} 0.3750 & 0.3750 & 0.0000 & 0.2500 & 0.0000 \\ 0.0044 & 0.4245 & 0.3195 & 0.2429 & 0.0088 \\ 0.0000 & 0.2956 & 0.3805 & 0.3136 & 0.0103 \\ 0.0021 & 0.2784 & 0.1897 & 0.5072 & 0.0227 \\ 0.1000 & 0.4500 & 0.1500 & 0.2500 & 0.0500 \end{bmatrix}$$

In Equity bank for instance the returns have a probability of 0.3750 of moving from $D_2$ to $D_1$ and the same chance of remaining to the same state. The returns are not likely to move to state U and $I_2$ but can move to $I_1$ with a probability of 0.25.

$$EABL_{TPM} = \begin{bmatrix} 0.0000 & 0.5000 & 0.0000 & 0.0000 & 0.5000 \\ 0.0017 & 0.4957 & 0.2147 & 0.2862 & 0.0017 \\ 0.0000 & 0.4089 & 0.2887 & 0.2955 & 0.0069 \\ 0.0021 & 0.3662 & 0.1734 & 0.4475 & 0.0107 \\ 0.0000 & 0.5000 & 0.1000 & 0.4000 & 0.0000 \end{bmatrix}$$

In EABL for instance, the returns have a probability of 0.0017 of moving from $D_1$ to $D_2$and the same chance of moving to $I_1$. The returns will remain in the same state $D_1$ with a probability of 0.4957, move to $U$ with a probability of 0.2147 and move to $I_1$ with a probability of 0.2862.

$$Kenya\ Airways_{TPM} = \begin{bmatrix} 0.3333 & 0.3333 & 0.0000 & 0.3333 & 0.0000 \\ 0.0017 & 0.4498 & 0.1903 & 0.3581 & 0.0000 \\ 0.0000 & 0.4319 & 0.2990 & 0.2658 & 0.0033 \\ 0.0000 & 0.4026 & 0.2141 & 0.3704 & 0.0129 \\ 0.1250 & 0.0000 & 0.0000 & 0.7500 & 0.1250 \end{bmatrix}$$

For Kenya Airways share price, a case of being in state $I_1$ was analysed. There is no chance of it going to $D_2$. The probability of moving from this state to state $D_1$ is 0.4026. The probability of moving to state $U$ is 0.2141, remaining in the same state with a probability of 0.3704 and a probability of 0.0129 of moving to state $I_2$.

The above share price transition movements can be illustrated clearly by a transition diagram as shown in the figures below for Centum, EABL, Equity and Kenya Airways respectively.

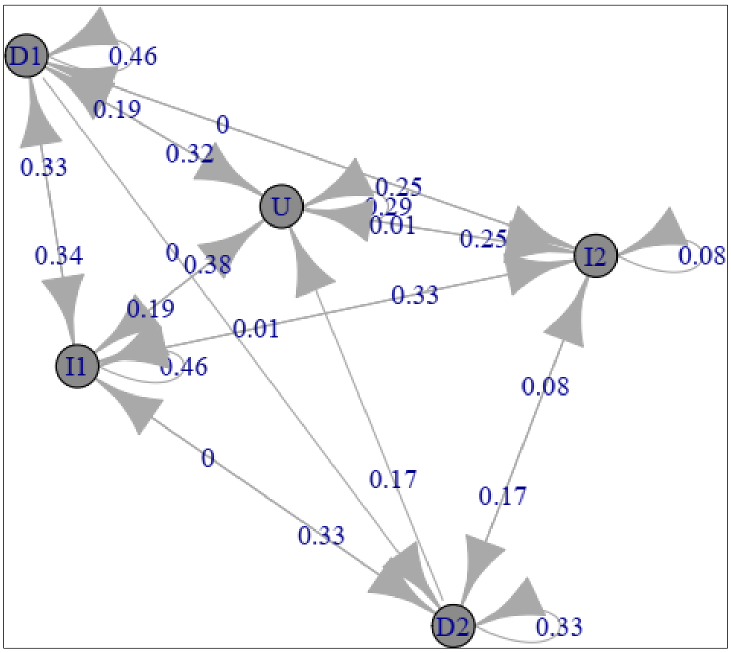


**Figure 4** Centum transition graph

For centum investment, there is a higher chance of share prices in state $D_2$ remaining in the same state or moving to state $I_1$ with a probability of 0.33. There is a higher chance of share prices in state $D_1$remaining in the same state with a probability of 0.46. There is a higher chance of share prices in state $U$ moving to state $I_1$ with a probability of 0.38. There is a higher chance of share prices in state $I_1$ moving to state $U$ with a probability of 0.46. There is a higher chance of state in $I_2$ moving to state $I_1$ with a probability of 0.33.

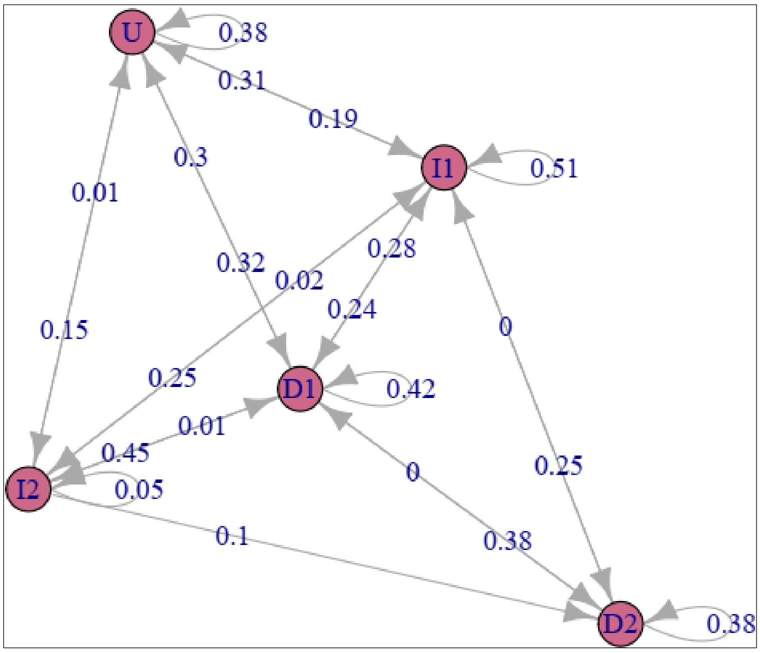


**Figure 5** Equity Bank transition graph

For equity bank, there is a higher chance of share prices in stat $D_2$ remaining in the same state or moving to state $D_1$ with a probability of 0.38. There is a higher chance of share prices in state $D_1$ remaining in the same state with a probability of 0.42. There is a probability of share prices in state $U$ remaining in the same state with a probability of 0.38. In state $I_1$, there is a higher chance of share prices remaining in the same state with a probability of 0.51. There is a chance in state $I_2$ moving to state 1 with a probability of 0.45

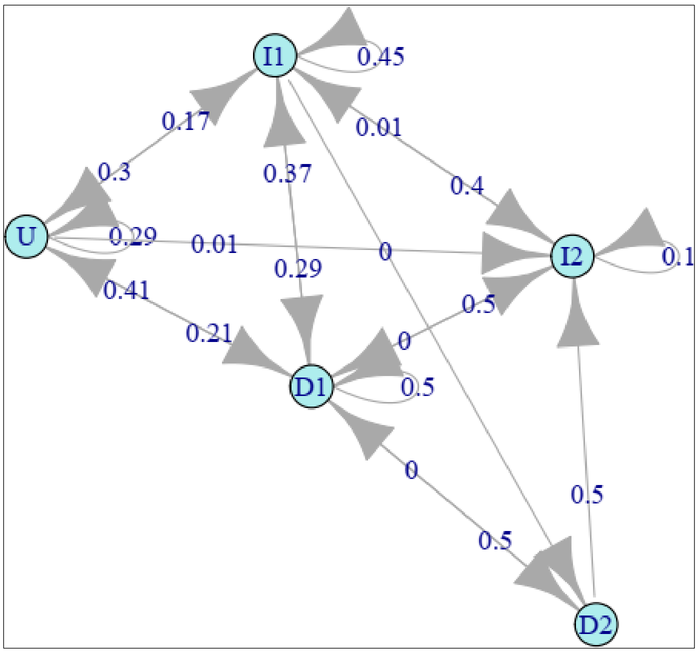

**Figure 6** EABL transition graph

For EABL, there is a chance of share prices in state $D_2$ moving to state $D_1$ and moving to state $I_2$ with a probability of 0.5. There is a chance of share prices in state $D_1$ remaining in the same state with a probability of 0.5. There is a probability of share prices in state $U$ moving to $D_1$ with a probability of 0.41. In state $I_1$, there is a chance of share prices remaining in the same state with a probability of 0.45. There is a chance in state $I_2$ moving to state $D_1$1 with a probability of 0.5.

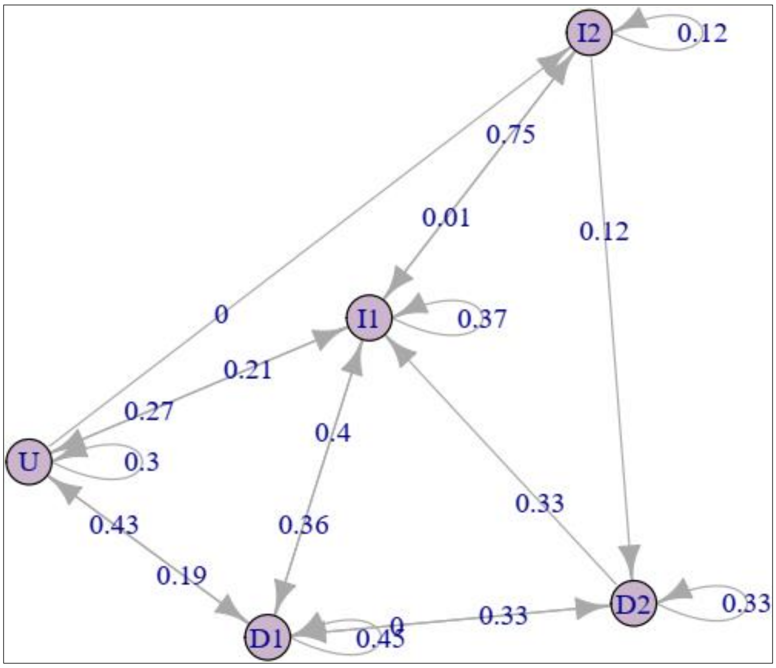

**Figure 7** Kenya Airways transition graph

For Kenya Airways, there is a chance of share prices in state $D_2$ remaining in the same state or moving to state $D_1$ or moving to state $I_2$ with a probability of 0.33. There is a chance of share prices in state $D_1$ remaining in the same state with a probability of 0.45. There is a probability of share prices in state $U$ moving to $D_1$ with a probability of 0.43. In state $I_1$, there is a chance of share prices moving to $D_1$ with a probability of 0.4. There is a chance in state $I_2$ moving to state $I_1$ with a probability of 0.75.

### 3.2. Testing Markovian Property

The table 2 below shows the result of the Markov property test performed. From the Markov property test performed, we observe for all companies that the maximum likelihood criterion statistic value is greater than the critical value, thus, the null hypothesis is rejected. This means that the Markov chain is of a first-order.

**Table 2** Tests for Markov Chain

| Company | Centum | Equity Bank | EABL | Kenya airways | $\chi^2$ Critical Value @5% |
|---|---|---|---|---|---|
| Statistic value | 2.9442 | 3.4349 | 2.0943 | 3.3956 | 1.33 |

### 3.3. Estimate of the Steady State Probabilities

The absolute probabilities at any stage $n$ was determined by the use of n-step transition probabilities. This is a higher order transition probability$p_{ij}^n$ of the transition matrix$p_{ij}$. The n-step matrix shows the behavior of share prices n-steps later. The elements of this matrix represent the probabilities that an object in a given state will be in the next state n-steps later. These repeated transitions were used to evaluate whether the transition probabilities converge over repeated iterations. The higher order transition probability $p_{ij}^n$ of the transition probability matrix $p_{ij}$ was calculated to observe the behavior of the share price and the results obtained using R Statistical Software are as shown below. From the above n-step transition matrix, it is noticed that after a period of 21 Trading days for Centum, 23 for equity, 12 for EABL and 21 Trading days for Kenya Airways, the matrix begins to approach some constant probabilities. After extending matrix multiplication to higher power we see all values converge indicating that equilibrium is attained. The fact that the transition matrix converges to a steady state system means that the Markov chain is ergodic. This is clear as the five states in the matrix each with five nonzero probabilities.

$$Centum\ p_{ij}^{21} = \begin{bmatrix} 0.0044 & 0.3788 & 0.2130 & 0.3950 & 0.0088 \\ 0.0044 & 0.3788 & 0.2130 & 0.3950 & 0.0088 \\ 0.0044 & 0.3788 & 0.2130 & 0.3950 & 0.0088 \\ 0.0044 & 0.3788 & 0.2130 & 0.3950 & 0.0088 \\ 0.0044 & 0.3788 & 0.2130 & 0.3950 & 0.0088 \end{bmatrix}$$

$$Equity\ Bank\ p_{ij}^{23} = \begin{bmatrix} 0.0059 & 0.3354 & 0.2861 & 0.3579 & 0.0147 \\ 0.0059 & 0.3354 & 0.2861 & 0.3579 & 0.0147 \\ 0.0059 & 0.3354 & 0.2861 & 0.3579 & 0.0147 \\ 0.0059 & 0.3354 & 0.2861 & 0.3579 & 0.0147 \\ 0.0059 & 0.3354 & 0.2861 & 0.3579 & 0.0147 \end{bmatrix}$$

$$EABL\ p_{ij}^{12} = \begin{bmatrix} 0.0015 & 0.4326 & 0.2144 & 0.3441 & 0.0074 \\ 0.0015 & 0.4326 & 0.2144 & 0.3441 & 0.0074 \\ 0.0015 & 0.4326 & 0.2144 & 0.3441 & 0.0074 \\ 0.0015 & 0.4326 & 0.2144 & 0.3441 & 0.0074 \\ 0.0015 & 0.4326 & 0.2144 & 0.3441 & 0.0074 \end{bmatrix}$$

$$Kenya\ Airways\ p_{ij}^{21} = \begin{bmatrix} 0.0022 & 0.4267 & 0.2210 & 0.3442 & 0.0059 \\ 0.0022 & 0.4267 & 0.2210 & 0.3442 & 0.0059 \\ 0.0022 & 0.4267 & 0.2210 & 0.3442 & 0.0059 \\ 0.0022 & 0.4267 & 0.2210 & 0.3442 & 0.0059 \\ 0.0022 & 0.4267 & 0.2210 & 0.3442 & 0.0059 \end{bmatrix}$$

### 3.4. Estimate of the Future Expected Daily Returns

The last research question sought to determine the prediction of one day future state which in this case the 1359th trading day. To address this, we apply an initial state vector to the transition matrix and predict what state that initial vector will transition to after one trading days. The 1359th trading day for Centum shows that there is a higher chance of the returns of the shares being in state $I_1$ i.e. appreciating up to the upper average. This is as shown below;

$$[0.0044 \quad 0.3785 \quad 0.2128 \quad 0.3954 \quad 0.0089]\begin{bmatrix} 0.0033 & 0.0000 & 0.1667 & 0.3333 & 0.1667 \\ 0.0039 & 0.4650 & 0.1946 & 0.3327 & 0.0039 \\ 0.0000 & 0.3183 & 0.2942 & 0.3806 & 0.0069 \\ 0.0019 & 0.3358 & 0.4646 & 0.1866 & 0.0112 \\ 0.0833 & 0.2500 & 0.2500 & 0.3333 & 0.0833 \end{bmatrix} = [0.0044 \quad 0.3787 \quad 0.2130 \quad 0.3950 \quad 0.0088]$$

The 1359th trading day for Equity bank shows that there is a higher chance of the returns of the shares being in state $I_1$ i.e. appreciating up to the upper average.

$$[0.0059 \quad 0.3366 \quad 0.2865 \quad 0.3579 \quad 0.0147]\begin{bmatrix} 0.3750 & 0.3750 & 0.0000 & 0.2500 & 0.0000 \\ 0.0044 & 0.4245 & 0.3195 & 0.2429 & 0.0088 \\ 0.0000 & 0.2956 & 0.3805 & 0.3136 & 0.0103 \\ 0.0021 & 0.2784 & 0.1897 & 0.5072 & 0.0227 \\ 0.1000 & 0.4500 & 0.1500 & 0.2500 & 0.0500 \end{bmatrix} = [0.0059 \quad 0.3360 \quad 0.2866 \quad 0.3583 \quad 0.0148]$$

The 1359th trading day for EABL shows that there is a higher chance of the returns of the shares being in state $D_1$ i.e. depreciating up to the lower average.

$$[0.0013 \quad 0.4330 \quad 0.2143 \quad 0.3439 \quad 0.0079]\begin{bmatrix} 0.0000 & 0.5000 & 0.0000 & 0.5000 & 0.0000 \\ 0.0017 & 0.4957 & 0.2147 & 0.2862 & 0.0017 \\ 0.0000 & 0.4089 & 0.2887 & 0.2955 & 0.0069 \\ 0.0021 & 0.3662 & 0.1734 & 0.4475 & 0.0107 \\ 0.0000 & 0.5000 & 0.4000 & 0.1000 & 0.0000 \end{bmatrix} = [0.0015 \quad 0.4326 \quad 0.2145 \quad 0.3441 \quad 0.0073]$$

The 1359th trading day for Kenya Airways shows that there is a higher chance of the returns of the shares being in state $I_1$ i.e. depreciating up to the lower average.

$$[0.0022 \quad 0.2791 \quad 0.2216 \quad 0.3439 \quad 0.0059]\begin{bmatrix} 0.0000 & 0.3333 & 0.0000 & 0.3333 & 0.0000 \\ 0.0017 & 0.4498 & 0.1903 & 0.3581 & 0.0000 \\ 0.0000 & 0.4319 & 0.2990 & 0.2658 & 0.0033 \\ 0.0000 & 0.4026 & 0.2141 & 0.3704 & 0.0129 \\ 0.1250 & 0.0000 & 0.0000 & 0.7500 & 0.1250 \end{bmatrix} = [0.0020 \quad 0.3604 \quad 0.1930 \quad 0.2914 \quad 0.0059]$$

## 4. Discussion

The study focused on the application of Markov chain to the daily returns of the Nairobi Securities Exchange. From the study, the analysis showed that the mean of the returns approximately equal to zero and with excess kurtosis in the four companies shows that the distribution of the daily returns data have heavy tails. This implies that the distribution of the daily returns of the Nairobi Securities market is a leptokurtic distribution. The Markov property tests carried out revealed that the returns of the four companies follow a first-order Markov chain which implies that a daily's return depends only on its preceding daily's return. This test satisfies the assumptions underlying the application of Markov Chain. The transition probability matrices obtained represented the probabilities of moving from one state to the other for the various companies. The convergence of transition matrix to a steady state implying that there is a limiting probability that the return states will be in a steady state condition after 21 days for Centum, 23 days for equity bank, 12 days for EABL and 21 days for Kenya Airways. The last objective was to determine the Markov model for forecasting share price in Nairobi Securities Exchange, it was concluded that the derived the transition matrix could be used to predict the states of share price.

## 5. Conclusion

Based on the first objective, it was concluded that based on the daily returns of the four companies are dependent on the immediate return only and independent of the past returns. The other objective was to determine the Markov model for forecasting the share prices in the Nairobi Securities Exchange, it was concluded that the derived initial state vector and the transition matrix could be used to predict the states of the share prices of Centum, EABL, Kenya Airways and Equity bank as confirmed by the prediction of the states of $1359^{th}$ trading days. Additionally, the convergence of the transition matrix to a steady state implying ergodicity is a characteristic of the stock market makes the model applicable.

Finally, In the long run, irrespective of the current state of share price, the model predicted that the share prices will decrease to a range of minimum and lower average, decrease to a range of lower average and zero, remain unchanged, increase to a range of zero and upper average and increase to a range of upper average and the maximum value with an approximate probability of 0.0044, 0.3788, 0.2130, 0.3950, 0.0088 for Centum, 0.0059, 0.3354, 0.2861, 0.3579, 0.0147 for equity 0.0015, 0.4326, 0.2144, 0.3441, 0.0074 for EABL and 0.0022, 0.4267, 0.2210, 0.3442, 0.0059 in Kenya Airways respectively.

The study focused on the application of the Markov chain to the daily returns of the Nairobi Securities Exchange. The study concluded that the analysis showed that the returns of the four companies follow a first-order Markov chain. The transition probability matrices obtained represented the probabilities of moving from one state to the other for the various companies. The convergence of the transition matrix to a steady state implies that there is a limiting probability that the return states will be in a steady state condition after 21 days for Centum, 23 days for equity bank, 12 days for EABL and 21 days for Kenya Airways. It was also concluded that the derived transition matrix could be used to predict one day of the share price. This study shows how the Markov model fits the data and can predict trends due to its memoryless property and random walk capability, in that each state can be reached directly by every other state in the transition matrix, consequently giving good results. Hence, this model will help both researchers and investors in identifying the future prices in stock markets in general thereby being able to make informed decisions regarding investment in the stock market.

## References


[1] Agwuegbo, SON, AP Adewole, and AN Maduegbuna (2010). "A random walk model for stock market prices". In: Journal of Mathematics and Statistics 9.10, pp. 113–119. DOI: DOI:10.1093/annonc/ mds173..

[2] Ching, WK, MK Ng, and ES Fung (2008). "Higher-order multivariate Markov chains and their applications". In: Linear Algebra and its Applications 6.10, pp. 26–31. DOI: DOI:10.1096-10(03)173..

[3] Choji, D N, S N Eduno, and G T Kassem (2013). "Markov chain model application on share price movement in stock market". In: mathematics and computation 4.10, pp. 84–95. DOI: DOI:10.10934/mds173..

[4] Doubleday, KJ and JN Esunge (2011). "Application of Markov chains to stock trends". In: Journal of Mathematics and Statistics 6.7, pp. 167–176. DOI: DOI:10.1116/S0002-7..

[5] Ghezzi, LL and C Piccardi (2003). "Stock valuation along a Markov chain". In: Applied mathematics and computation 3.9, pp. 188–200. DOI: DOI:10.1017-7090(03)-00183.

[6] Hassan, MR and B Nath (2005). "Stock market forecasting using hidden Markov model: a new approach". In: Systems Design and Applications, 2005. ...

[7] Hoek, J van der and RJ Elliott (2012). "Asset pricing using finite state Markov chain stochastic discount functions". In: Stochastic Analysis and Applications 5.8, pp. 66–70. DOI: DOI:10.S0001-9270..

[8] Mettle, Felix Okoe, Enoch Nii Boi Quaye, and Ravenhill Adjetey Laryea (2014). "A methodology for stochastic analysis of share prices as Markov chains with finite states." In: SpringerPlus 3.10,p. 657. ISSN: 2193-1801. DOI: 10.1186/2193-1801-3-657.

[9] Mitra, Shuchi S and Michael J. Riggieri (2011). "Predicting Stock Prices". In: Stochastic Analysis and Applications 6.8, pp. 81–91. DOI: DOI:10.1011/10083-7..

[10] Otieno, S, E O Otumba, and R N Nyabwanga (2015). "Aplication of markov chain to model and forecast stock market trend: A study of Safaricom shares in Nairobi securities exchange, Kenya". In: International Journal of Current Research 7.4, pp. 14712–14721. DOI: DOI:10.1-9270(03)00183-7..

[11] Shamshad, A, MA Bawadi, WMAW Hussin, and TA Majid (2005). "First and second order Markov chain models for synthetic generation of wind speed time series". In: Stochastic Analysis and Applications. 6.9, pp. 64–71. DOI: DOI:10.1016/S0002-9270(03)00183-7..

[12] Simonato, JG (2011). "Computing American option prices in the lognormal jump–diffusion framework with a Markov chain". In: Finance Research Letters 8.10, pp. 55–60. DOI: DOI:10.2147/CLEP.S58722..

[13] Svoboda, M and L Luka´s (2012). "Application of Markov chain analysis to trend prediction of stockˇ indices". In: Proceedings of the 30th International.

[14] Vasanthi, S, MV Subha, and ST Nambi (2011). "An empirical study on stock index trend prediction using markov chain analysis". In: Financial Services 5.10, pp. 78–90.

[15] Zhang, Deju and Xiaomin Zhang (2009a). "Study on Forecasting the Stock Market Trend Based on Stochastic Analysis Method". In: International Journal of Business and Management 3.6, pp. 101–105. DOI: DOI:10.1016-70183-7..

[16] Zhang, Y and L Wu (2009b). "Stock market prediction of SANDP 500 via combination of improved BCO approach and BP neural network". In: Expert systems with applications.

[17] Zhou, Qing-xin (2014). "Application of Weighted Markov Chain in Stock Price Forecasting 2 The Application of the Weighted Markov Chain in Forecasting the Stock Price". In: In stochastic analysis and Applications 53.Icm, pp. 241–244.