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Advanced method for solving the transportation problem

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Abstract

Transportation problem is one of the predominant areas of operations research. It is a type of linear programming problem designed to minimize the cost of distributing a single product from ***m*** sources to ***n*** destinations. Transportation problem is widely used as a decision making tool in different fields such as engineering, business management and many others. These types of problems can be solved by general network methods. In this paper The Advanced Method for Solving the Transportation Problem (AMSTP) is proposed to find initial basic feasible solution and optimal solution of transportation problem and compared its results with other existing methods.

Keywords: Transportation problem; Linear programming; Optimal solution; Initial basic feasible solution.

1. Introduction

The transportation problem in general is defined as an optimum model for distributing goods from a set of supply stations or centers that minimizes the total transportation cost, like workshops or manufactories, named sources, to various receiving stations, like storehouses, named destinations, with such a way kept the gross distribution costs minimal. The availability of transportation problem when every source can supply a constant number of output units, and the requirement is each destination has a fixed demand [2].

Also a company is used a Transportation models when wanted to choose where to determine a new facility. Attempting to reduce gross production and transportation costs for the whole system concerning facility location is a good financial decision. [8].

The first formulation of transportation problem was by [3] it was further developed by [4, 1]. Several extensions of transportation model and methods have been subsequently developed. The classic transportation problem in operational research is involved with finding the minimal cost of transmitting a single good or commodity from a given number of sources to a given number of destinations. Each source can supply fixed value units of goods generally known as the availability, and every destination has a fixed demand, is called requirement. The applications of transportation problem involve in determining how to optimally transport goods that is why it received this name. Transportation problem is a logistical problem for organizations especially for transport manufacturing and companies [6].

2. Types of Transportation problem

There are two types of Transportation Problem namely Balanced Transportation Problem and Unbalanced Transportation Problem [5]. The Transportation Problem is said to be balanced transportation problem if total number of supply is same as total number of demand, and a Transportation Problem is said to be unbalanced transportation problem if total number of supply is not same as total number of demand.

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2.1 Mathematical Model of a Transportation problem

Any solution $X_{ij} \geq 0$ is said to be a feasible solution of a transportation problem if it satisfies the constraints. The feasible solution is said to be basic feasible solution if the number of nonnegative allocations is equal to ***(m+n-1)*** while satisfying all rim requirements, i.e., it must satisfy requirement and availability constraint. There are three ways to get basic feasible solution.

- North West Corner Rule (NWCM).
- Least Cost Method (LCM).
- Vogel's Approximation Method or Regret Method (VAM).

A feasible solution of transportation problem is said to be optimal if it minimizes the total cost of transportation. There always exists an optimal solution to a balanced transportation problem. We start with initial basic feasible solution to reach optimal solution which is obtained from above three methods. We then check whether the number of allocated cells is exactly equal to ***(m+n-1)***, where ***m*** and ***n*** are number of rows and columns respectively. It works on the assumption that if the initial basic feasible solution is not basic, then there exists a loop. The methods used for optimality are: Modified Distribution (MODI) Method or (u - v) method and Stepping Stone method.

The mathematical formulation of the problem is as follows:

Using the following notations;

x_{ij} = the units numbers to be dispensed from ***i*** sources to ***j*** destinations
(***i*** = 1, 2, ..., m; ***j*** = 1, 2, ..., n)
s_i = supply from ***i*** sources
d_j = demand at ***j*** destination
c_{ij} = the units costs dispensed from ***i*** sources to ***j*** destinations

The mathematical model of linear programming can be applied to describe transportation problem and always it represents in a transportation matrix.

For any transportation problem the input can be briefed in a matrix format using a table called the transportation value or costs table (Table 2.1). The table presents the supply origins, the destinations with their demand and the transportation cost per cell.

Table 1 Transportation problem costs

Source	**Destination**				**Supply**
	1	**2**	**....**	**n**	
1	C_{11}	C_{12}	⋯	C_{1n}	S_1
2	C_{21}	C_{22}	⋯	C_{2n}	S_2
:	:	:	⋯	:	:
: **m**	C_{m1}	C_{m2}	⋯	C_{mn}	S_m
Demand	d_1	d_2	⋯	d_n	

In mathematical form this expressed as

$$\sum_{i=1}^{m} si = \sum_{j=1}^{n} dj \quad \ldots.(1)$$

Total supply = Total demand. This is known as a balanced problem.

The problem is modeled as a linear programming problem

$$Minimise\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij}\ \ldots.(2)$$

Subject to

$$\sum_{j=1}^{m} x_{ij} = s_i, for\ i = 1,2,\ldots,m\ \ \ldots(3)$$

$$\sum_{i=1}^{n} x_{ij} = d_j, for\ j = 1,2,\ldots,n\ \ \ldots.(4)$$

$$xij\ \geq 0, for\ all\ \boldsymbol{i}\ and\ \boldsymbol{j}$$

This special formula is of any linear programming problem of the transportation kind, regardless of its material context. In the framework for many applications the amounts of demand and supply will have integer values and enforcement will demand that integer's allocation amount. In the constraints the unit coefficient of the unknown variables guarantees integer values as an optimum solution.

2.2. Algorithm of modified distribution (MODI) method

Step 1: For an initial basic feasible solution with $(m+n-1)$ occupied (basic) cells, calculate u_i and v_j values for rows and columns respectively using the relationship $C_{ij} = u_i + v_j$ for all allocated cells only. To start with assume any one of the u_i or v_j to be zero.

Step 2: For the unoccupied (non-basic) cells, calculate the cell evaluations or the net evaluations as

$$\Delta_{ij} = C_{ij} - (u_i + v_j).$$

Step 3: a) If all $\Delta_{ij} > 0$, the current solution is optimal and unique. b) If any $\Delta_{ij} = 0$, the current solution is optimal, but an alternate solution exists. *c)* If any $\Delta_{ij} < 0$, then an improved solution can be obtained; by converting one of the basic cells to a non-basic cells and one of the non-basic cells to a basic cell. Go to step 4.

Step 4: select the cell corresponding to most negative cell evaluation. This cell is called the entering cell. Identify a closed path or a loop which starts and ends at the entering cell and connects some basic cells at every corner. It may be noted that right angle turns in this path are permitted.

Step 5: Put a + sign in the entering cell and mark the remaining corners of the loop alternately with – and + signs, with a plus sign at the cell being evaluated. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest one with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. This quantity is added to all the cells on the path marked with plus sign and subtract from those cells mark with minus sign. In this way the unoccupied cell under consideration becomes an occupied cell making one of the occupied cells as unoccupied cell. Repeat the whole procedure until an optimum solution is attained i.e. Δ_{ij} is positive or zero. Finally calculate new transportation cost [7].

2.3. Algorithm of (AMSTP) method

(Advanced Method for Solving the Transportation Problem)

Step 1: Construct the Transportation matrix from given transportation problem, deem rows as (sources) and columns as (destinations). Then form two columns, where first column represents sources and second column represents destinations.

Step 2: Below first column, write down the sources, say $X_1, X_2, X_3, \ldots, X_n$. Then find smallest or minimal value or unit cost for every row, whichever in the respecting column the minimal cost or value is available, chose the value and in terms of destinations record it below the second column.

Column 1	Column 2
X_1	Y_{min}
X_2	Y_{min}
X_3	Y_{min}
⋮	⋮
X_n	

Continue this process for each source. However, if any source has more than one same minimum values in different destination then write these destinations under column 2 and select the cell with minimum supply/demand.

Step 3: Select those rows under column1 which have unique destination then allocate minimum of demand and supply, subtract the allocated from demand and supply. Next delete that row/column where supply/demand exhausted. If tie at the place of minimum value in supply or demand then allocate at the maximum of supply or demand is observed.

However, if destinations are not unique then go to step 4.

Step 4: If destination under column2 is not unique then select those sources where destinations are identical. Next at that destination column find the difference between minimum and maximum unit cost for all those sources at that destination column. However if there is tie in difference for two and more than two destinations then further take the difference between minimum and next to maximum unit cost or if it is not available take the difference between minimum and zero.

Step 5: Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

Step 6: Repeat steps 3 and 4 for remaining sources and destinations till ***(m+n-1)*** cells are allocated.

Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

$$Minimise\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij} \qquad \dots (5)$$

3. Numerical Examples

3.1. Example

	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

3.1.1. Solution

Form two columns, where first column represents sources and second column represents destinations with minimum values.

Column 1 (Sources)	Column 2 (Destinations)	
A	F	
B	**D**	**unique**
C	F	

B has a unique destination at **D** directly allocate the minimum of demand/ supply

	D	**E**	**F**	**Supply**
A	6	4	1	50
B	**20** 3	8	7	40-20=20
C	4	4	2	60
Demand	20-20= 0	95	35	150

Then delete **D** column and again find the minimum destinations for sources.

Column 1	Column 2	Deference
A	F	7 – 1 = 6
B	F	7 – 7 = 0
C	F	7 – 2 = 5

A has maximum deference, allocate the minimum of demand/ supply.

	E	**F**	**Supply**
A	4	**35** 1	50-35=15
B	8	7	20
C	4	2	60
Demand	95	35-35=0	150

Then delete **F** column and again find the minimum destinations for sources.

Column 1	Column 2
A	E
B	E
C	E

	E	**Supply**
A	**15** 4	15
B	**20** 8	20
C	**60** 4	60
Demand	95	150

Number of allocations = *(m + n - 1)*

= 3 + 3 - 1

= 5

Optimal solution = 20 x 3 + 35 x 1 + 15 x 4 + 20 x 8 + 60 x 4

= 555

MODI = 555

3.2. Example

	1	**2**	**3**	**Supply**
A	8	6	9	20
B	6	3	8	30
C	10	7	9	70
Demand	90	20	10	

3.2.1. Solution

Form two columns, where first column represents sources and second column represents destinations with minimum values.

Column 1	**Column 2**	**Deference**
A	2	7 – 6 = 1
B	**2**	**7 – 3 = 4**
C	2	7 – 7 = 0

B has maximum deference, allocate the minimum of demand/ supply. Then delete **2** column and again find the minimum destinations for sources.

	1	**2**	**3**	**Supply**
A	8	6	9	20
B	6	**20** 3	8	30-20=10
C	10	7	9	70
Demand	90	20-20=0	10	

Column 1	Column 2
A	1
B	1
C	3 unique

C has a unique destination at **3** directly allocate the minimum of demand/ supply.

	1	**3**	**Supply**
A	8	9	20
B	6	8	10
C	10	**10** 9	70-10=60
Demand	90	10-10=0	

Then delete **3** column and again find the minimum destinations for sources.

Column 1	Column 2
A	1
B	1
C	1

	1	**Supply**
A	**20** 8	20
B	**10** 6	10
C	**60** 10	60
Demand	90	

Number of allocations = ($m + n - 1$)

= 3 + 3 - 1

= 5

Optimal solution = 20 x 3 +10 x 9 + 20 x 8 + 10 x 6 + 60 x 10

= 970

MODI = 970

3.3. Example

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
F_1	3	1	7	4	300
F_2	2	6	5	9	400
F_3	8	3	3	2	500
Demand	250	350	400	200	

3.3.1. Solution

Column 1	Column 2	
F_1	D_2	unique
F_2	D_1	unique
F_3	D_4	unique

F1, F2 and F3 have a unique destination at D2, D1 and D4 respectively, directly allocate the minimum of demand/ supply.

And delete F1 because it has zero supply and demand.

	D_1	D_2	D_3	D_4	Supply
F_1	3	**300** 1	7	4	300-300=0
F_2	**250** 2	6	5	9	400-250=150
F_3	8	3	3	**200** 2	500-200=300
Demand	250-250=0	350-300=50	400	200-200=0	

Then delete D1, D4 and F1, column and again find the minimum destinations for remaining sources.

Column 1	Column 2
F_2	D_3
F_3	D_2 , D_3

	D_2	D_3	Supply
F_2	6	5	150
F_3	**50** 3	3	300 – 50 = 250
Demand	50 – 50 =0	400	

Delete **D2** and again find the minimum destinations for sources.

Column 1	Column 2
F_2	D_3
F_3	D_3

	D_3	Supply
F_2	**150** 5	150
F_3	**250** 3	250
Demand	350	

Number of allocations = $(m + n - 1)$

$= 3 + 4 - 1$

$= 6$

Optimal solution = 300 x 1 + 250 x 2 + 200 x 2 + 50 x 3 +150 x 5 + 250 x 3

= 2850

MODI = 2850

4. Result and discussion

The comparison among the results of MODI Method and Advanced Method for Solving the Transportation problem is presented in the following table.

Table 2 Results of balanced and unbalanced transportation problems obtained by MODI and AMSTP.

Problems	Problem Dimension	MODI Method	(AMSTP)
1	3X3	555	555
2	3X3	970	970
3	3X4	2850	2850
4	3X4	3500	3500
5	3X3	3900	3900
6	3X3	1320	1200

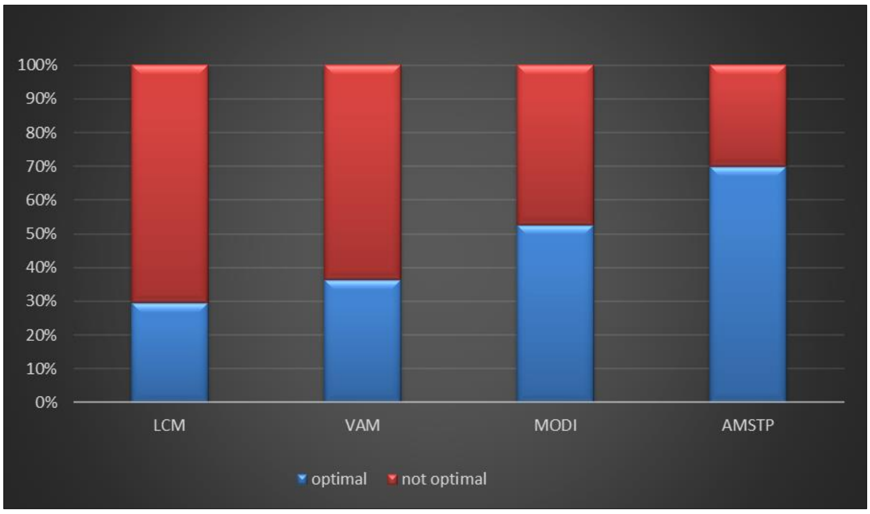

Figure 1 Optimal and non-optimal % comparison of existing methods and Advanced Method for Solving the Transportation Problem (AMSTP)

5. Conclusion

In this study, after successfully applied developed Advanced Method for Solving the Transportation Problem (AMSTP) in different cases of transportation problems balanced and unbalanced numerical examples the conclusion is that it works more efficiency than other methods.

When comparing the results of MODI method and Advanced Method for Solving the Transportation Problem (AMSTP) it is found that MODI methods takes much iterations and time to find optimum solution, therefore the advanced method has been proposed to reduce iterations, save time, and find optimum solution of transportation problems by easy way to understand and to calculate and it getting an optimal solution without solving the Initial Basic Feasible Solution. It is a much faster and more effective tool to treat the transportation problem and provide an optimal solution same as MODI Method and better than other existent methods Figure 4.1.

Advanced Method for Solving the Transportation Problem (AMSTP) can be applied for efficient optimum distribution of combination problems and limited resources, which are related to combinatorial optimization; hence it is possible to solve real-world major scale problems.

Compliance with ethical standards

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Disclosure of conflict of interest

There is no conflict of interest regarding this article.

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