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Evaluating the characteristics of functionally graded plate: A Review study

Raghad Azeez Neamah [1], Zainab Abboud M. [2], Husam Jawad Abdulsamad [1,*], Luay S. Al-Ansari [1] and Marwah Ghazi Kareem [3]

[1] University of Kufa, College of Engineering, Department of Mechanical Engineering, Najaf, Iraq.
[2] AL-Furat Al-Awsat Technical University, Faculty of Kufa Institute, Department of Mechanics, Kufa,Iraq.
[3] Al-Qadisiyah university /College of Engineering/ Department of Materials Engineering, Al-Qadisiyah, Iraq.

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Abstract

Functionally graded materials (FGMs) were developed as heat barrier materials for aircraft structural applications and fusion reactors. They are now being designed for broad usage as structural components in highly harsh settings. FGMs are one of the most efficient and effective materials for achieving long-term industrial development. The present article examines FGMs in general, their applications, and existing processing approaches. It also demonstrates the behavior of FGM-generated plates under five distinct conditions: buckling, bending, vibration, theories, and loadings.

Keywords: Functionally graded material; Bending behavior; Vibration; FGM under different theories; FGM under different loads

1. Introduction

Functionally graded materials (FGMs) have a microstructure that is not uniform, which means that the properties of the materials are constantly changing. FGMs have gained a considerable deal of interest from academics in numerous industries, including aerospace, biomaterials, and engineering, among others, in recent decades due to their unique graded material features. Functionally graded materials (FGMs) have a location-dependent microstructure, chemical composition, or atomic order, which may cause a continuous change in material characteristics such as mechanical, electrical, and thermal properties with position [1].

The significant properties of FGM have made them popular in practically all human endeavors. Functionally graded materials are now employed in various sectors, with considerable potential for future usage in additional applications. FG materials are increasingly recognized as one of the most significant, and efficient materials for improving industrial sustainability. The initial use of FGM was for spacecraft bodies. The use of this innovative material in the aerospace sector has grown throughout the years. Most structures and aeronautical equipment are now constructed using FGM. These included the structure of spacecraft truss, components of rocket engine, heat exchange panels, and specific structures such as solar panels, reflectors, camera housing, turbine wheels and blade coatings, etc. These materials, which combine thermal and acoustic insulation capabilities, are also employed for structural walls. Automobiles are another sector that uses functionally graded materials [2]. The FGM's ability to deliver penetration-resistant qualities by blocking fracture propagation is a significant selling point in the military sector. Bulletproof jackets, traditional Japanese swords, and armor plates are just a few of the defense industry applications for functionally graded materials. Functionally graded materials are also widely used for bulletproof car bodies [3]. The energy sector is always in need of various kinds of functionally graded materials. The inside of nuclear reactors, the thermoelectric converter for changing energy, the solar cells and panel, tubes, and pressure vessels, the graded electrode for making solid oxide fuel

* Corresponding author: Husam Jawad Abdulsamad https://orcid.org/0000-0001-9999-5682

and fuel cell, the dielectric, thermal barrier coat and turbine blade coatings are all places where these materials are used [4].

Due to the high cost of producing functionally graded materials, the application of functionally graded materials in the vehicle industry remains restricted for the time being. However, the material is employed in critical elements of automobiles where the current high cost justifies its usage. Moreover, FGM are used to make better coatings for car bodies, such as graded coatings that contain particles like dioxide or mica [5].

In other side, Bones and teeth are two examples of the functionally graded elements of the human body. These are the most common human body parts to be replaced due to injury or simple wear and tear with age. Biocompatible engineering materials are employed as their substitutes. These synthetics are designed to be direct replacements for naturally occurring components with specific functions. This is why the biomedical industry relies so heavily on functionally graded materials [6]. Many types of athletic equipment, including golf clubs, tennis racquets, and skis, employ functionally graded materials. FGM are used in their fabrication [7]. The electronic and electrical sectors find several applications for functionally graded materials. Among them are the fabrication of sensors, diodes, semiconductors, insulators and the field stress relaxation at the electrode and field-spacer interface [8]. The thermal-shielding parts in the micro-electronics are likewise manufactured from carbon nanotube functionally graded materials.

To produce the thin films of FG coatings, there are many types of deposition processes that are used. These include the chemical, physical vapor deposition process,and (SHS) process, or a combination of some of these process[9].

In this paper, the previous studies will classified according to several important points such as mechanical responses, applied load, boundary condition and theory or method of analysis.

2. Modal Behavior of FGM Plates under Buckling

The equilibrium and stability equations of rectangular functionally graded plates were derived by B. A. Samsam Shariat and M.R. Eslami [10]. The derivations originated from the 3rd order shear deformation theory and linear combination of the component materials. The buckling such plates under various thermal and mechanical stresses, different load ratio, and different aspect ratio is studied. They determine that the critical buckling stresses of FG plate typically drop as the aspect ratio (a/b) rises, but they increase as the relative thickness (h/b) grows. The plate exhibits more buckling with temperature variations throughout its thickness than a homogeneous temperature increase.

The behavior of thermal buckling for aluminum alumina plates with rectangular cross section in a thermal environment is examined by Bouazza MOKHTAR et al. [11]. They used first-order shear deformation theory for their analysis. The thermal load is considered to be uniformly distributed, with a linear and sinusoidal temperature increase in the direction of thickness. During this analysis, the raising in geometric parameter significantly reduces the temperature gradient. Decreasing the aspect ratio reduces the required temperature.

S.M. Kazerouni [12] provided a precise mathematical method for analyzing the thermal buckling of thin rectangular plates made of functionally graded (FG) materials. The researcher derives the stability equations by utilizing the concept of the lowest total potential energy, which is based on the classical plate theory. The stability equations in FG materials, including both in-plane and out-of-plane displacements, are interconnected due to the dependence of material characteristics on the coordinates, particularly the thickness. A novel analytical technique transforms the linked stability equations into separate, autonomous ones. The effect of aspect ratio, index value, and boundary conditions on the critical buckling temperature of FG rectangular plates is studied.

Hamid Mozafari and Amran Ayob [13] investigated the buckling load of FG plate with a thickness that varies linearly. They conducted the derivations using the first and third-order shear deformation theories basing on the power law model. The results showed that the increase in the index value reduces the critical buckling load of FG plate. Furthermore, the increase in aspect ratio leads to an augmented acute buckling stress of FG plate.

The analytical formulations and solutions for the buckling analysis of supported functionally graded plates (FGPs) is provided by B. Sidda Reddy et al. [14]. They used the higher-order shear deformation theory (HSDT) and did not impose zero transverse shear stresses on the top and bottom surfaces of the plate. Shear-correction factors are not necessary. Shear stresses exhibit a parabolic distribution throughout the thickness. The volume fractions of its parts are expected to determine a power law distribution in the thickness direction for the plate's material characteristics. They obtain the equations of motion and boundary conditions using the virtual work concept. Analytical solutions for FGPs are obtained using Navier's method to produce closed-form solutions. Comparative investigations validate the accuracy and efficiency of the current findings, leading to the conclusion that the proposed theory effectively predicts the buckling behavior of functionally graded plates. The study also looks at and talks about how the critical buckling load of FGPs is affected by the aspect ratio, modulus ratio, volume fraction exponent, side-to-thickness ratio, and loading conditions.

Z.X. Lei et al. [15] looked at how FG carbon nanotube-reinforced composite plates buckled when put under different mechanical stresses. An investigation to assess the effectiveness and precision of the current approach in analyzing the buckling behavior of composite plates reinforced with single-walled carbon nanotubes (SWCNTs) is presented. The investigation evaluates the effectiveness and precision of the current approach for analyzing the buckling behavior of composite plates reinforced with single-walled carbon nanotubes (SWCNTs) through comparative research and numerical simulations using different parameters. The findings show that changes in the temperature, the loading condition, the ratio of plate width to thickness, the aspect ratio of the plate, and the volume percentage of carbon nanotubes have a big effect on the buckling strength. Tahar Hassaine et al. [16] introduced a streamlined and straightforwardly improved theory for analyzing the buckling behavior of functionally graded plates. The theory is a lot like classical plate theory in a lot of ways. It explains why transverse shear strains change in a quadratic way across the thickness of the plate. Power law model is used to distribute the material in thickness direction. They obtain the governing equations by utilizing the concept of minimal total potential energy. Researchers derive analytical solutions for rectangular plates. They conduct comparison studies to validate the accuracy of the current findings. They investigated and discussed how loading circumstances and changes in the power of functionally graded material, modulus ratio, aspect ratio, and thickness ratio impact the critical buckling load of functionally graded plates. Their study found that the current theories are accurate and efficient in predicting the critical buckling loads of FGM plates.

Korosh Khorshidi and Abolfazl Fallah [17] analyzed the buckling behavior of a nanoplate made of functionally graded material using the exponential shear deformation theory. The hypothesis in this study is based on the traditional plate theory. This theory utilizes exponential functions concerning the thickness coordinate, including the influence of transverse shear deformation and rotating inertia. The number of unknown displacement variables in the suggested theory is equivalent to that in the first-order shear deformation theory. The study uses nonlocal elasticity theory to look at how small scales affect the way a rectangular nanoplate with functional grading bends. The Power Law equation suggests that the plate's material characteristics exhibit variation in the thickness direction. The implementation of Hamilton's principle obtains the governing equations and accompanying boundary conditions. The study examines the impact of many factors, such as the nonlocal parameter, Power Law indexes, aspect ratio, and thickness ratio, on the rectangular FG nanoplate's non-dimensional critical buckling stress. The findings indicate that the critical buckling load decreases as the nonlocal parameter value increases. Additionally, an increase in the power law index leads to a decrease in the non-dimensional critical buckling. Furthermore, an increase in the aspect ratio increases the critical buckling load. At the same time, an increase in the length-to-thickness ratio also increases the critical buckling load.

Wang Xin et al. [18] derived the equation of the critical load of dynamic buckling of FG plates. The findings indicate that the dynamic buckling essential experiences an exponential drop as the length increases. In contrast, the buckling critical load reduces with a rise in the gradient index k. The buckling critical load is significantly affected by the gradient index k while the boundary conditions have a big effect on the mode of buckling of FG plate.

The analytical and numerical methods to look at how FG plate buckles when it is compressed in one direction are used by Elias Y. Ali et al. [19]. These properties can be explained by power law, sigmoid, and exponential functions, which are based on the volume percent of the metal and ceramic materials that make up the plate. The buckling analysis of FGM plates uses an analytical and theoretical approach using the first-order shear deformation theory (FSDT). The Galerkin method, a reliable technique for solving differential equations, is used to solve the eigenvalue issue and determine the stability of the FGM plate. They verified and validated the mathematical formulation using numerical analysis with ABAQUS. After that, parametric analyses examine various material models, aspect ratios, length-to-thickness ratios, and plate boundary conditions. Numerical simulations determined the optimal material composition of the functionally graded plates. The findings of this study showcase the possible utilization of FGMs as thin-walled structural elements in the future advancement of durable and eco-friendly structural components and systems.

Nihat Can et al. [20]calculated the critical buckling loads of rectangular (FG) plates under different boundary circumstances. The finite element method is used to analyze the stability of plates and to determine the critical buckling loads of a plate under two different types of boundary conditions: (a) CFFC, where two parallel edges are clamp and leave the other two free; (b) FFFC, where they clamp one edge and leave all the others free. The longitudinal and transverse variations in the FG plate's mechanical characteristics have been considered—a case where the rectangular plate is uniform throughout—to substantiate the suggested model. The critical buckling loads for the CFFC design were lower than those for the FFFC arrangement. The latter is more prone to instability. This applies to plates with the same composition throughout and to plates with different compositions in different regions.

The dynamic buckling analysis on an arch of a composite material reinforced FG graphene nano platelets GPLs is studied by Zhicheng Yang et al. [21]. The arch was subjected to a step-central point stress applied at its center. The arch consists of many layers strengthened by graphene nanoplatelets. Each layer uniformly disperses these GPLs, but the weight percentage of GPLs varies along the thickness direction from layer to layer. The Halpin-Tsai micromechanics model forecasts the practical material characteristics of each layer in the GPLRC. The results showed that all quantities of GPLs as reinforcing nanofillers significantly enhances the emotional stability of the arch.

Leila Monajati [22] presented a technique that utilizes the airy stress function and refined plate theory of FG plates. The vibration and buckling characteristics for this plate is studied. The most important thing about the suggested method is that it only looks at two variables in the displacement field. This makes it easier to see how shear and bending affect the overall transverse displacement. They showed that the using the Airy stress function decreases the number of variables, hence streamlining the dynamic model. Therefore, it can be used to comprehensively investigate the static and dynamic characteristics of FG plates.

The Summary of the previous papers are listed in Table (1).

Table 1 Summary of FGM Plates under Buckling Load

Ref. No.	Variable	Theory	Load and parameters
[10]	the Mechanical and thermal buckling of rectangular functionally graded plates	shear deformation theory	Different types of mechanical and thermal loads. Different load ratio and different aspect ratio.
[11]	thermal buckling of aluminum-alumina plates with rectangular cross section	classical plate theory	Uniformly distributed, with a linear and sinusoidal temperature. Aspect ratio.
[12]	thermal buckling of thin rectangular plates	classical plate theory	Aspect ratio, index value, and boundary conditions on the critical buckling temperature.
[13]	mechanical buckling load of FG plate	first and third-order shear deformation theories	Power law index and aspect ratio
[14]	mechanical buckling analysis of supported functionally graded plates	higher-order shear deformation theory	aspect ratio, modulus ratio, volume fraction exponent, side-to-thickness ratio, and loading conditions
[15]	the buckling behavior of composite plates reinforced with single-walled carbon nanotubes	Eshelby–Mori–Tanaka approach and numerical simulation	temperature, loading condition, the ratio of plate width to thickness, the aspect ratio of the plate, and the volume percentage of carbon nanotubes
[16]	Mechanic buckling behavior of functionally graded plates	classical plate theory	power -law, modulus ratio, aspect ratio, and thickness ratio
[17]	buckling behavior of a nanoplate made of functionally graded material	exponential shear deformation theory	Power Law indexes, aspect ratio, and thickness ratio, on the rectangular FG nanoplate's non-dimensional critical buckling stress
[18]	the critical load of dynamic buckling of FG plates	trial function Method	critical length, gradient index, modal order, and the elastic modulus of constituent materials and boundary conditions
[19]	The buckling analysis of FGM plates	analytical and numerical methods	Various material models, aspect ratios, length-to-thickness ratios, and plate boundary conditions.
[20]	The critical buckling loads of rectangular (FG) plates	finite element method	Two different types of boundary conditions: CFFC and FFFC,

[21]	the dynamic buckling analysis on an arch of a composite material reinforced FG graphene nano platelets	Halpin-Tsai micromechanics model and energy-based methods	critical geometric parameters, FG graphene nano platelets distribution, concentration, and dimension of FG graphene nano platelets
[22]	buckling characteristics of FG plates	The airy stress function and refined plate theory	shear and bending affect the overall transverse displacement

3. Modal Behavior of FGM Plates under Bending

Using a combination of independent geometric analysis (IGA) and higher-order shear deformation theory (HSDT), Tran et al. [2] derived the equilibrium and stability equations for a functionally graded material (FGM) plate under heat conditions. The FGM plate consists of two separate components, with material qualities that continuously change in thickness according to a power-law distribution and depend on temperature. The temperature field is considered to be constant in any plane and uniform, linear, and nonlinear along the thickness of the plate, respectively. The von Karman assumption and thermal effects contribute to the nonlinearity of the governing equation. The current plate model requires strict C1 continuity, easily achieved using a NURBS-based geometric finite element formulation. The impact of gradient indices, boundary conditions, temperature distributions, material qualities, and length-to-thickness ratios on the behavior of FGM plates is thoroughly examined.

The numerical findings show the exceptional performance of the current methodology. Zhan Zhao et al. [23] used the finite element technique to study the bending and vibration characteristics of a new kind of trapezoidal plate. These plates were reinforced with graphene nanoplatelets (GPLs) and had functionally graded properties. The modified Halpin-Tsai model and the rule of mixing are used to ascertain the effective material characteristics of the nanocomposites, such as Young's modulus, mass density, and Poisson's ratio. An extensive parametric analysis is carried out to investigate the impact of the distribution, concentration, and dimensions of graphene platelets (GPL), as well as the geometry of the plate, on the static and dynamic characteristics of functionally graded trapezoidal plates reinforced with GPL. The findings indicate that using a modest quantity of GPLs as reinforcing Nano fillers may significantly improve the rigidity of the plate. The optimal reinforcing effect is produced by dispersing a greater number of GPLs with a more extensive surface area along the upper and lower surfaces of the plate. More than that, this distribution pattern makes the bending and vibration properties of trapezoidal plates more sensitive to both the weight percent of GPL and the shape of the plate when compared to other distribution patterns. Furthermore, reducing either of the two base angles decreases plate static and dynamic deflections. Mitao Song et al. [24] analyzed the static bending and compressive buckling of composite plates made of functionally graded multilayer graphene Nano platelets (GPL) and polymers. These analyses were carried out using first-order shear deformation theory. The GPL weight fraction of exhibits a gradual variation in thickness direction, with GPLs evenly distributed throughout the polymer matrix in each layer. The Halpin-Tsai micromechanics model estimates the nanocomposites' effective Young's modulus, while the rule of mixing establishes their effective Poisson's ratio. The Navier solution approach gives mathematical answers for the static deflection and critical buckling load of GPL/polymer plates that are supported and functionally graded. The shape, size, weight percentage, and distribution pattern of the GPL have a big effect on how the functionally graded GPL-reinforced nanocomposite plate bends and buckles.

Ashraf M. Zenkour and Rabab A. Alghanmi [25] studied how a functionally graded plate with two supported edges that are opposite each other bends using a new quasi-3D theory that accounts for both shear and normal deformation with a third-order shape function. The current theory accounts for the inclusion of transverse normal strains. In contrast to conventional ordinary and shear deformation theories, the current theory involves four unknowns. The use of the notion of virtual labor obtains the equilibrium equations. Merdaci Slimane [26] used a high-order shear deformation theory to look into how porous ceramic-metal functionally graded (FG) rectangular plates bend. The proposed theory differs from prior theories in that it involves four unknowns instead of five. However, it effectively verifies the boundary conditions without limiting the upper and lower plate surfaces. The proposed theory incorporates shear strain and normal deformation, making a shear correction factor unnecessary. He employs Navier's methodology to obtain the equilibrium equations for the ceramic-metal porous FG plates. He solves the issue by employing Navier's methodology. He has documented and juxtaposed numerical findings with existing data about impermeable plates in the public domain. He conducted an investigation on the effects of the exponent grade and porosity variables.

The summary of the previous papers listed in Table (2).

Table 2 Summary of FGM Plates under Bending

Ref. No.	Variable	Theory	Load and parameters
[2]	Bending behavior of FG Plate considering Thermal effect	Independent geometric analysis (IGA), higher-order shear deformation theory (HSDT) and NURBS-based geometric finite element formulation.	gradient indices, boundary conditions, temperature distributions, material qualities, and length-to-thickness ratio
[23]	the bending and vibration characteristics of a new kind of trapezoidal functionally graded plate	modified Halpin-Tsai model and finite element technique	weight percent of graphene platelets and the shape of the plate
[24]	The static bending and compressive buckling of composite plates made of functionally graded multilayer graphene Nano platelets (GPL) and polymers	The Halpin-Tsai micromechanics model, Navier solution approach and first-order shear deformation theory	The shape, size, weight percentage, and distribution pattern of the GPL
[25]	transverse normal strains	new quasi-3D theory	Supporting type
[26]	Bending behavior of porous ceramic-metal functionally graded rectangular plates	high-order shear deformation theory	The boundary condition, exponent grade and porosity variables

4. Modal Behavior of FGM Plates under Vibration

Senthil S. Vel and R.C. Batra [27] discussed the oscillations of functionally graded rectangular plates supported at both ends when they occur naturally and when external forces induce them. Displacement functions that fulfill boundary conditions reduce the equations controlling steady-state vibrations on a plate to a set of coupled ordinary differential equations. They solve these equations using the power series technique. The precise answer applies to thick and thin plates and arbitrary variations in material characteristics in the thickness direction. The study shows that the volume fractions of metal and ceramic components in rectangular plates change in a way that follows a power law across the thickness of the plates. One can estimate the practical material characteristics at a specific position using either the Mori-Tanaka or the self-consistent techniques. The correctness of the classical plate theory, the first-order shear deformation theory, and a third-order shear deformation theory for functionally graded plates is judged by comparing their exact natural frequencies, displacements, and stresses. Parametric analyses investigate the effects of different ceramic volume fractions, volume fraction profiles, and length-to-thickness ratios. The researchers calculate the results for a functionally graded plate with a changing microstructure in the thickness direction using Mori-Tanaka and self-consistent techniques. Examining the oscillations of a plate under a sinusoidal pressure distribution applied to its upper surface. S. Natarajan [28] investigated the natural frequency of a FG plate with cracks using the extended finite element technique. The material is presumed to exhibit temperature dependency and is graded toward its thickness. Computational simulations were performed to investigate the impact of index, aspect ratio, crack length, orientation, and positio on the natural frequency of the FG plate. The study also investigates the impact of several fractures and their respective direction on the natural frequency. The findings demonstrated that elongating the fracture length leads to a reduction in the natural frequency.

JANGHORBAN Maziar and ROSTAMSOWLAT Iman [29] used the finite element approach to examine the natural oscillation of functionally graded plates with several circular and non-circular cuts. The volume fraction of the material ingredients follows a straightforward power law distribution. The criteria under consideration include the dimensions of the cutout, its placement, and the quantity of cutouts. To date, no investigation has been conducted on the phenomenon of free vibration in FG plates, which includes rectangular, skew, trapezoidal, and circular plates with multiple cuts. Therefore, the findings presented in this study might serve as reference points for future research.

Holm Altenbach and Victor A. Eremeyev [30]examined how transverse shear stiffness affects the static behavior of FGM plates. They conduct the analysis of FGM plates by considering both the transverse shear stiffness and the rotatory inertia in a dynamic manner. Specifically, non-zero rotatory inertia accurately characterizes the highly oscillating solutions. The limits for functionally graded material's (FGM) natural frequencies are determined. The proposed method for modeling FGM plates, based on a 5-parametric theory of plates, offers an advantage over theories that deal with sandwich or laminated plates. These other theories do not consider transverse shear stiffness and rotatory inertia or rely on a fixed shear correction. The presented approach has the advantage of providing more accurate results and allowing for obtaining results that cannot be achieved using the classical approach.

Lei et al. [31] looked at the natural vibrations of functionally graded nanocomposites that were made stronger by single-walled carbon nanotubes (SWCNTs). They used the element-free kp-Ritz technique for this research. They are considering various distributions of uniaxially aligned single-walled carbon nanotubes (SWCNTs). FG-CNTRCs exhibit different properties along the thickness direction, which depend on the distribution of volume percentage of carbon nanotubes. They get the governing equations from the first-order shear deformation plate theory and use mesh-free kernel particle functions to get close to the two-dimensional displacement fields. The researchers did convergence and comparative experiments to make sure that our current method for looking at the free vibration of different types of CNTRC plates is stable and accurate. There are many examples in the computer simulation that show how changes in temperature, the number of carbon nanotubes in the plate, the ratio of plate width to thickness, and the ratio of plate aspect ratio can affect the natural frequencies and mode shapes of different types of FG-CNTRC plates. In addition, the study includes results for uniformly distributed (UD) CNTRC plates for comparison. They also investigate the impact of boundary conditions.

Mitao Song et al. [32] studied the free and forced vibrations behave of FG multilayer graphene nanoplatelet polymer composite plates. The difference in the weight percentage of GPL nanofillers along the thickness direction is studied. The effect of distribution of pattern, weight percentage, shape, size, and layers number of GPL is studied extensively.

Avadesh K. Sharma et al. [33]conducted a study on analyzing free vibrations in functionally graded plates with several circular cuts. They used the finite element approach to analyze the natural oscillation of functionally graded plates. The volume fraction of the components is expected to determine the power law distribution of the plates' material characteristics. Researchers conducted a study to examine how the volume fraction index, thickness ratio, and various boundary conditions affect the natural frequencies of plates. The results show that the natural frequency of rectangular, trapezoidal, and circular FG plates with circular cuts goes down as the volume fraction index goes up.

Ankit Guptaa et al. [34], developed the numerical solution for analyzing the free vibration of FG plates supported on an elastic base. They characterize the foundation using the Pasternak model. In thickness direction, material distributed based on power law model. He effect of index, geomety, and foundation parameters is studied. The findings demonstrate that the Pasternak elastic basis substantially impacts the inherent frequency.

Amir Nasirmanesh and Soheil Mohammadi, both aged [35], analyzed the vibration of cracked FGM shells. Through the simulation of different problems, it showed that this method can be address the problem of vibration analysis in FG plate and shell structures.

In their study, Yazdani Sarvestani et al. [36] used the couple stress theory and FSDT to examine how material and geometrical characteristics impact the free and forced vibration of size-dependent FG (functionalally graded) doubly-curved panels. They also took into account various sets of boundary conditions. Researchers use the classical mechanics homogenization approach to evaluate the effective mechanical characteristics of FG doubly curved panels. The nano-panels under consideration include spherical, saddle-shaped, cylindrical, and flat panels. Different materials, aspect ratios, curvatures, and length scales are looked at in this study to see how they affect the fundamental frequencies of size-dependent doubly-curved panels made of FG materials. The aim is to provide a reliable foundation for building advanced panels at the nano/micro scale by considering various boundary conditions. The results show that for size-dependent functionally graded doubly-curved panels, a longer scale leads to a higher fundamental frequency. Two-phase functionally graded materials (FG) panels with square inclusions have up to 3% higher fundamental frequencies than FG panels with circular inclusions. The difference is due to the size of the inclusions.

A study by A. Aliour Ghasabí et al. [37] used a nonlocal elasticity-based method to look at the free vibration of rectangular nanoplates with different functional properties. The presented technique may effectively consider the spatial variability of the nonlocal parameter. They obtain the governing equations and boundary conditions by using the variational technique and implementing Hamilton's principle. The derivations imply that all material characteristics, including the nonlocal parameter, vary as functions of the thickness coordinate. The formulation generates findings for three plate theories by coherently expressing the displacement field. They solve the obtained equations using the extended differential quadrature technique. The numbers show how the dimensionless plate length, the nonlocal parameter ratio, and the power-law index affect the natural vibration frequencies of nanoplates that are supported and those that are cantilevered. The vibration frequencies calculated for the three different plate theories show that the KPT theory predicts the nanoplate to have higher natural frequencies and behave more rigidly. The findings obtained for the

Timoshenko shear deformation theory (TSDT) fall within the range of values computed for the Kirchhoff and Mindlin plate theories. When the plate length without dimensions extends in all scenarios, the vibration frequencies rise and converge toward stable levels.

Şeref Doğuşcan Akbaş is [38] examined the natural oscillation and fixed deformation of a functionally graded (FG) plate with porosity supported at both ends. The plate's material qualities associated with its alteration vary depending on its location. By using Hamilton's principle within the framework of the first-order shear deformation plate theory, they derive the governing equations of the FG plate. The author resolves the issue using the Navier solution. This research examines the impact of porosity and material distribution characteristics on the static and vibration responses of the FG plate. It provides a detailed analysis and discussion of the findings. The findings indicate that the material distribution and porosity parameters play a crucial role in determining the static and vibration responses of the FG porous plates. The disparity between the uniform and uneven porosity models widens as k and parameters grow.

A.K. Sharma et al. [39] conducted a comprehensive examination of the harmonic and vibration properties of FG plates. They used the finite element approach for this purpose. Initially, the researchers assume that the plates have uniform characteristics in all directions, and they expect the material properties to smoothly vary across the thickness based on a power-law distribution. Four plate are examined in this study to determine their suitability for use in high-temperature environments and to reduce vibration amplitudes in applications. The modal analysis shows that the natural frequencies of the Ti–6Al–4V/Aluminum oxide and SUS304/Si3N4 plates are very similar. The Al/Al2O3 plate has the highest frequencies, while the Al/ZrO2 plate has the lowest. The vibration amplitude grows proportionally to the volume fraction index for all boundary conditions. The harmonic analysis results show that the plate made of Al/ZrO2 has the highest amplitude values, while the plate made of Al/Al2O3 has the lowest values. Furthermore, providing the damping constant results in a decrease in the maximum amplitude at the resonance point, with this reduction becoming more pronounced as the damping constant increases. Therefore, it is crucial to take into account the damping constant while designing the plate. It aids in mitigating vibrations, reducing noise, enhancing system stability, and improving resilience to fatigue and impact.

Ta Duy Hien and Nguyen Ngoc Lam [40]looked at how functionally graded (FG) rectangular plates on a viscoelastic foundation moved when loads were put on them. The plate's material qualities change constantly in the direction of thickness, following a power-law relationship. Applying Hamilton's principle, which considers the influence of the higher-order shear deformation in the plate, yields the governing equations. The authors analyze the transient reactions of simply supported rectangular plates made of functionally graded (FG) materials using state-space approaches. The text provides several instances of displacement and stresses in plates with different structural factors and analyzes the impact of these parameters. The findings suggest that the damping coefficient of the foundation has a substantial impact on the dynamic response of the FG plate. The foundation's damping coefficient facilitates the disposal of energy, hence reducing ambient vibration during forced and free vibrations.

In their study, Narayanan, N. I., et al. [41]used the finite element method to analyze the vibrations of FG plates. They conducted the analysis by constructing a computer program using MATLAB. They use a 9-noded heterosis element to simulate the FGM plate because it has better properties than the 8-noded serendipity and 9-noded Lagrange elements. The heterosis element has superior attributes in comparison to the 8-noded serendipity and 9-noded Lagrange elements. It provides a high degree of precision for exceedingly thin plate designs. They conducted a convergence analysis to ensure the numerical findings converged. They obtained the numerical findings using Abaqus CAE and S8R5 shell elements, which demonstrated excellent agreement with the designed elements. The purpose of doing free vibration analysis is to examine the various modes and frequencies. Carbon plates exhibit minimal free vibration response, while silicon carbide plates demonstrate maximal response. The center deflection of the plate rises proportionally with the increase in volume fraction index, regardless of the kind of boundary circumstances.

In their study Hoang X. et al. [42] the strain gradient theory is used to incorporate a single material length component and an extra micro-inertia term to represent the impact of size. The refined plate theory describes the movement of the plates using four unknowns. The XIGA method incorporates enrichment functions to accurately forecast the behavior of microplates with cracks. The is geometric analysis (IGA) method ensures accurate and effective fulfillment of the strain gradient theory's requirements for more continuity by utilizing very smooth non-uniform rational B-spline (NURBS) basis functions. The benchmark numerical findings show a substantial deviation from those examined using classical continuum elasticity. The conventional theory does not account for the significant impact of microstructural properties on the vibration responses of microplates. These impacts grow more prominent as the size of the plates approaches the material length parameter.

Bourihane et al. [43]examined the dynamic behavior of FG plate under forced nonlinear conditions. They used a high-order implicit algorithm to study the plate's response to external dynamic stresses, both numerically and theoretically. They use the TSDT model to create the plate kinematics, including third-order shear deformation. They write the formulation in this work without employing any homogenization approach, such as the rule of mixing or treating Functionally Graded Materials (FGM) as a laminated composite. To address integrals involving inhomogeneous Functionally Graded Materials (FGMs), they divide the stress vector into four components and express them in a dimensionless manner. The authors use the Hamiltonian principle to derive the equations of motion. The theoretical aspect of this contribution is shown by using the Third-Order Shear Deformation Plate Theory (TSDT). They perform the numerical resolution using a high-order implicit algorithm based on the concepts of the ANM approach. A comparison of the obtained findings with those found in the literature and laminate composite modeling substantiates the performance and efficacy of the suggested approach.

Laith O. Mazahreh and Ibrahim M. Abu-Alshaikh [44] under various boundary conditions, the free vibration analysis of two-dimensional FG plate is studied. This plate consisted from three layers. The middle layer is represented using the third-order shear deformation theory, while the upper and lower isotropic layers are characterized using the first-order shear deformation theory. For this purpose, a quadrilateral element with 8 nodes and 13 degrees of freedom per node is utilized. The researchers conducted an analysis to examine the effects of several plate characteristics, such as applied boundary conditions, volume fraction exponents, and plate side to thickness ratio.

J.R. Cho [45]conducted an investigation on the free-vibration characteristics of a sandwich plate with a homogenous core made of FG materials. The study focuses on building a reliable and efficient numerical approach. The numerical technique used hierarchical models derived from the spectrum model accuracy and the 2-D natural element method (NEM). Using the principles of 3-D elasticity, the hierarchical models were obtained, while the NEM distinguished itself through the use of very smooth interpolation functions. Based on the verification trials, the suggested technique demonstrates a strong correlation with the reference and a consistent convergence to the 3-D elasticity.

V. Kumar et al. [46] introduced a non-polynomial hyperbolic theory to analyze the free vibration behavior of a rectangular FG plate with a linearly changing thickness. Exponentially, material qualities vary in the thickness direction. Galerkin's approach is used to solve the Eigen value issue of the proposed model. They examined the effects of thickness change on a plate, assuming it is both simply supported and clamped, using different span ratios, aspect ratios, taper ratios, and foundation stiffness. When the taper and aspect ratio rise, the frequency also increases dramatically. Moreover, the escalation of edge limitations results in a substantial augmentation of frequency parameters. Another significant discovery about elastic foundations is that the impact of Pasternak's foundation is more significant than that of Winkler's foundation.

The summary of the previous papers are listed in Table (3).

Table 3 Summary of FGM Plates under Vibration Behavior

Ref. No.	Variable	Theory	Load and parameters
[27]	the oscillations of functionally graded rectangular plates	the power series technique, the Mori-Tanaka and self-consistent techniques, classical plate theory, the first-order shear deformation theory, and a third-order shear deformation theory	different ceramic volume fractions, volume fraction profiles, and length-to-thickness ratios.
[28]	the natural frequency of a FG plate with cracks	The extended finite element technique.	Material index, aspect ratio, the length, orientation position of crack
[29]	the natural oscillation of functionally graded plates several circular and non-circular cuts	finite element approach	several circular and non-circular cuts
[30]	transverse shear stiffness and a rotatory inertia effect on the static and dynamic behavior of FGM plates	5-parametric laminated plate theory	comparison

[31]	the natural vibrations of functionally graded nanocomposites	the element-free kp-Ritz technique, first-order shear deformation plate theory,	Temperature effect, the number of carbon nanotubes in the plate, the width to thickness ratio, the aspect ratio, and boundary conditions
[32]	free and forced vibrations behave of FG multilayer graphene nanoplatelet polymer composite plates	modified Halpin-Tsai model and Navier solution based technique	distribution of pattern, weight percentage, shape, size, and layers number of graphene nanoplatelet (GPL)
[33]	free vibrations in functionally graded plates with several circular cuts		the volume fraction index, thickness ratio, and various boundary conditions
[34]	free vibration of FG plates supported on an elastic base	Pasternak model and finite element approach	Material index, geometry, and foundation parameters
[35]	vibration of cracked FGM shells	Extended Finite Element Method	the length and angle of the crack and different distribution patterns of material stiffness and density across the thickness of the shell
[36]	free and forced vibration of size-dependent FG (functionalally graded) doubly-curved panels.	couple stress theory and first-order shear deformation theory	Size of panel and boundary conditions
[37]	the free vibration of rectangular nanoplates	nonlocal elasticity-based method, extended differential quadrature technique, Timoshenko shear deformation, Kirchhoff theory, Mindlin plate theory, variational technique and Hamilton's principle	the dimensionless plate length, the nonlocal parameter ratio, and the power-law index
[38]	the natural frequency and fixed deformation of a functionally graded plate with porosity supported at both ends.	Hamilton's principle, the first-order shear deformation plate theory and Navier solution	the material distribution and porosity parameters
[39]	harmonic and vibration properties of FG plates	finite element approach	comprehensive study
[40]	forced and free vibration of functionally graded (FG) rectangular plates on a viscoelastic foundation	Hamilton's principle, state-space methods, and higher-order shear deformation theory	damping coefficient of the foundation
[41]	the vibrations of functionally graded plate	the finite element method	volume fraction index and kind of boundary circumstances
[42]	vibration responses of micro-plates	strain gradient theory, refined plate theory	material length parameter
[43]	dynamic behavior of FG plate under forced nonlinear conditions	Third-Order Shear Deformation Plate Theory, high-order implicit algorithm	the performance and efficacy of the suggested approach
[44]	the free vibration analysis of two-dimensional FG plate	third-order shear deformation theory, first-order shear deformation theory and the finite element method	boundary conditions, volume fraction exponents, and plate side to thickness ratio.
[45]	the free-vibration characteristics of a sandwich	2-D natural element method	Convergence the numerical method

	plate with a homogenous core made of FG materials.		
[46]	the free vibration behavior of a rectangular FG plate with a linearly changing thickness.	non-polynomial hyperbolic theory, Galerkin's approach	Boundary conditions, span ratios, aspect ratios, taper ratios, and foundation stiffness.

5. Modal Behavior of FGM Plates under different theories

A study by Ramu I and Mohanty S.C[47] used the Finite Element Method (FEM) to find the natural frequencies and mode shapes of a functionally graded material (FGM) plate. Functionally graded material is characterized by a gradual variation in composition and structure across its volume, resulting in matching changes in its component materials. A Functionally Graded Material (FGM) plate exhibits continuous variation in its mechanical characteristics throughout its thickness direction, following a power law relationship. A MATLAB application has been built to do modal analysis on a plate made of Functionally Graded Material (FGM). Several situations have been resolved, and their outcomes are compared to the existing findings in the literature. The authors determine the mode shape and natural frequencies of a rectangular plate made of functionally graded material (FGM) under various boundary circumstances. They noted the impact of the volume fraction index, which represents the proportion of ceramic and metal components in the FGM. Additionally, the authors examine the influence of the power law index on the natural frequency and mode forms of the FGM plate under various boundary conditions.

J. N. Reddy et al. [48] created a finite element models to analyze the axisymmetric bending of circular plates. These models look at the strain gradient effects, the von Karman nonlinearity, and the through-thickness power-law variation of a two-constituent material. Researchers have developed the models for both classical and first-order theory kinematics. The updated couple stress theory incorporates the strain gradient effects by introducing a single material length scale parameter. This parameter effectively accounts for the size impact on a functionally graded material plate. Geometric nonlinearity, power-law index, and microstructure-dependent constitutive relations are assessed for their impact on the bending behavior of functionally graded circular plates using the implemented finite element models.

Table 4 Summary of FGM Plates using different theories

Ref. No.	Variable	Theory	Load and parameters
[48]	the bending behavior of functionally graded circular plates	classical theory, first-order theory, updated couple stress theory and finite element models	Geometric nonlinearity, power-law index, and microstructure-dependent constitutive relations
[49]	the static analysis of functionally graded plates	hyperbolic shear deformation theory, finite element solution	Boundary conditions, aspect ratios, power law indexes, and span-to-thickness ratios.
[50]	thermal stresses of simply supported FG plates		deflection, stress, strain, axial force, and bending momen
[51]	the fracture of a FG plate	Boundary Element Method	the effectiveness of evaluating the stress intensity factor for flat plates

Kamlesh Kulkarni and colleagues [49]conducted a numerical study on the static analysis of functionally graded plates using inverse hyperbolic shear deformation theory. The theory has excellent performance even when used on thick plates. The finite element solution allows applying it to various boundary conditions and loading circumstances. The current approach may also be used to examine the influence of boundary conditions, aspect ratios, power law indexes, and span-to-thickness ratios. The findings generated by IHSDT exhibit a high degree of accuracy when compared to other two-dimensional theories.

Yen-Ling Chung [50] analyzed simply supported FG plates that subjected to thermal stresses. They used the idea of medium-thick plates as their starting point. The main goal of this study is to look at the FGM plate's deflection, stress, strain, axial force, and bending moment when it is heated in both the thickness and axial directions. FGM causes the highest level of stress to shift from the upper or lower surface towards the interior region of the FGM plate, resulting in a substantial reduction in the maximum stress experienced by the plates. Furthermore, careful selection of the appropriate material gradation can immediately determine the closed-form solution generated, allowing for zero bending of the FGM plate despite the thermal stress applied in the direction of its thickness.

Simone Palladino et al. [51] analyzed the fracture of a FG plate by using Field Boundary Element Method. The objective was to determine the Mode I Stress Intensity Factor (SIF). The transversely isotropic plane plate serves as the subject of research in this Field Boundary Element Method instance. The elasticity tensor of the material exhibits an exponential variation that is dependent on a scalar function of position. The approach of employing a parametric model of the structural reaction demonstrates the effectiveness of evaluating the stress intensity factor for flat plates made of functionally graded materials, even when dealing with more intricate geometries. The summary of the previous papers are listed in Table (4).

6. Modal Behavior of FGM Plates Under different loading

Yen-Ling Chung and Wei-Ting Chen (2007) [52] analyzed the bending of FG plates that subjected to a uniform load. Fourier series expansions that used in this study to find an analytical solution to the problem of FGM plate bending, which agrees very well with a finite element calculation. The analysis shows that the lower region of the FGM plates experiences the highest tensile stresses. Nevertheless, the highest compressive stresses migrate towards the interior of the FGM plates.

In their study, X.R. Liu et al[53] examined all metallic sandwich plates' dynamic responses and blast resistance with functionally graded close-celled aluminum foam cores. They conducted finite element simulations and compared the results with those obtained from ungraded single-layer sandwich plates. After confirming the numerical method by comparing it to existing experimental data and presenting the current computational model, they examine a series of graded sandwich plates subjected to air blasts. The analysis focuses on the deformation and blast resistance of these plates. They conduct a quantification of the impacts of face-sheet configurations and interfacial adhesion strength between various foam layers. The results show that when the same amount of air blast is applied to graded plates versus regular ungraded plates, the graded plates have less center transverse deflection and better blast resistance. Furthermore, optimizing the layout of the foam core can further enhance its performance. The study also investigates the blast resistance of both graded and ungraded sandwich plates, ensuring they have the same mass.

Manish Bhandari and Kamlesh Purohit [54] analyzed the mechanical deformation of functionally graded ceramic-metal plates under different boundary circumstances. This project included conducting parametric studies by manipulating the distribution of volume fractions and boundary conditions. The authors subjected the functional gradient material plate to static analysis using the sigmoid law and compared its results to the published findings for verification. Altering the mesh size and layer size achieves the optimization of the convergence study. They use the power law and exponential law to describe the behavior of the same substance under certain circumstances. Observers notice that the bending response of the functionally graded plate falls between that of the metal plate and that of the ceramic plate. The bending response of the functionally graded plate remains consistent regardless of boundary circumstances. The bending response of S-FGM is more consistent across different values of "n" compared to P-FGM. The flexural response of E-FGM closely resembles the behavior of P-FGM. They can expand the work to accommodate variations in stress, loading patterns, and different combinations of ceramic and metal materials. In addition to mechanical loads, the experiment may also subject the material to varying temperature environments.

Manish Bhandari aged [55], created a model and conducted simulations of a plate made of FG under a transverse mechanical force. The task involves altering the ratio of the side dimensions of the FGM plate and modifying the volume fraction index 'n.' The findings demonstrated that the plate's layered construction yields reasonably precise results. The non-dimensional deflection values of functionally graded material (FGM) plates, where the parameter n ranges from 0 to infinity, are lower than those of pure metal plates. The dimensionless tensile stress is most significant for pure metal ($n = \infty$) and the smallest for pure ceramic ($n = 0$). A plate made entirely of metal ($n = \infty$) experiences the greatest magnitude of shear strain, while a plate wholly made of ceramic ($n = 0$) experiences the smallest magnitude. FG plates have exceptional resistance to bending forces.

In their study, Kulkarni and Pendhari [56] aimed to accurately measure the temperature distribution over the whole thickness of the laminate using a heat conduction formulation. they conducted a stress study using a semi-analytical formulation to analyze both an accurate temperature profile and a power law variable temperature profile. Documentation exists comparing the temperature profiles of several material sets, aspect ratios, power indices, and width-to-length ratios ranging from thick to thin rectangular laminate. This comparison includes both precise and assumed profiles. The present research has observed the influence of material properties on the thermal profile while finding that the aspect ratio does not impact it. The thermal stress analysis highlights the critical significance of accurately determining the thermal profile. This is because various factors, such as material properties, aspect ratios, and power indices (k), influence the stress analysis. There was a notable disparity in numerical outcomes between thermal stress assessments when comparing the actual and assumed temperature profiles. The semi-analytical methodology described in this study combines the benefits of both analytical and numerical approaches, achieving simplicity and accuracy simultaneously. Consequently, it allows for the avoidance of intricate 3D solutions. The current formulation can only handle laminates that are supported all around and subjected to dispersed loads. It may be feasible to expand the proposed formulation to include FG laminates with different boundary conditions, multidimensional volume fraction changes, and so on. Furthermore, it is possible to manipulate additional intelligent materials such as composites, sandwich structures, and piezoelectric materials using the provided formulation via a series of mathematical operations. The summary of the previous papers is listed in Table (5).

Table 5 Summary of FGM Plates Under different loading

Ref. No.	Variable	Theory	Load and parameters
[52]	he bending of FG plates that subjected to a uniform load	Fourier series expansions, finite element method	tensile and compressive stresses
[53]	dynamic response of metallic sandwich plates	finite element method	transverse deflection and optimizing the layout of the foam core
[54]	the mechanical deformation of functionally graded ceramic-metal plates	Finite Element method	the distribution of volume fractions and boundary conditions
[55]	bending deflection of functionally graded material (FGM) plates	Numerical Method	Power law index and boundary conditions
[56]	Temperature profile and Thermal stress analysis of FGM plate	two-point boundary value problem, Semi-analytical formulation for 3-dimensional heat conduction and basic linear theory of elasticity	material sets, aspect ratios, power indices, and width-to-length ratios

7. Conclusion

From previous survey, the FG plate under different mechanical behaviors can analyzed analytically using the classical shear deformation theories with approximately simple consideration to vary the material properties in plate thickness. In other side, the finite element method is the suitable numerical method which deals with the material properties variation in thickness of FG plate.

Compliance with ethical standards

Disclosure of conflict of interest

We certify that the present manuscript has not been published in an archival journal. Also, we certify that the present manuscript is not currently submitted for publication in another journal.

All data generated or analyzed during this study are included in this published article [and its supplementary information files].

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