International Journal of Science and Research Archive

eISSN: 2582-8185

Cross Ref DOI: 10.30574/ijsra

Journal homepage: https://ijsra.net/

(Review Article)

Check for updates

Role of Relativistic Charged Perfect Fluid in Bianchi type-III Space-time in Brans-Dicke theory of gravitation

V. G.Mete [1,*], K. R.Mule [2], S. N. Bayaskar [3] and V.S. Bawane [4]

[1] *Department of Mathematics, R.D.I.K. & K.D. College, Badnera-Amravati-444701, India.*
[2] *Department of Mathematics, S. D. M. Burungale Science and Arts College, Shegaon-444203, India.*
[3] *Department of Mathematics, Adarsha Science, J. B. Art and Birla Commerce College, Dhamangaon Rly- 444709, India.*
[4] *Department of Mathematics, M. F. Arts, Comm. & S. C. Science Mahavidyalaya, Warud-444906, India*

International Journal of Science and Research Archive, 2024, 11(02), 035–044

Publication history: Received on 13 January 2024; revised on 24 February 2024; accepted on 27 February 2024

Article DOI: https://doi.org/10.30574/ijsra.2024.11.2.0355

Abstract

In this paper, we investigate the role of relativistic charged perfect fluid in Bianchi type-III cosmological model in Brans-Dicke theory of gravitation. Solutions of the model are obtained by volumetric exponential expansion, power law expansion and power law relation between scalar field ϕ and the scale factor a . Some physical and kinematical properties of the model also are studied.

Keywords: Bianchi type-III universe; Brans-Dicke theory of gravitation; Electromagnetic field; Perfect fluid; Constant vector potentials

1. Introduction

Brans - Dicke theory of gravitation is a well-known modified version of Einstein's theory. Brans and Dicke [1] formulated a theory of gravitation in which, besides a gravitational part, a dynamical scalar field is introduced to account for variable gravitational constant and to incorporate Mach's Principle in Einstein's theory. In this theory, the scalar field has the dimension of the universe of a gravitational constant and its role is confined to its effect on gravitational field equations. Brans-Dicke scalar-tensor theory of gravitation is quite important in view of the fact that scalar fields play a vital role in inflationary cosmology. There has been a renewed interest in gravitational constants in recent years. "The new inflationary models [2], the potential problem of "graceful exit" [3] and extended chaotic inflation [4] are based on the gravity theory of Brans-Dicke.

The scalar-tensor theories have been the subject of considerable interest in the study of various cosmological models due to their relevance for the inflationary expansion of the universe and to solve many outstanding problems in cosmology. Several aspects of Brans-Dicke theory have been widely examined by many authors. Bardeen et al. [5] explored the inflationary universe models which provide a mechanism for galaxy formation by generating small scale density fluctuation in the universe, Bianchi type-I string cosmological models with and without a source-free magnetic field have been examined by Banerjee et al.[6]. Johari and Desikan [7] have investigated cosmological models with constant deceleration parameter in Nordtvedt's theory. In Brans-Dicke theory of gravity, Bianchi type-III cosmological model with a negative constant deceleration parameter in presence of perfect fluid have been studied by Adhav et al.[8], Katore et al. [9] explored a plane symmetric space-time filled with dark energy models in Brans-Dicke theory, Bhoyar et al.[10] studied Bianchi type-III and Kantowski Sachs cosmological model containing a magnetic field with variable cosmological constant. Lorenz-Petzold [11], Kumar et al.[12], Pawar et al. [13], Rao et al.[14], Naidu et al.[15], Kandalkar et al. [16],Mete et al. [17],Sireesha et al. [18],Hegazy et al.[19],Trivedi et al.[20] are some of the authors who have

*Corresponding author: V. G.Mete

investigated several aspects of Brans-Dicke theory of gravitation. In the presence of the magnetic field, Tikekar and Patel [21] have acquired some exact Bianchi type-III cosmological solutions of massive string. Singh and Shri Ram [22] have presented a technique to generate new exact Bianchi type-III cosmological solutions of massive strings in the presence and absence of the magnetic field. Tripathy et al. [23] and Pradhan [24] studied string cosmological models in the presence of the electromagnetic field. A detailed discussion of Brans-Dicke cosmology is given by Singh et al. [25,26] have investigated some Bianchi type-II cosmological models in Brans-Dicke theory, Shamir et al. [27] explored anisotropic dark energy Bianchi type-III cosmological models in Brans-Dicke theory of gravity, Solanke and Karade [28,29] have investigated Bianchi type-I and III universe field with perfect fluid and scalar field coupled with electromagnetic fields in theory of gravity. Katore et al. [30] have investigated Bianchi type-I dark energy cosmological model with power-law relation in Brans-Dicke theory of gravitation. Recently Jumi Bharali et al. [31] studied bulk viscous magnetized locally rotationally symmetric Bianchi type- I cosmological model in general relativity. In the context of the Brans-Dicke theory of gravitation, Nimkar et al. [32] investigated the Bianchi type-VI Cosmological model andstudiedsome observational parameters such as jerk parameter, redshift, Look-back time, Luminosity distance redshift, and angular diameter distance.

Inspired by the aforementioned study, we examined the Bianchi type-III perfect fluid cosmological model in the Brans-Dicke theory of gravitation with an electromagnetic field. We also discussed some physical and kinematical properties of the model.

2. The metric and field equation

We consider a spatially homogeneous Bianchi Type-III space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2dz^2 , \qquad (1)$$

where A , B and C are functions of t and m is a constant.

Brans-Dicke field equations for the combined scalar and tensor fields are

$$G_j^i = \frac{-8\pi}{\phi}T_j^i - \frac{\omega}{\phi^2}\left(g^{ii}\phi_{,i} - \frac{1}{2}g_j^i\phi_{,k}\phi^{,k}\right) - \frac{1}{\phi}\left(g^{ii}\phi_{i;j} - g_j^i\phi_{;k}^{,k}\right), \qquad (2)$$

where G_j^i is Einstein's tensor, ϕ is a dimensionless coupling constant and T_j^i is energy momentum tensor for perfect fluid with conservation equation

$$\phi_{;k}^{;k} = \frac{1}{\sqrt{-g}}\left[\sqrt{-g}\phi^k\right]_{,k} \qquad (3)$$

2.1 Energy Momentum Sources

The energy momentum tensor for matter under discussion given by

$$T_j^i = {}^pT_j^i + E_j^i , \qquad (4)$$

where ${}^pT_j^i$ is energy momentum tensor for perfect fluid and E_j^i is energy momentum tensor for electromagnetic field is given by

$$E_{ij} = \frac{1}{4}F_{ab}F^{ab}g_{ij} - F_{ai}F_{bj}g^{ab} \qquad (5)$$

Here the electromagnetic field tensor F_{ij} has the expression

$$F_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}. \qquad \text{..............} (6)$$

where A_i is a four potential vector.

To achieve the compatibility with the metric (1), we assume electromagnetic vector potential as

$$A_i = [\lambda(x)v_1(t), v_2(t), v_3(t), v_4(t)] \text{...............} (7)$$

From equations (6) and (7), we deduce

$$F_{14} = \lambda \dot{v}_1,\ F_{24} = \dot{v}_2,\ F_{34} = \dot{v}_3,\ F_{43} = -\dot{v}_3 \text{..............} (8)$$

Using equations (6), (7) and (8), we obtained

$$F_{ab}F^{ab} = -2\left[\frac{\lambda^2 \dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{\dot{v}_3^2}{C^2}\right], \qquad \text{..............} (9)$$

The energy momentum tensor for a perfect fluid is given by

$${}^pT_j^i = (\rho + p)u^i u_j - p\delta_i^j, \text{...............}(10)$$

where ρ is the density, p is the pressure of perfect fluid and four velocity u_i is given by

$$g_{ij}u^i u^j = -1.$$

For co-moving coordinate system, we have

$$u_x = 0, u_y = 0, u_z = 0, u_t \neq 0 \text{...............} (11)$$

Accordingly equation (10), provides

$${}^pT_1^1 = {}^pT_2^2 = {}^pT_3^3 = -p,\ {}^pT_4^4 = \rho,\ T_i^j = 0 \quad \forall i, j \text{..............} (12)$$

Using equations (5), (9) and (12), we obtained

$${}^pT_1^1 + E_1^1 = \frac{1}{2}\frac{\lambda^2 \dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2^{\,2}}{B^2 e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3^{\,2}}{C^2} - p, \text{...............} (13)$$

$${}^pT_2^1 + E_2^1 = {}^pT_1^2 + E_1^2 = \frac{\lambda \dot{v}_1 \dot{v}_2}{A^2}, \text{...............} (14)$$

$${}^pT_3^1 + E_3^1 = {}^pT_1^3 + E_1^3 = \frac{\lambda \dot{v}_1 \dot{v}_3}{A^2}, \text{...............} (15)$$

$${}^pT_2^2 + E_2^2 = -\frac{1}{2}\frac{\lambda^2 \dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} - \frac{1}{2}\frac{\dot{v}_3^2}{C^2} - p, \text{...............} (16)$$

$$ {}^{p}T_{3}^{2} + E_{3}^{2} = {}^{p}T_{2}^{3} + E_{2}^{3} = \frac{\dot{v}_2 \dot{v}_3}{B^2 e^{-2mx}} ,\ldots\ldots\ldots\ldots\ldots (17) $$

$$ {}^{p}T_{3}^{3} + E_{3}^{3} = -\frac{1}{2}\frac{\lambda^2 \dot{v}_1^2}{A^2} - \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} - p \ldots\ldots\ldots\ldots\ldots (18) $$

$$ {}^{p}T_{4}^{4} + E_{4}^{4} = \frac{1}{2}\frac{\lambda^2 \dot{v}_1^2}{A^2} + \frac{1}{2}\frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} + \rho . \ldots\ldots\ldots\ldots\ldots (19) $$

2.1. Conservation Law:

The Conservation equation is given by

$$ \frac{\partial}{\partial x^i}\left(\sqrt{-g}F^{ij}\right) = 0 \ldots\ldots\ldots\ldots\ldots (20) $$

Equation (20) with different combination of i and j gives following equations

$$ \left[\frac{\dot{v}_1}{v_1}\right]^{\bullet} + \frac{\dot{v}_1^{\,2}}{v_1^{\,2}} + \frac{\dot{v}_1}{v_1}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right] = 0 ,\ldots\ldots\ldots\ldots\ldots (21) $$

$$ \left[\frac{\dot{v}_2}{v_2}\right]^{\bullet} + \frac{\dot{v}_2^2}{v_2^2} + \frac{\dot{v}_2}{v_2}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0 ,\ldots\ldots\ldots\ldots\ldots (22) $$

$$ \left[\frac{\dot{v}_3}{v_3}\right]^{\bullet} + \frac{\dot{v}_3^2}{v_3^2} + \frac{\dot{v}_3}{v_3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] = 0 ,\ldots\ldots\ldots\ldots\ldots (23) $$

$$ \phi_{;k}^{;k} = -\ddot{\phi} - \ddot{\phi}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] \ldots\ldots\ldots\ldots\ldots (24) $$

From the vanishing components of Einstein's tensor and using (5) and (7), we deduce

$$ \frac{\dot{v}_1 \dot{v}_2}{v_1 v_2} = \frac{\dot{v}_1 \dot{v}_3}{v_1 v_3} = \frac{\dot{v}_2 \dot{v}_3}{v_2 v_3} = 0 ,\ldots\ldots\ldots\ldots\ldots (25) $$

$$ \frac{\dot{v}_1}{v_1} = \frac{\dot{v}_2}{v_2} = \frac{\dot{v}_3}{v_3} = \frac{\dot{D}}{D} ,\ldots\ldots\ldots\ldots\ldots (26) $$

where D is an unknown function of t

Integrating equation (26) with respect to t, we get

$$ v_1 = k_1 D,\ v_2 = k_2 D,\ v_3 = k_3 D, \qquad \ldots\ldots\ldots\ldots\ldots (27) $$

where k_1, k_2 and k_3 are constants.

Using equations (25) and (27), we get

$$\left(\frac{\dot{D}}{D}\right)^2 = 0 . \quad \text{.............. (28)}$$

With an aid of equation (27), we can write the equations (21) to (23) as

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right) = 0 , \quad \text{.............. (29)}$$

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right) = 0 , \quad \text{.............. (30)}$$

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 . \quad \text{..............(31)}$$

We attempt to express the component of T_j^i in terms of T_4^4 already used [28,29] .

For this, we consider the expression as

$$\frac{\lambda^2 \dot{v}_1^{\ 2}}{A^2} + \frac{\dot{v}_2^{\ 2}}{B^2 e^{-2mx}} + \frac{\dot{v}_3^2}{C^2} = \left[\frac{\lambda^2 v_1^{\ 2}}{A^2}\left(\frac{\dot{D}}{D}\right)^2 + \frac{v_2^{\ 2}}{B^2 e^{-2mx}}\left(\frac{\dot{D}}{D}\right)^2 + \frac{v_3^{\ 2}}{C^2}\left(\frac{\dot{D}}{D}\right)^2\right] = 0 \quad \text{.................. (32)}$$

$$\frac{\lambda^2 \dot{v}_1^{\ 2}}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2mx}} + \frac{\dot{v}_3^2}{C^2} = \left[\frac{\lambda^2 v_1^{\ 2}}{A^2} + \frac{v_2^{\ 2}}{B^2 e^{-2mx}} + \frac{v_3^{\ 2}}{C^2}\right]\left(\frac{\dot{D}}{D}\right)^2 = 0$$

$$T_4^{\ 4} = \frac{1}{2}\frac{\lambda^2 \dot{v}_1^{\ 2}}{A^2} + \frac{1}{2}\frac{\dot{v}_2^{\ 2}}{B^2 e^{-2mx}} + \frac{1}{2}\frac{\dot{v}_3^2}{C^2} + \rho = \rho , \quad \text{.................. (33)}$$

$$T_1^1 = -p = T_2^2 = T_3^3 , \quad \text{.................. (34)}$$

3. Solution of field equations

Considering the non-vanishing component of Einstein's tensor from equation (3), we derive

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \frac{8\pi\, p}{\phi} - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad \text{.................. (35)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \frac{8\pi p}{\phi} - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right) - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right), \quad \text{.................. (36)}$$

$$-\frac{m^2}{A^2} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi\, p}{\phi} - \frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \quad \text{.................. (37)}$$

$$-\frac{m^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=\frac{8\pi\rho}{\phi}+\frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \quad \text{.................. (38)}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0. \quad \text{.................. (39)}$$

Integrating (39) with respect to t, we get

$$A=k_4B, \quad \text{.................. (40)}$$

where k_4 is a constant of integration. Without loss of generality let us assume k_4 as unity so that equation (40) written as

$$A=B. \quad \text{.................. (41)}$$

Using equations (41) and (35) to (38), we yield

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\frac{8\pi p}{\phi}-\frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \quad \text{.................. (42)}$$

$$-\frac{m^2}{B^2}+\frac{2\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=\frac{8\pi p}{\phi}-\frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right), \quad \text{.................. (43)}$$

$$-\frac{m^2}{B^2}+\frac{2\dot{A}^2}{A^2}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=\frac{8\pi\rho}{\phi}+\frac{1}{2}\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right). \quad \text{.................. (44)}$$

Since the field equations are highly nonlinear with three equations and five unknown. Therefore two extra conditions can be considered to solve the field equations.

Let us choose power law form of metric potential [30] given by

$$B=\alpha t^n \text{, and } C=\beta t^n \quad \text{.................. (45)}$$

The power law relation between scalar field ϕ and the scale factor a has already been used by Johri and Desikan [7] in the context of Robertson Walker Brans-Dicke models. Thus the power law relation between ϕ and the scale factor a is $\phi=\eta a^m$, where a is constant of proportionality. The average scale factor is given by

$$a=(ABC)^{\frac{1}{3}}=k_5t^n, \quad \text{.................. (46)}$$

where $k_5=\alpha^{\frac{2}{3}}\beta^{\frac{1}{3}}$ is a constant.

Hence scalar field ϕ is obtained as

$$\phi=k_5t^{mn}=k_5t^s,$$

where $s = mn$ is a constant.

Using equations (24) and (25), we have

$$\frac{\dot{D}}{D} = k_6 , \qquad \text{.................. (47)}$$

where k_6 is a constant of integration.

Which on integration, yield

$$D = e^{k_6 t} . \qquad \text{.................. (48)}$$

Using equations (46) and (27), we have

$$v_1 = k_1 e^{k_6 t} , \qquad \text{.................. (49)}$$

$$v_2 = k_2 e^{k_6 t} , \qquad \text{.................. (50)}$$

$$v_3 = k_3 e^{k_6 t} , \qquad \text{.................. (51)}$$

v_4 remained undetermined.

The metric in (1), with the help of (45), can be redefined in the form

$$ds^2 = \alpha^2 t^{2n} (dx^2 + e^{-2mx}) dy^2 + \beta^2 t^{2n} dz^2 - dt^2 . \qquad \text{.................. (52)}$$

3.1. Physical and Kinematical Properties of the Model:

The physical and kinematical properties of the model (52) are obtained as follows.

For the investigated model, the pressure p and the density ρ are given by

$$p = \frac{k_5 t^s}{8\pi} \left[\frac{2n(n-1) + n^2}{t^2} + \omega \frac{s^2}{2t^2} + \frac{s^2 (s-1)}{t^2} + \frac{2sn}{t^2} \right], \qquad \text{.................. (53)}$$

$$\rho = \frac{k_5 t^s}{8\pi} \left[\frac{4n^2}{t^2} - \frac{m^2}{\alpha t^n} - \omega \frac{s^2}{2t^2} + \frac{3sn}{t^2} \right], \qquad \text{.................. (54)}$$

where k_5 is a constant of integration.

The physical quantities of observational interest in cosmology are,

The spatial volume is obtained as

$$V = \sqrt{-g} = \left(\alpha^2 \beta t^{3n}\right) e^{-mx} . \qquad \text{.................. (55)}$$

The expansion scalar becomes

$$\theta = 3H = \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{3n}{t}, \qquad \text{.................. (56)}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{6} = 0 \qquad \text{.................. (57)}$$

The mean anisotropic parameter A_m as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 0 \qquad \text{.................. (58)}$$

The mean Hubble parameter

$$H = \frac{n}{t}. \qquad \text{.................. (59)}$$

The deceleration parameter is given by

$$q = \frac{d}{dt}(H) - 1 = \frac{1-n}{n} \qquad \text{.................. (60)}$$

The cosmic Jerk parameter is given as,

$$J = q + 2q^2 - \frac{\dot{q}}{H}$$

$$= \frac{(n-1)(n-2)}{n^2} \qquad \text{.................. (61)}$$

4. Conclusion

In this paper, we have presented Bianchi Type-III charged fluid universe in Brans-Dicke theory of gravitation in presence of perfect fluid with electromagnetic field. The spatial volume V of the model increases with time showing the spatial expansion of the universe. It is observed that Hubble's parameter H vanishes with extremely large value and continues to decrease with time. The scalar expansion and the physical parameters energy density and pressure diverge at $t = 0$ and they all vanish as t approaches to infinity. The scalar field increases with time and at $t = 0$, it vanishes. The recent observations of type Ia supernovae reveal that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$. It follows that one can choose the value of n in the range $0 < n < 1$ to ensure that the derived model is consistent with observations. It is also seen that the value of the cosmic jerk parameter is positive throughout the entire history of this model. This shows that our model strongly agrees with present day observations.

Compliance with ethical standards

Disclosure of conflict of interest

The authors certify that they have no Conflict of Interest in the subject matter or materials discussed in this manuscript.

References

[1] Brans, H and Dicke, R. H. (1961). Mach's Principle and a relativistic theory of gravitation. Physicalreview A, Vol.124, No. 3, 925-935. doi:10.1103/PhysRev.124.925

[2] Mathiazhagan, C and Johri, V. B.(1984). An inflationary universe in Brans-Dicke theory: a hopeful sign of theoretical estimation of the gravitational constant. Classical and Quantum Gravity, Vol.1, No. 2 , L29-L32. doi:10.1088/0264-9381/1/2/005.

[3] Pimentel, L. O.(1997).New Exact Vacuum Solutions in Brans–Dicke Theory, Modern Physics Letters A, Vol.12, No.25 ,1865-1870. doi:10.1142/S0217732397001904.

[4] Linde, A. D.(1990). Extended chaotic inflation and spatial variations of the gravitational constant, Physics Letters B, Vol. 238, No. 2-4 ,160-165. doi:10.1016/0370-2693(90)91713-L.

[5] Bardeen, M., Steinhardt, P. J. and Turner M.S.(1983). Spontaneous creation of almost scale-free density in an inflationary universe, Phys. Rev.Vol.28,No. 4, 670-679. https://doi.org/10.1103/PhysRevD.28.679

[6] Banerjee, A., Sanyal, A. K and ChakrabortyS.(1990). String cosmology in Bianchi type-I space-time, Pramana J. of Phys.Vol.34 (1), 1-11. DOI:10.1007/BF02846104

[7] Johri, V.B and Desikan Kalyani (1994). Cosmological models with constant deceleration parameter inNordtvedt's theory, Pramana. J. of Phys., Vol.42, No. 6, 473-481. https://doi.org/10.1007/BF02847129

[8] Adhav, K.S.,Nimkar,A.S., Ugale M.R.(2008). Dawande, M.V., Bianchi Type-III Cosmological Model with Negative Constant Deceleration Parameter with Wet Dark Fluid in Brans-Dicke Theory of Gravitation, Int. J. of Theor. Phys.47(3),634-639. DOI: 10.1007/s10773-007-9487-y

[9] Katore, S. D and Shaikh, A.Y.(2012).Plane Symmetric Dark Energy Model in Brans-DickeTheory of Gravitation, Bulgarian J. of Physics, Vol. 39 (3), 241-247.

[10] Boyar, S.R and Chirde, V.R.(2018). Magnetized anti-stiff fluid cosmological models with Variable cosmological constant, Int. J. of Scientific research in mathematical and Statistical Sci.,Vol.5, No.18,11-18.https://doi.org/10.26438/ijsrmss/v5i1.1118

[11] Lorenz-Petzold D.(1982).Tilted Electromagnetic Bianchi type-III cosmological solution, Astrophysics and space science,Vol. 85, 59-61.https://doi.org/10.1007/BF00653432

[12] Kumar S and Singh, C.P.(2008). Exact Bianchi type-I cosmological models in scalar tensor theory, Int. J. of Theor. Physics, Vol. 47, No. 6, 1722-1730.https://doi.org/10.1007/s10773-007-9614-9.

[13] Pawar, D. D., Bayaskar, S. N and Patil, V. R. (2009). Plane Symmetric Cosmological Model with Thick Domain Walls in Brans-Dicke Theory of Gravitation, Bulgarian J. ofPhysics, Vol. 36, No. 4, 68-75.

[14] Rao,V U M., Santhi,M V. (2011) .Bianchi Type-I,VIII & IX Perfect Fluid Cosmological Models in Brans Dicke Theory of Gravitation , J. of Modern Phys.2, 1222-1228. DOI:10.4236/jmp.2011.210151.

[15] Naidu, R. L, Satyanaryan B and Reddy, D.R.K. (2012). Bianchi Type-III Dark Energy Model in a Saez-Ballester Scalar-Tensor Theory, Int. J.of Phys., 51,No.5,2857- 2862. DOI 10.1007/s10773-012-1161-3.

[16] Kandalkar, S.P and Samdurkar, S. (2014). Bianchi type-V cosmological model with linearequation of state in Brans-Dicke theory of gravitation, Int. J. of Astronomy and Astrophysics, Vol.4 (3), 429-436. DOI:10.4236/ijaa.2014.43038

[17] Mete, V.G and Mule, K.R.(2018).Bianchi type-III charged fluid universe in Brans-Dicke theory of gravitation, Int. J.of Sci.Res. in Phy and Apl. Sci. Vol.6, No. 5, 57-61. https://doi.org/10.26438/ijsrpas/v6i5.5761.

[18] Sireesha, K.V.S., Rao ,V.U.M. (2019).Bianchi type -II, VIII & IX Holographic Darkenergy Cosmological Models in Brans-Dicke theory of gravitation, African Rev. Phys. 14: 0010.

[19] Hegazy, E.A., Rahaman, F. (2020).Bianchi type VI0 cosmological model with Electromagnetic field and variable deceleration parameter in general theory of relativity. Indian JPhys. 94, 1847–1852. https://doi.org/10.1007/s12648-019-01614-4.

[20] Trivedi, D., Bhabor, A. K. (2021).Higher dimensional Bianchi type-III stringCosmological models with dark energy in Brans-Dicke scalar-tensor theory of gravitation. NewAstronomy, 89, 101658. https://doi.org/10.1016/j.newast.2021.101658.

[21] Tikekar, R and Patel, L. K. (1992).Some exact solutions of string cosmology in Bianchi- III space-time, Gen. Relativity Gravit., Vol.24, No. 5, 397-404. https://doi.org/10.1007/BF00760415.

[22] Singh, J. K and Shri Ram(1996).String cosmological models of Bianchi type-III, Astrophys.Space Science, Vol. 246, 65-72. https://doi.org/10.1007/BF00637400.

[23] Tripathy, S.K., Nayak,S.K., Sahu,S.K. and Routray, T.R. (2009).Bulk viscous string Cosmological models with electromagnetic field, Astrophys. Space Sci.,Vol.321, 247-252.https://doi.org/10.1007/s10509-009-0033-7.

[24] Pradhan,A., Rai, A and Singh, S.K. (2007). Cylindrically symmetric inhomogeneous Universe with electromagnetic field in string cosmology, Astrophys. Space Sci. 312, 261-265. https://doi.org/10.1007/s10509-007-9681-7.

[25] Singh, J. K , Sharma, N. K. (2010). Some Bianchi type-II cosmological models in Brans–Dicke theory, Astrophysics. Space Sci. Vol.327,293-298.https://doi.org/10.1007/s10509-010-0319-9.

[26] Singh, J. K , Sharma, N. K. (2010a).Some Exact Solutions of String Cosmological Models inBianchi Type-II Space-Time, Int. J. Theor. Phys. Vol.49, 2902-2909. DOI:10.1007/s10773-010-0481-4.

[27] Shamir, M. F , Bhatti, A. A. (2012).Anisotropic Dark Energy Bianchi Type-IIICosmologicalModels in Brans Dicke Theory of Gravity, Can. J. of Phys., Vol. 90 (2), 193-198.DOI:10.1139/P2012-007

[28] Solanke, D. T ,Karade, T. M. (2017). Bianchi type I universe filled with scalar field Coupled with electromagnetic fields in $f(R,T)$ theory of gravity, Indian J.of Phys., Vol.91,Issue11, 1457-1466. 10.1007/s12648-017-1029-8

[29] Solanke, D. T and Karade, T. M. (2016).Bianchi Type-III Universe Filled withCombination of Perfect Fluid & Scalar Field Coupled with Electromagnetic Fields in $f(R,T)$ Theory ofGravity, Int. J.of Math. Archive, Vol.7, No. 7, 151-162.

[30] Katore, S. D, Kapse , D.V. (2018). Bianchi type-I dark energy cosmology with power-law relation in Brans-Dicke theory of gravitation, Advances in High Energy Physics,Vol.10 (4), 1-11. https://doi.org/10.1155/2018/2854567

[31] JumiBharali and Kanika Das (2020).Magnetized anisotropic modified holographic Ricci dark energy viscous cosmological model, African Rev. Phys. 15: 0019.

[32] Nimkar, A.S., Hadole, S.R. & Wath, J.S.(2023).Cosmological model in Brans–Dicke theory of gravitation. Indian J Phys 97, 1633–1640.https://doi.org/10.1007/s12648-022-02500-2