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(Review Article)

Minimally interacting two fluid cosmological model in the framework of scale covariant theory

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Abstract

In this paper we have studied minimally interacting two fluid cosmological model in the framework of scale covariant theory by considering Bianchi type III metric in the presence of matter and radiation field. Here, we have assumed the exponential volumetric law to construct this model and considered the equation of state $p_m = \gamma \rho_m$ to find matter density, radiation density and parameters of matter and radiation density. Lastly, we have discussed some physical and kinematical parameters.

Keywords: Bianchi Type III; Two Fluid; Scale Covariant Theory; EoS.

1. Introduction

In recent years from various observational data it is observed that the universe is undergoing an accelerated expansion due to which there has been considerable interest in deriving the cosmological models for various theories of gravitation. The general theory of relativity provides mathematically precise and physically sound theory of gravitation for constructing the cosmological models of the universe. But, it is not sufficient to explain the current phase of the universe. So, various attempts have been made to modify the Einstein's field equations in which alternating theories and modified theories of gravitation are introduced. In recent years there has been lot of interest of researchers in constructing the cosmological models using alternative theories of gravitation such as Lyra Geometry [1], Brans Dicke Theory [2], Barber's first and second self-creation theory [3], Saez and Ballester theory [4]. The Scale covariant theory introduced by Canuto et. al. [5] which is optional to Einstein's general theory of gravitation. It provides the necessary theoretical framework in which it becomes sensible to discuss the possible variation of the gravitational constant G. A. Beesham [6] have examined the Bianchi type I cosmological model in scale covariant theory. Also, S. Ram et. al. [7] investigated spatially homogeneous Bianchi type V cosmological model in the scale covariant theory. The scale covariant theory of gravitation is derived by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space time distances. In Scale covariant theory Einsten's field equations are in gravitational units and physical quantities are in atomic units. D. R. K. Reddy et. al. [8] investigated Exact Bianchi type II, VIII and IX cosmological model in scale covariant theory, Reddt et. al. [9] studied five dimensional minimally interacting holographic dark energy model in Brans-Dicke theory of gravitation, Naidu et. al.[10] investigated Bianchi type-III dark energy model in a Saez-Ballester scalar-tensor theory, [11] M. Zeyauddin et. al. investigated Bianchi type VI cosmological models in Scale-Covariant theory, R. Venkateswarlu [12] investigated Cylindrically symmetric cosmic strings in scale covariant theory of gravitation, Katore et. al. [13] studied Magnetized dark energy cosmological models in scale covariant theory, Katore et. al. [14] studied Bianchi type III dark energy cosmological model in scalar tensor

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theory of gravitation, Y. Aditya, et. al. [15] Anisotropic new holographic dark energy model in Saez-Ballester theory of gravitation

Two fluid models, including energy densities of radiation and matter, are cosmologically important. Cosmological observations suggest that the radiation frame and the matter frame of the universe may not coincide. The radiating fluid is modeling a cosmic microwave background. The matter-fluid modeling the observed matter content of the universe. Recently, researchers have been taking interest in two fluid cosmological models. Adhav et. al. [16] investigated Bianchi type V two- fluid cosmological model, Adhav et. al. [17] examined interacting cosmic fluids in LRS Bianchi type-I cosmological models, Mete et. al. [18] studied two-fluid cosmological models in Bianchi type-V space-time in higher dimensions, Hatkar et. al. [19] studied the Bianchi type I metric with two-fluid cosmological model. S. Oli [20] studied two fluid cosmological models in Bianchi type I space time. Pant and Oli [21] have examined the Bianchi type II space-time with a two-fluid cosmological model. Coley and Dunn [22] have investigated the two fluids source of Bianchi type VI0 Models. Also, Amirhashchi et. al. [23] has evaluated interacting two –fluid dark energy models in a non-flat universe.

Motivated by above research we have studied minimally interacting two fluid cosmological model in scale covariant theory.

This paper is organized as follows: section 2 contains metric and field equations; in section 3 Cosmological model is obtained by considering different Equation of state. In section 4 Case I: Dust Model $(\gamma = 0)$, Case II: Zeldovich Universe $(\gamma = 1)$ and some physical and Kinematical Properties of the models are discussed. Section 5: Discussion and conclusion.

2. Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi Type III metric in the following form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 dz^2 \quad \text{.................} (1)$$

The field equation in the scale covariant theory is given by

$$R_j^i - \frac{1}{2} R g_j^i + f_j^i(\phi) = -8\pi G T_j^i \quad \text{....................}(2)$$

Where, ϕ is function of t only.

$$\phi^2 f_{ij} = 2\phi\phi_{;ij} - 4\phi_{;i}\phi_{;i} - g_{ij}(2\phi\phi^k_{;k} - \phi^{'k}\phi_k) \quad \text{................} (3)$$

Here, ϕ is the scalar function of t only and other symbols have their usual meanings as in Riemannian geometry. The energy momentum tensor for two fluid is given by

$$T_{ij} = (T^m)_{ij} + (T^r)_{ij} \quad \text{................} (4)$$

Where, $(T^m)_{ij}$ is the energy momentum tensor for matter field and $(T^m)_{ij}$ is the energy momentum tensor for radiation field which is given by

$$(T^m)_{ij} = (p_m + \rho_m) u_i^m u_j^m - p_m g_{ij} \quad \text{................} (5)$$

$$(T^r)_{ij} = \frac{4}{3}\rho_r u_i^r u_j^r - \frac{1}{2}\rho_r g_{ij} \quad \text{................} (6)$$

Where, ρ_m is the energy density of matter, p_m is the pressure of the matter and ρ_r is the energy density of radiation with

$$g^{ij}u_i^m u_j^m = 1 \; and \; g^{ij}u_i^r u_j^r = 1 \quad \text{................ (7)}$$

We assume that the matter and radiation are both commoving which imply that $u_i^m = (0,0,0,1),\ u_i^r = (0,0,0,1)$

Then the energy momentum tensor takes the form

$$T_1^1 = T_2^2 = T_3^3 = -(p_m + \frac{1}{3}\rho_r),\ T_4^4 = (\rho_m + \rho_r),\ T_4^1 = 0 \quad \text{................ (8)}$$

Using equation (1), (2), (3) and (8) the field equations of the scale covariant can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{\phi}\dot{A}}{\phi A} + \frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\dot{\phi}\dot{C}}{\phi C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi} = 8\pi G(p_m + \frac{1}{3}\rho_r) \quad \text{................ (9)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{\phi}\dot{A}}{\phi A} - \frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\dot{\phi}\dot{C}}{\phi C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi} = 8\pi G(p_m + \frac{1}{3}\rho_r) \quad \text{................ (10)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{\dot{\phi}\dot{A}}{\phi A} + \frac{\dot{\phi}\dot{B}}{\phi B} - \frac{\dot{\phi}\dot{C}}{\phi C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi} = 8\pi G(p_m + \frac{1}{3}\rho_r) \quad \text{................ (11)}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} + \frac{\dot{\phi}\dot{A}}{\phi A} + \frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\dot{\phi}\dot{C}}{\phi C} - \frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}^2}{\phi} = 8\pi G(\rho_m + \rho_r) \quad \text{................ (12)}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \quad \text{................ (13)}$$

Where, the overhead dot denotes differentiation with respect to t.

Using equation (13) equations (9) to (11) becomes

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{\phi}\dot{C}}{\phi C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi} = 8\pi G(p_m + \frac{1}{3}\rho_r) \quad \text{................ (14)}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} + 2\frac{\dot{\phi}\dot{B}}{\phi B} - \frac{\dot{\phi}\dot{C}}{\phi C} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi} = 8\pi G(p_m + \frac{1}{3}\rho_r) \quad \text{................ (15)}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{1}{B^2} + 2\frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\dot{\phi}\dot{C}}{\phi C} - \frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}^2}{\phi} = 8\pi G(\rho_m + \rho_r) \quad \text{................ (16)}$$

The field equations (14) to (16) are the system of four linearly independent equations with $p_m, \rho_m, \rho_r, B, C, \phi, G$ six unknown parameters.

3. Solution of the field equations and Cosmological Model

To get the solution of the field equations we assume,

i) The relation between the metric potentials given by physical condition that shear is proportional to expansion scalar.

$$C = B^n \quad \text{................} \ (17)$$

ii) The volumetric expansion law is given by

$$V = a^3 = c_1 e^{3mt} \quad \text{................} \ (18)$$

The gravitational term G is assumed to be time dependent. We establish that G is decreasing function of time. However the possibility of increasing the function of time cannot be neglected. We assume the most simple and useful form of G as

$$G = \alpha t \quad \text{................} \ (19)$$

Where α is the proportionality constant.

We consider the scale function as $\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}$, $\varepsilon = \pm 1, \pm \frac{1}{2}$

Where t_0 is constant. The most suitable form of guage function to fit in the observational data is

$$\phi(t) \approx t^{\frac{1}{2}} \quad \text{................} \ (20)$$

Using equations (14) and (15) we get,

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{C}}{BC} - \frac{1}{B^2} + 2\frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \quad \text{................} \ (21)$$

Equation (21) can be written as

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(2\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) - \frac{1}{B^2} + 2\frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \quad \text{................} \ (22)$$

Let,

$$V = ABCe^{-2x} \quad \text{................} \ (23)$$

Using equation (23) in equation (22), we get

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = \frac{1}{B^2} - \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(\frac{\dot{V}}{V} + 2\frac{\dot{\phi}}{\phi}\right) \quad \text{................} \ (24)$$

Solving we get,

$$\frac{B}{C} = \frac{A}{C} = f_1 e^{\int \frac{\phi^2}{V} dt} \quad \text{............... (25)}$$

Solving equation (25) for A, B, C, we get,

$$A = B = D_1 V^{\frac{1}{3}} e^{\int \frac{\phi^2}{V} dt} \quad \text{............... (26)}$$

$$C = D_2 V^{\frac{1}{3}} e^{\int \frac{\phi^2}{V} dt} \quad \text{............... (27)}$$

Using equation (18) and (20) we get,

$$A = B = D_1 e^{(mk-k)e^{-mt}} \quad \text{............... (28)}$$

$$C = D_2 e^{(nmk-nk)e^{-mt}} \quad \text{............... (29)}$$

Where, D_1, D_2, n, m, k are constants.

Using equations (28) and (29) the metric in equation (1) becomes

$$ds^2 = dt^2 - \left(D_1 e^{(mt-k)e^{-mt}}\right)^2 dx^2 - \left(D_1 e^{(mt-k)e^{-mt}}\right)^2 e^{-2x} dy^2 - \left(D_1 e^{(nmt-nk)e^{-mt}}\right)^2 dz^2 \quad \text{............... (30)}$$

4. Physical and Kinematical Properties

We assume the relation between pressure and density of matter field through the "gamma-law" equation of state which is given by

$$p_m = \gamma \rho_m \qquad 0 \le \gamma \le 1 \quad \text{............... (31)}$$

We get the density of matter, energy density of radiation and matter and radiation density parameters.

4.1. Matter Density

$$\rho_m = \frac{1}{\alpha \pi t(\gamma - 1)} \left[\left\{ \frac{1}{6t^2} + \frac{m^2}{3} - \frac{nm}{12} - \frac{nm^2}{12} - \frac{1}{16t} \right\} + \left\{ \begin{array}{l} \left(\frac{D_1 + ke^{-mt}}{4} \right) + \left(\frac{mk + k^2 e^{-mt}}{6} \right) - \\ \frac{nk}{12} - \frac{2nmk}{12} - \frac{nk^2 e^{-mt}}{12} \end{array} \right\} e^{-mt} - \frac{x_1^2 \exp(-2(mt-k)\exp(2mt))}{12} \right] \text{...........(32)}$$

4.2. Radiation Density

$$\rho_r = \frac{1}{\alpha \pi t(\gamma - 1)} \left[\begin{array}{l} m^2 n \left(\frac{2 + 3(\gamma - 1)}{24} \right) + m^2 \left(\frac{3(\gamma - 1) - 8}{24} \right) + \left(\frac{3(\gamma - 1) - 8}{48t^2} \right) + nm \left(\frac{2 + 3(\gamma - 1)}{24} \right) - \frac{m}{12} + \frac{1}{16t} - \\ \left\{ \begin{array}{l} \left(\frac{(12(\gamma - 1) - 1)mke^{-mt}}{6} \right) + \left(\frac{(6(\gamma - 1) - 1)k^2 e^{-mt}}{6} \right) + \\ \frac{(\gamma - 1) - t}{t} + \frac{(k(\gamma - 1) - t)e^{-mt}}{t} + \\ \frac{nk(12(\gamma - 1) - 1)}{12} - \frac{nmk}{6} - \frac{nk^2 e^{-mt}}{12} \end{array} \right\} e^{-mt} - \frac{x_1^2 \exp(-(mt-k)\exp(2mt))}{4} \end{array} \right] \text{...............(33)}$$

4.3. Matter Density Parameter

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{3}{(n+2)^2(m+ke^{-mt})^2\alpha\,\pi\,t(\gamma-1)}\left[\begin{array}{l}\left\{\frac{1}{6t^2}-\frac{m^2}{16t}+\frac{4m-m(n+1)}{12m}\right\}+\left\{\frac{\begin{array}{l}3D_1+3ke^{-mt}+2mk+2k^2e^{-mt}+\\ k(1-n-2nm-nke^{-mt})\end{array}}{12}\right\}e^{-mt}-\\ \frac{x_1^2e^{-(2mt-k)\exp 2mt}}{12}\end{array}\right] \quad \text{...............(34)}$$

4.4. Radiation Density Parameter

$$\Omega = \Omega_m + \Omega_r = \frac{1}{(n+2)^2(m+ke^{-mt})^2\alpha\,\pi\,t(\gamma-1)}\left[\begin{array}{l}\frac{3(\gamma-1)-1}{48t^2}-\frac{m[(4m-1)-n(m+1)]}{12}+\left\{\begin{array}{l}\frac{3D_1+2mk+(3+3k-nk)ke^{-mt}+k(1-n(m+1+e^{-mt})}{12}+\\ \frac{nk(12(\gamma-1)-1)}{12}\end{array}\right\}e^{-mt}+\\ \left\{\begin{array}{l}\frac{mke^{-mt}(12(\gamma-1)-1)-nk}{6}+\frac{k^2e^{-mt}(6(\gamma-1)-1)}{6}+\\ \frac{(ke^{-mt}+1)((\gamma-1)-t)}{t}-\frac{2+ke^{-mt}}{12}\end{array}\right\}e^{-mt}-\frac{4x_1\left\{e^{-(mt-k)e^{2mt}}\right\}}{12}\end{array}\right]$$

4.5. Case I: Dust Model

In order to investigate the physical behavior of the fluid parameters we consider the particular case of dust, when $\gamma = 0$.

The Hubble parameter, Expansion scalar, deceleration parameter, anisotropic parameter, shear scalar are given by

$$H = \frac{1}{3}(n+2)(m+ke^{-mt})$$

$$\theta = 3H = (n+2)(m+ke^{-mt})$$

$$q = -1 \quad \text{............... (37)}$$

$$A_m = \frac{n^2+2n+5}{n^2+4n+4}$$

$$\sigma^2 = \left(\frac{n^2+2n+5}{n^2+4n+4}\right)(n+2)(m+ke^{-mt})$$

The energy density and density parameters are

$$\rho_m = \frac{-1}{\alpha\pi t}\left[\begin{array}{l}\left\{\frac{1}{6t^2}-\frac{1}{16t}+\left(\frac{m(4m-n-nm)}{12}\right)\right\}+\left\{\left(\frac{D_1+ke^{-mt}}{4}\right)+\left(\frac{mk+k^2e^{-mt}}{6}\right)-\frac{nk}{12}-\frac{2nmk}{12}-\frac{nk^2e^{-mt}}{12}\right\}e^{-mt}-\\ \frac{x_1^2\exp\left(-2(mt-k)\exp(2mt)\right)}{12}\end{array}\right]$$

$$\rho_r = \frac{-1}{\alpha\pi t}\left[\begin{array}{l} \dfrac{1}{16t} - \dfrac{11}{48t^2} + \dfrac{2m - nm - m^2 n - 11m^2}{24} + \left\{\begin{array}{l} \left(\dfrac{13mke^{-mt}}{6}\right) + \left(\dfrac{-7k^2 e^{-mt}}{6}\right) - \dfrac{1+t}{t} - \\ \dfrac{(k+t)e^{-mt}}{t} - \\ \dfrac{13nk}{12} - \dfrac{nmk}{6} - \dfrac{nk^2 e^{-mt}}{12} \end{array}\right\} e^{-mt} - \\ \dfrac{x_1^2 \exp\left(-(mt-k)\exp(2mt)\right)}{4} \end{array}\right]$$

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{-3}{(n+2)^2(m+ke^{-mt})^2\alpha\pi t}\left[\left\{\frac{1}{6t^2} - \frac{m^2}{16t} + \frac{4m - m(n+1)}{12m}\right\} + \left\{\begin{array}{l} 3D_1 + 3ke^{-mt} + 2mk + 2k^2 e^{-mt} + \\ \dfrac{k(1 - n - 2nm - nke^{-mt})}{12} \end{array}\right\} e^{-mt} - \frac{x_1^2 e^{-(2mt-k)\exp 2mt}}{12}\right]$$

$$\Omega_r = \frac{\rho_r}{3H^2} = \frac{-3}{(n+2)^2(m+ke^{-mt})^2\alpha\pi t}\left[\left(\frac{1}{16t}\right) - \left(\frac{11}{48t^2}\right) - \frac{m + 6nm + nm^2 + 11}{24} - \left\{\begin{array}{l} \dfrac{mnk}{6} - \dfrac{13mke^{-mt}}{6} - \dfrac{k^2 e^{-mt}}{6} - \dfrac{1+t}{t} - \\ \dfrac{ke^{-2mt} + tke^{-mt}}{t} - \dfrac{13nk}{12} - \dfrac{nk^2}{12} \end{array}\right\} e^{-mt} - \frac{x_1^2 e^{-(mt-k)\exp 2mt}}{4}\right]$$

$$\Omega = \Omega_m + \Omega_r = \frac{-3}{(n+2)^2(m+ke^{-mt})^2\alpha\pi t}\left[\begin{array}{l} \left\{\begin{array}{l} 3D_1 + 2mk + (3 + 3k - nk)ke^{-mt} + \\ \dfrac{k(1 - n(m+1+e^{-mt})}{12} - \dfrac{13nk}{12} \end{array}\right\} e^{-mt} - \left\{\begin{array}{l} \dfrac{13mke^{-mt} - nk + 7k^2 e^{-mt}}{6} - \\ \dfrac{(ke^{-mt}+1)(1+t)}{t} - \dfrac{2 + ke^{-mt}}{12} \end{array}\right\} e^{-mt} - \\ \left(\dfrac{1}{12t^2} + \dfrac{m[(4m-1) - n(m+1)]}{12}\right) - \dfrac{4x_1\left\{e^{-(mt-k)e^{2mt}}\right\}}{12} \end{array}\right]$$

Here, ρ_m *and* ρ_r are negative.

4.6. Case II: Zeldovich Universe

$\gamma = 1$ In this case, the Hubble parameter, Expansion scalar, deceleration parameter, anisotropic parameter, shear scalar are given by

$$H = \frac{1}{3}(n+2)(m+ke^{-mt})$$

$$\theta = 3H = (n+2)(m+ke^{-mt})$$

$$q = -1 \qquad \text{…………….. (38)}$$

$$A_m = \frac{n^2 + 2n + 5}{n^2 + 4n + 4}$$

$$\sigma^2 = \left(\frac{n^2 + 2n + 5}{n^2 + 4n + 4}\right)(n+2)(m+ke^{-mt})$$

The energy density and density parameters for $\gamma = 1$ are

$$\rho_m = \infty, \rho_r = \infty, \Omega_m = \infty \text{ and } \Omega_r = \infty$$

5. Conclusion

The constructed cosmological model is singularity free also we have discussed two different cases of the universe case I for dust model when $\gamma = 0$ we observe that in this case H is constant for $t \to \infty$ and deceleration parameter indicates q = -1 indicates that the expansion of the universe is accelerated which is in good agreement with observational data of the present phase of the universe. Average mean parameter A_m is also nonzero constant which shows that the model is anisotropic. Also, the energy density for matter and radiation vanishes for t tends to infinity and corresponding density parameters are vanishes. In case II we obtain Zeldowich universe for $\gamma = 1$. For this universe also the Hubble parameter and expansion scalar are constant as t tends to infinity. The deceleration parameter q=-1 showing that the universe is accelerating. The anisotropic mean parameter A_m shows that the model is anisotropic, the energy density and density parameter of this model tends to infinity. $\frac{\sigma^2}{\theta^2} = \frac{(n+1)^2 + 4}{3(n+2)^3(m + k\exp(-mt))} \neq 0$ in all the cases. Thus in all the cases the model is anisotropic and accelerating which can be thought as of realistic model of the universe.

Compliance with ethical standards

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Disclosure of conflict of interest

No conflict of interest to be disclosed.

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