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Statefinder Diagnostic of Interacting Dark Fluid in Bianchi Type-IX Universe

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Abstract

In this paper, we deal with spatially homogeneous and anisotropic Bianchi types-IX and Universe filled with Holographic dark energy and cold dark matter. Assuming the condition that the shear scalar is proportional to expansion scalar, we have obtained solutions of Einstein field equations. The Statefinder diagnostic pair i.e. $\{r, s\}$ is adopted to distinguish our dark energy models from other dark energy models. Some important physical features of the models have been discussed.

Keywords: Bianchi types-IX space-time; Interacting dark fluids; Statefinder parameters; Dark Energy; Dark Matter

1. Introduction

Present cosmological observations [1]–[8] suggest that the universe is expanding in an accelerating manner. The reason behind this cosmic acceleration is the unknown form of energy called as dark energy (DE) having negative pressure. The most important result coming from these observations is the fact that only ≈4% of the total energy density of the universe is in the form of baryonic matter, ≈23% is Dark matter (DM), and almost ≈73% is DE. Many cosmologists believe that the simplest candidate for the DE is the cosmological constant (Λ) or vacuum energy since it fits the observational data well. During the cosmological evolution, the Λ-term has the constant energy density and pressure p (de) = −ρ (de). However, one has the reason to dislike the cosmological constant since it always suffers from the theoretical problems such as the ―fine-tuning‖ and ―cosmic coincidence‖ puzzles [9]. That is why, the different forms of dynamically changing DE with an effective equation of state (EoS), ω (de) = p (de)/ρ(de) < −1/3, have been proposed in the literature. Other possible forms of DE include quintessence (ω (de) > −1)[10], phantom (ω (de) < −1) [11] etc. While the possibility ω (de) << −1 is ruled out by current cosmological data from SNeIa (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) [5, 12], CMBR (WMAP, BOOMERANG) [13, 14] and large scale structure (Sloan Digital Sky Survey) [15] data, the dynamically evolving DE crossing the phantom divide line (PDL) (ω (de) = −1) is mildly favored. Some other limits obtained from the observational results coming from SN Ia data [16] collaborated with CMBR anisotropy and galaxy clustering statistics[17] are −1.67 < ω(de) < −0.62 and −1.33 < ω(de) < −0.79 respectively.

In this paper we will discuss Bianchi type-IX cosmological model filled with interacting cold dark matter and dark energy in Einstein's general theory of gravitation. Some physical and kinematical properties of the models are also discussed.

The physical parameters that are of cosmological importance for Bianchi types-IX space-time are

The mean Hubble parameter: $$H = \frac{1}{3}\frac{\dot{V}}{V} \quad \text{............ (1)}$$

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The deceleration parameter: $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$ ………… (2)

The Shear Scalar: $\sigma^2 = \frac{1}{2}(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2)$ ………… (3)

The mean anisotropy parameter: $A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$, ………… (4)

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{A}}{A}, H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of x, y, z axes respectively.

2. Metric and Field Equations:

The spatially homogeneous and anisotropic Bianchi types-IX metric can be written as

$$ds^2 = -dt^2 + A^2(d\theta^2 + sin^2(\theta)d\varphi^2) + B^2(d\phi + \cos\theta\, d\varphi)^2, \quad \text{…………} (5)$$

where θ, ϕ and φ are Eulerian angles, Also A and B are the scale factors and functions of the cosmic time t only .

The Einstein's field equations are ($8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = -({}^{m}T_{ij} + {}^{\Lambda}T_{ij}), \quad \text{…………} (6)$$

$$\text{where } {}^{m}T_{ij} = \rho_m u_i u_j \text{ and } {}^{\Lambda}T_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j + g_{ij} p_\Lambda, \quad \text{…………} (7)$$

are matter tensor for cold dark matter (pressureless i.e. $w_m = 0$) and holographic dark energy. Here ρ_m is the energy density of dark matter and ρ_Λ and p_Λ are the energy density and pressure of Holographic dark energy.

The Einstein's field equations (6) for metric (5) with the help of equations (7) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = -p_\Lambda \quad \text{…………} (8)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2 + 1}{A^2} - \frac{3B^2}{4A^4} = -p_\Lambda, \quad \text{…………} (9)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2 + 1}{A^2} - \frac{B^2}{4A^4} = \rho_\Lambda + \rho_m, \quad \text{…………} (10)$$

Where overhead dot (·) represents derivative with respect to time t.

Further we assume that the interaction between two perfect fluid, cold (pressureless) dark matter and holographic dark energy. We consider exchange of energy between these component in a such matter that continuity equation for holographic dark energy and cold dark matter are given by

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q \quad \text{..........} (11)$$

,

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1+w_\Lambda)\rho_\Lambda = -Q \quad \text{..........} (12)$$

The over dot denotes the derivative with respect to commoving time. ρ_m and ρ_Λ are cold dark matter and holographic dark energy densities respectively. w_Λ is the equation of state parameter for holographic dark energy and is given by $w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$. The quantity Q stands for the interacting term. The direction of transfer of energy depends upon the sign of Q. We assume that Q be positive, which represents energy transfer from dark energy to dark matter. Wetterich [18] and Horvat [19] have introduced the interaction between DE and DM. From continuity equation (11) and (12) imply that the interacting term (Q) should be a function of a quantity with unit of inverse time and the expression for interacting looks purely phonological. It can be expressed phenomenological in form as [20]-[23]

$$Q = 3b^2 H\rho_m = b^2 \frac{\dot{V}}{V}\rho_m, \quad \text{..........} (13)$$

where b^2 is coupling constant. Same relation for interacting phantom dark energy and dark matter has been considered to avoid the coincidence problem [24].

From equations (11) and (12), we get the energy density of dark matter as

$$\rho_m = \rho_0 V^{(b^2-1)} \quad \text{..........} (14)$$

where $\rho_0 > 0$ is a real constant of integration.

Using equations (11) and (14), we get the interacting term as

$$Q = 3\,\rho_0 b^2 H V^{(b^2-1)}. \quad \text{..........} (15)$$

3. Cosmological Solutions:

Now, we assume that the shear scalar (σ) in the models proportional to expansion scalar (θ)

$$B = A^n \quad \text{..........} (16)$$

where A and B are the metric potentials and $n > 0$, $n \neq 1$ is constant.

$$\frac{\ddot{A}}{A} + c_1\frac{\dot{A}^2}{A^2} - \frac{1}{(n-1)A^2} + \frac{A^{2n-4}}{(n-1)} = 0,\ n \neq 1, \quad \text{..........} (17)$$

From equation (17), for n=2 and with suitable substitution, we obtain

$$\dot{A}^2 = c_2^2 - c_3^2 A^2, \quad \text{..........} (18)$$

where c_2^2 and c_2^3 are real constants of integration.

From equation (18), we obtain

$$A = \left(\frac{c_2}{c_3}\right)\sin(c_3 t) \quad \text{.}\ldots\ldots\ldots\ldots (19)$$

From equations (19) and (16), we get

$$B = \left[\left(\frac{c_2}{c_3}\right)\sin(c_3 t)\right]^2 \quad \text{.}\ldots\ldots\ldots\ldots (20)$$

The volume scale factor V is defined and obtained as,

$$V = A^2 B = \left[\left(\frac{c_2}{c_3}\right)\sin(c_3 t)\right]^4 \quad \text{.}\ldots\ldots\ldots\ldots (21)$$

Mean Hubble parameter, Deceleration parameter, Shear Scalar and Mean anisotropy parameter obtained as,

$$H = \frac{4}{3} c_3 \cot(c_3 t) \quad \ldots\ldots\ldots\ldots (22)$$

$$q = \frac{3}{4}\sec^2(c_3 t) - 1 \quad \ldots\ldots\ldots\ldots (23)$$

$$\sigma^2 = \frac{c_3^2 \cot^2(c_3 t)}{3} \quad \ldots\ldots\ldots\ldots (24)$$

$$A_m = \frac{1}{8} \quad \text{.}\ldots\ldots\ldots\ldots (25)$$

Using equation (21) in equations (14) and (15), we get energy density of dark matter and interacting term as,

$$\rho_m = \rho_0 \left(\frac{c_2}{c_3}\sin(c_3 t)\right)^{4(b^2-1)} \quad \ldots\ldots\ldots\ldots (26)$$

$$Q = 4b^2 \rho_0 c_3 \cot(c_3 t)\left(\frac{c_2}{c_3}\sin(c_3 t)\right)^{4(b^2-1)} \quad \text{.}\ldots\ldots\ldots\ldots (27)$$

Using equations (19), (20) and (26) in the equation (10), we obtain the energy density of holographic dark energy as,

$$\rho_\Lambda = 5c_3^2 \cot^2(c_3 t) + \frac{c_3^2}{c_2^2}\operatorname{cosec}^2(c_3 t) - \frac{1}{4} - \rho_0 \left(\frac{c_2}{c_3}\sin(c_3 t)\right)^{4(b^2-1)} \quad \text{.}\ldots\ldots\ldots\ldots (28)$$

Using equations (19), (20) in the equation (8), we obtain the pressure of holographic dark energy as,

$$p_\Lambda = 3c_3^2 - 4c_3^2 \cot^2(c_3 t) - 1/4 \;.\;\;\ldots\ldots\ldots\ldots (29)$$

The EoS parameter of holographic dark energy is given by,

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda},$$

$$w_\Lambda = \frac{3c_3^2 - 4c_3^2 \cot^2(c_3 t) - 1/4}{5c_3^2 \cot^2(c_3 t) + \frac{c_3^2}{c_2^2} \operatorname{cosec}^2(c_3 t) - \frac{1}{4} - \rho_0 \left(\frac{c_2}{c_3} \sin(c_3 t)\right)^{4(b^2-1)}} \;.\;\;\ldots\ldots\ldots\ldots (30)$$

The coincidence parameter $\bar{r} = \rho_m / \rho_\Lambda$, which is the ratio of two energies density i.e. the ratio of dark matter energy density to the dark energy density is given by,

$$\bar{r} = \frac{\rho_0 \left(\frac{c_2}{c_3} \sin(c_3 t)\right)^{4(b^2-1)}}{5c_3^2 \cot^2(c_3 t) + \frac{c_3^2}{c_2^2} \operatorname{cosec}^2(c_3 t) - \frac{1}{4} - \rho_0 \left(\frac{c_2}{c_3} \sin(c_3 t)\right)^{4(b^2-1)}} \;.\;\;\ldots\ldots\ldots\ldots (31)$$

A new cosmological diagnostic pair $\{r, s\}$ called as statefinder parameters is defined as [25]

$$r = \frac{\dddot{a}}{aH^3} = \frac{-(1 + 9\tan^2(c_3 t))}{8} \;\;\ldots\ldots\ldots\ldots (67)$$

$$s = \frac{r-1}{3(q-1/2)} = \frac{-\sec^2(c_3 t)}{2(\tan^2 c_3 t - 1)} \;.\;\;\ldots\ldots\ldots\ldots (68)$$

$$s = \frac{2(1-r)}{(4r+5)} \;\;\ldots\ldots\ldots\ldots (69)$$

4. Discussion

In the present paper we have studies the anisotropic and homogeneous Bianchi type-IX with interacting dark energy and dark matter. Also, we have studied the statefinder parameters. Our model is oscillating as deceleration parameter oscillates from negative to positive and vice versa. The statefinder diagnostic is tool is applied in order to distinguish our model with other DE models. The universe is anisotropic throughout the evolution of the universe.

5. Conclusion

The accelerating universe due to the Dark energy is clearly observed from our model also due to interacting dark fluids the is possibility of no coincidence problem. This Study will help to understand the dynamical nature of Dark Energy.

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