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Common Fixed Point Theorems in Bipolar Metric Spaces

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Abstract

In this paper, a common fixed point theorem for generalized contractions in bipolar metric spaces is proved. Additionally, these theorems expand and apply a number of intriguing findings from metric fixed point theory to the bipolar metric setting. I also provide a few instances to illustrate my theorems.

Keywords: Common fixed point; Iterative methods; Contraction; Bipolar metric spaces

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1. Introduction

Due to its numerous applications in the fields of applied mathematics and the sciences, metric fixed point theory is becoming more and more important in mathematics. The study of non-linear phenomena greatly benefits from the use of fixed-point theory. It is an interdisciplinary area of mathematics that has applications in many different areas of mathematics as well as in other disciplines, such as biology, chemistry, physics, engineering, game theory, mathematical economics, optimization issues, approximation theory, initial and boundary value issues in ordinary and partial differential equations, and variational inequalities.

I can think about the distances between points in a set, in a classical or non-classical sense, using metric spaces and many of their generalizations. Instead of arising between points of a particular set, distances may occur between components of two different sets. Due to a lack of information, distances between the same kinds of sites in these situations are either undefined or undetermined. For instance, knowing only the distances between a food delivery service's sites and the available delivery addresses would be sufficient if it were assumed that each deliveryman could only carry one order at a time. This would avoid managing the enormous amount of information related to the distances between delivery addresses. In science and mathematics, there are numerous examples of these kinds of distances.

The conventional idea of a metric space has undergone a variety of generalizations. A bipolar metric space that Ali Mutlu and Utku Gurdal developed and investigated [4] is one such generalization. The most important finding in fixed point theory, which had an impact on many scholars, was made in 1922 by the Polish mathematician Stefan Banach [1].

The following fixed point theorem was established by Kannan [3] in 1968.

1.1. Theorem 1.1

Let $(\mathfrak{D}, \varrho)$ be a complete metric space and let $\Gamma: \mathfrak{D} \to \mathfrak{D}$be a mappingsuch that there exists $k < \frac{1}{2}$ satisfying

$$\varrho(\Gamma\sigma, \Gamma\varsigma) \leq k[\varrho(\sigma, \Gamma\sigma) + c\varrho(\varsigma, \Gamma\varsigma)] \ldots\ldots (1.1)$$

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for all $\sigma, \varsigma \in \mathfrak{D}$. Then, Γ has a unique fixed point $\omega \in \mathfrak{D}$, and for any $\sigma \in \mathfrak{D}$ the sequence of iterates $\{\Gamma^n\sigma\}$ converges to ω and

$$\varrho(\Gamma^{n+1}\sigma, \omega) \leq k\left(\frac{k}{1-k}\right)^n \varrho(\omega, \Gamma\omega), n = 0, 1, 2, \ldots$$

The fixed point theorem of Kannan and some of its generalizations are examined in [9-13].

I shall discuss a common fixed-point theorem for generalized contractions in bipolar metric spaces in this article. Our findings extend the contractions of the metric space to a bipolar metric space by Banach's contraction and Kannan's contraction. Additionally, we demonstrate how our findings expand upon, generalize, and improve upon earlier findings in the literature on bipolar metric spaces.

2. Preliminaries

$\mathbb{N}$ and $\mathbb{R}$ refer to the set of all positive integers and the set of all real numbers, respectively, throughout this essay. We specifically write $\mathbb{R}^+ = [0, +\infty)$ to represent the set of all non-negative reals. To make this paper self-sufficient, we review some mathematical fundamentals and concepts.

2.1. *Definition* 2.1 (see [3])

Let $\mathfrak{D}$ and $\mathfrak{E}$be non-empty sets.A bipolar metric on the pair $(\mathfrak{D}, \mathfrak{E})$is a function $\varrho: \mathfrak{D} \times \mathfrak{E} \to \mathbb{R}^+$ complying with following axioms:

(bm1) $\sigma = \varsigma$, if $\varrho(\sigma, \varsigma) = 0; \forall(\sigma, \varsigma) \in (\mathfrak{D}, \mathfrak{E})$.

(bm2) $\varrho(\sigma, \varsigma) = 0$, if $\sigma = \varsigma; \forall(\sigma, \varsigma) \in (\mathfrak{D}, \mathfrak{E})$.

(bm3) $\varrho(\sigma, \varsigma) = \varrho(\varsigma, \sigma)$, if $\sigma, \varsigma \in \mathfrak{D} \cap \mathfrak{E}$.

(bm4) $\varrho(\sigma_1, \varsigma_2) \leq \varrho(\sigma_1, \varsigma_1) + \varrho(\sigma_2, \varsigma_1) + \varrho(\sigma_2, \varsigma_2), \forall\sigma_1, \sigma_2 \in \mathfrak{D}$ and , $\forall\varsigma_1, \varsigma_2 \in \mathfrak{E}$.

The triple$(\mathfrak{D}, \mathfrak{E}, \varrho)$is called a bipolar metric space.

On the pair $(\mathfrak{D}, \mathfrak{E})$, ϱ is referred to as a bipolar pseudo-semi metric if (bm2) and (bm3) hold. It is referred to as a bipolar pseudo-metric if is a bipolar pseudo-semi metric satisfying (bm4). A bipolar metric is a bipolar pseudo-metric satisfying (bm1). A triple $(\mathfrak{D}, \mathfrak{E}, \varrho)$ is a bipolar (pseudo-(semi)) metric space, where ϱ is a bipolar (pseudo-(semi)) metric on $(\mathfrak{D}, \mathfrak{E})$. In specifically, a space is said to be disjointed if $\mathfrak{D} \cap \mathfrak{E} = \emptyset$, and joint otherwise. The left pole and the right pole of $(\mathfrak{D}, \mathfrak{E}, \varrho)$ are the sets $\mathfrak{D}$ and $\mathfrak{E}$, respectively.

Example 2.2(see [3]) Consider the case when $(\mathfrak{D}, \varrho)$ is a (pseudo-(semi)) metric space. Consequently, $(\mathfrak{D}, \mathfrak{D}, \varrho)$ is a bipolar (pseudo-(semi)) metric space. But if $(\mathfrak{D}, \mathfrak{E}, \varrho)$ is a bipolar (pseudo-(semi)) metric space with $\mathfrak{D} = \mathfrak{E}$, then $(\mathfrak{D}, \varrho)$ is a (pseudo-(semi))metric space.

2.2. Definition 2.3(see [3])

Let $(\mathfrak{D}_1, \mathfrak{E}_1)$ and $(\mathfrak{D}_2, \mathfrak{E}_2)$ be two pair of sets. A map $\Gamma: \mathfrak{D}_1 \cup \mathfrak{E}_1 \to \mathfrak{D}_2 \cup \mathfrak{E}_2$ is called

- Covariant if $\Gamma(\mathfrak{D}_1) \subseteq \mathfrak{D}_2$And $\Gamma(\mathfrak{E}_1) \subseteq \mathfrak{E}_2$, and it is denoted as$\gamma: (\mathfrak{D}_1, \mathfrak{E}_1) \rightrightarrows (\mathfrak{D}_2, \mathfrak{E}_2)$.
- Contravariant if$\gamma(\mathfrak{D}_1) \subseteq \mathfrak{E}_2$And $\Gamma(\mathfrak{E}_1) \subseteq \mathfrak{D}_2$, and it is denoted as $\Gamma: (\mathfrak{D}_1, \mathfrak{E}_1) \rightleftarrows (\mathfrak{D}_2, \mathfrak{E}_2)$.

2.3. Definition 2.4(see [3])

Let $(\mathfrak{D}, \mathfrak{E}, \varrho)$ be a bipolar metric space. Then,

- $\mathfrak{D}$ = set of left points; $\mathfrak{E}$ = set of right points; $\mathfrak{D} \cap \mathfrak{E}$ = set of central points. In particular, if$\mathfrak{D} \cap \mathfrak{E} = \emptyset$, the space is called disjoint, and otherwise it is called joint. Unless otherwise stated, we shall work with joint spaces.
- A sequence (σ_n) on the set $\mathfrak{D}$ is called a left sequence, and a sequence (ς_n)on $\mathfrak{E}$ is called a right sequence. In a bipolar metric space, a left or a right sequence is called simply a sequence.

- A sequence (σ_n) is said to be convergent to a point σ if and only if (σ_n) is a left sequence, $\lim_{n\to\infty} \varrho(\sigma_n, \sigma) = 0$ and $\sigma \in \mathfrak{E}$, or (σ_n) is a right sequence, $\lim_{n\to\infty} \varrho(\sigma, \sigma_n) = 0$ and $\sigma \in \mathfrak{D}$.
- A bisequence (σ_n, ς_n) on $(\mathfrak{D}, \mathfrak{E}, \varrho)$ is a sequence on the set $\mathfrak{D} \times \mathfrak{E}$. Furthermore, if the sequences (σ_n) and (ς_n) are convergent, then the bisequence (σ_n, ς_n) is said to be convergent. In addition, if (σ_n) and (ς_n) converge to a common point $\in \mathfrak{D} \cap \mathfrak{E}$, then (σ_n, ς_n) is called biconvergent.
- A bisequence (σ_n, ς_n) is a Cauchy bisequence if $\lim_{n\to\infty} \varrho(\sigma_n, \varsigma_n) = 0$.

2.3.1. Remark 2.5(see [3])

In a bipolar metric space, every convergent Cauchy bisequence is biconvergent.

2.4. Definition 2.6(see [3])

A bipolar metric space is called complete if every Cauchy bisequence is convergent, hence biconvergent.

Example 2.7(see [3]) Assume that $\mathfrak{E}$ is the class of all nonempty compact subsets of $\mathbb{R}$ and that $\mathfrak{D}$ is the class of all singleton subsets of $\mathbb{R}$. We define $\varrho: \mathfrak{D} \times \mathfrak{E} \to \mathbb{R}^+$ as $\varrho(\sigma, A) = |\sigma - \inf(A)| + |\sigma - \sup(A)|$. The triple $(\mathfrak{D}, \mathfrak{E}, \varrho)$ is a complete bipolar metric space.

2.5. Definition 2.8(see [3])

A covariant or a contravariant map S from the bipolar metric space $(\mathfrak{D}_1, \mathfrak{E}_1, \varrho_1)$ to the bipolar metric space $(\mathfrak{D}_2, \mathfrak{E}_2, \varrho_2)$ is continuous, if and only if $\sigma_n \to \varsigma$ on $(\mathfrak{D}_1, \mathfrak{E}_1, \varrho_1)$ implies $\Gamma(\sigma_n) \to \Gamma(\varsigma)$ on $(\mathfrak{D}_2, \mathfrak{E}_2, \varrho_2)$.

3. Main Results

I present a common fixed theorem based on bipolar metric spaces in this section.

3.1. Theorem 3.1

Let $(\mathfrak{D}, \mathfrak{E}, \varrho)$ be a complete bipolar metric space and $\Gamma, \Delta: (\mathfrak{D}, \mathfrak{E}, \varrho) \rightleftarrows (\mathfrak{D}, \mathfrak{E}, \varrho)$ be contravariant mappingssuch that there exist constant $a, b, c \geq 0$ with $a + b + c < 1$ satisfying

$$\varrho(\Delta\varsigma, \Gamma\sigma) \leq a\varrho(\sigma, \varsigma) + b\varrho(\sigma, \Gamma\sigma) + c\varrho(\Delta\varsigma, \varsigma) \ldots\ldots\ldots(3.1)$$

for all $(\sigma, \varsigma) \in \mathfrak{D} \times \mathfrak{E}$, with $\sigma \neq \varsigma$. Then, $\Gamma, \Delta: \mathfrak{D} \cup \mathfrak{E} \to \mathfrak{D} \cup \mathfrak{E}$ have a unique common fixed point, provided that Γ and Δ are continuous in $(\mathfrak{D}, \mathfrak{E})$.

Proof :Let $\sigma_0 \in \mathfrak{D}$ and $\varsigma_0 \in \mathfrak{E}$. We employ one of the iterative approaches described below to define sequences $\{\sigma_n\}$ and $\{\varsigma_n\}$ for each $n \in \mathbb{N} \cup \{0\}$:

$$\Delta\sigma_{2n} = \varsigma_{2n}, \Gamma\sigma_{2n+1} = \varsigma_{2n+1}, \Delta\varsigma_{2n} = \sigma_{2n+1}, \Gamma\varsigma_{2n+1} = \sigma_{2n+2} \ldots\ldots(3.2)$$

By (3.1), we now obtain

$$\varrho(\sigma_{2n+1}, \varsigma_{2n+1}) = \varrho(\Delta\varsigma_{2n}, \Gamma\sigma_{2n+1}) \ldots\ldots\ldots(3.3)$$

$$\leq a\varrho(\sigma_{2n+1}, \varsigma_{2n}) + b\varrho(\sigma_{2n+1}, \Gamma\sigma_{2n+1}) + c\varrho(\Delta\varsigma_{2n}, \varsigma_{2n})$$

$$= a\varrho(\sigma_{2n+1}, \varsigma_{2n}) + b\varrho(\sigma_{2n+1}, \varsigma_{2n+1}) + c\varrho(\sigma_{2n+1}, \varsigma_{2n})$$

The last inequality provides

$$\varrho(\sigma_{2n+1}, \varsigma_{2n+1}) \leq \left(\frac{a+c}{1-b}\right) \varrho(\sigma_{2n+1}, \varsigma_{2n}) \ldots\ldots\ldots(3.4)$$

We also acquire

$$\varrho(\sigma_{2n+1}, \varsigma_{2n}) = \varrho(\Delta\varsigma_{2n}, \Delta\sigma_{2n}) \ldots\ldots\ldots(3.5)$$

$$\leq a\varrho(\sigma_{2n},\varsigma_{2n}) + b\varrho(\sigma_{2n},\Delta\sigma_{2n}) + c\varrho(\Delta\varsigma_{2n},\varsigma_{2n})$$

$$= a\varrho(\sigma_{2n},\varsigma_{2n}) + b\varrho(\sigma_{2n},\varsigma_{2n}) + c\varrho(\sigma_{2n+1},\varsigma_{2n})$$

The last inequality gives

$$\varrho(\sigma_{2n+1},\varsigma_{2n}) \leq \left(\frac{a+b}{1-c}\right)\varrho(\sigma_{2n},\varsigma_{2n})\text{.........(3.6)}$$

Let k be the maximum of $\frac{a+c}{1-b}$, and $\frac{a+b}{1-c}$. Then, $k < 1$ and based on (3.4) and (3.6), we deduce that

$$\varrho(\sigma_{2n+1},\varsigma_{2n+1}) \leq k^{4n+2}\varrho(\sigma_0,\varsigma_0)$$

and

$$\varrho(\sigma_{2n+1},\varsigma_{2n}) \leq k^{4n+1}\varrho(\sigma_0,\varsigma_0)$$

Now, for each $n \in \mathbb{N}$, we may obtain that

$$\varrho(\sigma_{n+1},\varsigma_{n+1}) \leq k^{2n+2}\varrho(\sigma_0,\varsigma_0),$$

$$\varrho(\sigma_{n+1},\varsigma_{n}) \leq k^{2n+1}\varrho(\sigma_0,\varsigma_0),$$

$$\varrho(\sigma_{n},\varsigma_{n}) \leq k^{2n}\varrho(\sigma_0,\varsigma_0).$$

We also take into account the following scenarios for all $m, n \in \mathbb{N}$:

3.2. Case 1

If $m > n$, we have

$$\varrho(\sigma_n,\varsigma_m) \leq \varrho(\sigma_n,\varsigma_n) + \varrho(\sigma_{n+1},\varsigma_n) + \varrho(\sigma_{n+1},\varsigma_m)$$

$$\leq k^{2n}\varrho(\sigma_0,\varsigma_0) + k^{2n+1}\varrho(\sigma_0,\varsigma_0) + \varrho(\sigma_{n+1},\varsigma_m)$$

$$\leq (k^{2n} + k^{2n+1})\varrho(\sigma_0,\varsigma_0) + \varrho(\sigma_{n+1},\varsigma_{n+1}) + \varrho(\sigma_{n+2},\varsigma_{n+1}) + \varrho(\sigma_{n+2},\varsigma_m)$$

$$\leq (k^{2n} + k^{2n+1})\varrho(\sigma_0,\varsigma_0) + (k^{2n+2} + k^{2n+3})\varrho(\sigma_0,\varsigma_0) + \varrho(\sigma_{n+2},\varsigma_m)$$

$$\leq k^{2n}\left(1 + k + k^2 + k^3 + \cdots + k^{2(m-n)}\right)\varrho(\sigma_0,\varsigma_0)$$

$$\leq k^{2n}\left(\frac{1 - k^{2(m-n)+1}}{1-k}\right)\varrho(\sigma_0,\varsigma_0)$$

$$\text{Hence, } \lim_{m,n\to\infty}\varrho(\sigma_n,\varsigma_m) = 0.$$

3.3. Case 2

If $m < n$, we have

$$\varrho(\sigma_n,\varsigma_m) \leq \varrho(\sigma_{m+1},\varsigma_m) + \varrho(\sigma_{m+1},\varsigma_{m+1}) + \varrho(\sigma_n,\varsigma_{m+1})$$

$$\leq k^{2m+1}\varrho(\sigma_0,\varsigma_0) + k^{2m+2}\varrho(\sigma_0,\varsigma_0) + \varrho(\sigma_n,\varsigma_{m+1})$$

$$\leq (k^{2m+1} + k^{2m+2})\varrho(\sigma_0,\varsigma_0) + \varrho(\sigma_{m+2},\varsigma_{m+1})$$

$$+\varrho(\sigma_{m+2},\varsigma_{m+2}) + \varrho(\sigma_n,\varsigma_{m+2})$$

$$\leq (k^{2m+1} + k^{2m+2})\varrho(\sigma_0, \varsigma_0) + (k^{2m+3} + k^{2m+4})\varrho(\sigma_0, \varsigma_0) + \varrho(\sigma_n, \varsigma_{m+2})$$

$$\leq k^{2m+1}\left(1 + k + k^2 + k^3 + \cdots + k^{2(m-n-1)}\right)\varrho(\sigma_0, \varsigma_0)$$

$$\leq k^{2m+1}\left(\frac{1 - k^{2(m-n)-1}}{1-k}\right)\varrho(\sigma_0, \varsigma_0)$$

Since, $k < 1$, hence, $\lim_{m,n\to\infty} \varrho(\sigma_n, \varsigma_m) = 0$.This indicates that $\varrho(\sigma_n, \varsigma_m)$can be made arbitrarily small by large m and n, and hence (σ_n, ς_m) is a Cauchybisequence in $(\mathfrak{D}, \mathfrak{E})$.The bisequence$(\sigma_n, \varsigma_m)$biconverges to some $\sigma^* \in \mathfrak{D} \cap \mathfrak{E}$ such that $\lim_{n\to\infty} \sigma_n = \lim_{n\to\infty} \varsigma_n = \sigma^*$ due to the completeness of $(\mathfrak{D}, \mathfrak{E}, \varrho)$.Also $\lim_{n\to\infty} \Delta\sigma_{2n} = \lim_{n\to\infty} \varsigma_{2n} = \sigma^* \in \mathfrak{D} \cap \mathfrak{E}$ implies that $\Delta\sigma_{2n}$has a unique limit σ^*, and $\sigma_n \to \sigma^*$ implies that $\sigma_{2n} \to \sigma^*$. Now, $\Delta\sigma_{2n} \to \Delta\sigma^*$ is implied by the continuity of Δ. Consequently, $\Delta\sigma^* = \sigma^*$.

In a similar way, the statements $\Gamma\varsigma_{2n+1} = \sigma_{2n+2} = \sigma^* \in \mathfrak{D} \cap \mathfrak{E}$ implies that $\Gamma\varsigma_{2n+1}$ has a unique limitσ^*, and $\varsigma_n \to \sigma^*$ implies that $\varsigma_{2n+1} \to \sigma^*$. Now, $\Gamma\varsigma_{2n+1} \to \Gamma\sigma^*$ follows from the continuity of Γ. Consequently, $\Gamma\sigma^* = \sigma^*$. Thus, $\Delta\sigma^* = \Gamma\sigma^* = \sigma^*$.

We shall now prove the uniqueness of the common fixed point. If$\varsigma^* \in \mathfrak{D} \cap \mathfrak{E}$is another common fixed point of ΔandΓ, that is,$\Delta\varsigma^* = \Gamma\varsigma^* = \varsigma^*$,then we get

$$\varrho(\varsigma^*, \sigma^*) = \varrho(\Delta\varsigma^*, \Gamma\sigma^*) \leq a\varrho(\sigma^*, \varsigma^*) + b\varrho(\sigma^*, \Gamma\sigma^*) + c\varrho(\Delta\varsigma^*, \varsigma^*)$$

$$\leq a\varrho(\sigma^*, \varsigma^*) + b\varrho(\sigma^*, \sigma^*) + c\varrho(\varsigma^*, \varsigma^*).$$

Therefore, $\varrho(\varsigma^*, \sigma^*) \leq a\varrho(\sigma^*, \varsigma^*)$,which is contradictory, and hence, $\sigma^* = \varsigma^*$.

Theorem 3.1 can now be verified by the example given below.

Example 3.2 Let $\mathfrak{D} = \{7, 8, 11, 17\}$ and $\mathfrak{E} = \{2, 4, 17, 18\}$. Define $\varrho: \mathfrak{D} \times \mathfrak{E} \to \mathbb{R}^+$ as the usualmetric, $\varrho(\sigma, \varsigma) = |\sigma - \varsigma|$. Then, the triple $(\mathfrak{D}, \mathfrak{E}, \varrho)$ is a complete bipolar metric space. The contravariant mappings $\Gamma, \Delta: (\mathfrak{D}, \mathfrak{E}, \varrho) \rightleftarrows (\mathfrak{D}, \mathfrak{E}, \varrho)$defined by

$$\Gamma\sigma = \begin{cases} 17, & \sigma \in \mathfrak{D} \cup \{18\} \\ 18, & \text{otherwise} \end{cases}$$

and

$$\Delta\sigma = \begin{cases} 17, & \sigma \in \{17,18\} \\ 18, & \text{otherwise} \end{cases}$$

Satisfy the inequality of Theorem 3.1 for $a = \frac{1}{4}, b = \frac{1}{5}$ and $c = \frac{1}{5}$and $147 \in \mathfrak{D} \cap \mathfrak{E}$ is the only commonfixed point of Γ and Δ.

4. Conclusion

In this work, we have taken advantage of the notion of bipolar metric to present new contraction conditions. Next, we proved a common fixed point theorems in the context of bipolar metric spaces. An example is provided to show the validity and usefulness of our findings.

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