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LRS Bianchi type-I model with polytropic EoS in $f(R)$ gravity

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Abstract

In the present work we have focused on investigated dynamical aspects of LRS Bianchi type –I metric within the frame work of $f(R)$ theory of gravitation. The solution of the metric is accelerating universe are derived by assuming the negative constant deceleration parameter and utilizing a power law relation. Additionally, we have visually analyzed the metric's dynamical and geometrical properties through graphical representations.

Keywords: LRS Bianchi Model; Polytropic EoS; $f(R)$ gravity; Deceleration parameter

1. Introduction

In the beginning stage of evolution of the universe, the nature of the universe remains a mystery. After a Big-bang, there are several phase changes as the temperature dropped down. The recent discovery of the modern cosmos such as Type Ia supernovae, large scale structure gives indirect proofs of the universe undergoing exponential acceleration [1-5]. Adhav [6] shows the expanding universe with respect to cosmic time t by using Kantowski–Sachs string cosmological model in $f(R)$ theory of gravity.

The $f(R)$ theory of gravity is generalization of general theory of gravity by considering the function of Ricci scalar R. Researcher did their work to study the actual nature of the universe by using $f(R)$ theory of gravity. Santhi *et al.* [7] studied Bianchi type-II, VII and IX bulk viscous string cosmological models in context of $f(R)$ gravity and shows that how the cosmos changes from early deceleration to the acceleration. LRS Bianchi type-I metric in $f(R)$ gravity has been studied by Aditya and Reddy [8] and he got the result that at late time the anisotropic effect disappears. Hatkar *et al.* [9] explore Kasner type of Bianchi type-I model with non-interacting string and HDE in $f(R)$ theory of gravitation and also found that the string like phase in the cosmos existed early in the history of universe. Also different authors studied different conceptual phenomenon in the context of $f(R)$ theory of gravity [10, 21].

In the General Theory of Relativity the Adhav *et al.* [22] investigate "higher dimensional spherically symmetric universe with polytropic equation of state" by considering the five dimensional static spherically star. The physical aspect of Kantowski-Sachs universe has been studied by Samdurkar *et al.* [23] with variable deceleration parameter and polytropic EoS $p = \omega\rho^2 - \rho$ for polynomial and exponential scenario. In $f(R)$ theory of gravity polytropic EoS has been studied in different kind of context. The behavior of flat geometric anisotropic spheres with polytropic EoS in $f(R)$ theory with the help of Schwarzschild metric and electromagnetic field as source has been examined with their dynamical properties by Bhatti and Tariq [24], Cooney *et al.* [25] examined the composition and characteristics of neutron stars in the context of $f(R)$ gravity theories which incorporates perturbative constraints by considering

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polytropic equation of state. Maeda *et al.* [26], Chirde *et al.* [27] and Adhav *et al*. [28] put their efforts to study the different kind of equation of state in $f(R)$ theory of gravity.

Motivated by the above-mentioned study, our current research focusses on LRS Bianchi type-I universe with polytropic EoS in $f(R)$ gravity. The work is divided into different sections and arranged as introduction, basic formation of $f(R)$ gravity, the solution of field equation with cosmological model, discussion of dynamical and geometrical properties, graphical approach, result and references.

2. Basic formation of $f(R)$ gravity

The $f(R)$ theory of gravitation was proposed by Buchdahl [29] in 1970 in which he used ϕ instead of f . The Generalization in General Theory of Relativity is $f(R)$ theory of Gravity. The “Metric Approach”, “Palatine Approach” and “Affine $f(R)$ gravity” are the three main approaches in $f(R)$ theory of gravity.

The action for $f(R)$ gravity is given by

$$S=\frac{1}{2}\int\sqrt{-g}\,\left(f(R)+L_M\left(g_{ij},\psi\right)\right)d^4x \qquad ,..........(1)$$

Where $f(R)$ is a general function of the Ricci scalar R , g is the determinant of metric g_{ij} and L_m is metric Lagrangian which is depends on the metric g_{ij} and ψ the matter field.

The field equations in metric formalism for $f(R)$ gravity by varying the action given in equation (1) with respect to the metric g_{ij} is given by

$$F(R)R_{ij}-\frac{1}{2}f(R)\,g_{ij}+[g_{ij}g^{ij}\nabla_i\nabla_j-\nabla_i\nabla_j]F(R)\;=T_{ij},..........(2)$$

where $F=\frac{df}{dR}$, ∇ is an covariant derivative, $g^{ij}\nabla_i\nabla_j$ is an D’Alemberts operator and T_{ij} is standard energy momentum tensor obtained by using Lagrangian L_m given by

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{ij}}\;,..............(3)$$

For the perfect fluid the energy momentum tensor is,

$$T_{ij}=\left(p+\rho\right)u_iu_j-pg_{ij},..........(4)$$

with $u^i=\left(1,0,0,0\right)$ being the four velocity, for which $u^iu_i=1,\;u^i\nabla_ju_i=0$.

3. Metric and dynamical parameter

Here we studied LRS Bianchi type-I cosmological model given by

$$ds^2=dt^2-A_1^2(t)\,dx^2-A_2^2(t)\left(dy^2+dz^2\right),.................(5)$$

The Dynamical parameters related to cosmological model (5) are defined as follows.

The special volume V in the form of average scale factor $a(t)$ of model is defined as

$$V = a(t)^3 = A_1 A_2^2 \quad ,............ (6)$$

The mean Hubble parameter H is defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\left(\frac{\dot{A}_1}{A_1} + 2\frac{\dot{A}_2}{A_2}\right) ....(7)$$

where H_x, H_y, H_z are directional Hubble parameter in the direction of x, y, z respectively.

To discuss the anisotropy of universe the anisotropic parameter Δ is define as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 ....(8)$$

The expansion parameter θ is defined as

$$\theta = 3H ....(9)$$

The deceleration parameter $q(t)$ is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, ... (10)$$

It grants the acceleration of the universe for $-1 \le q(t) < 0$, deceleration of the universe for the $q(t) > 0$ and constant rate of expansion for $q(t) = 0$.

The Ricci scalar R of model is given by

$$R = 2\left[\frac{\ddot{A}_1}{A_1} + 2\frac{\ddot{A}_2}{A_2} + 2\frac{\dot{A}_1\dot{A}_2}{A_1A_2} + \left(\frac{\dot{A}_2}{A_2}\right)^2\right] .... (11)$$

3.1. State finding Parameter

The pair of state finding parameter (r, s) is given by [30],

$$r = \frac{\dddot{a}}{a}\frac{1}{H^3}, \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)H^3}, ... (12)$$

Where r is jerk parameter, H is a mean Hubble parameter given in equation (7), q is deceleration parameter, a is average scale factor and over headed triple dots denotes the derivative with respect to cosmic time t.

4. Solution of field equation

In the presence of perfect fluid as source given in equation (4), the field equation (2) analogous with metric given in equation (5) gives the set of linearly independent equations as

$$\frac{\ddot{A}_1}{A_1}+2\frac{\dot{A}_1\dot{A}_2}{A_1A_2}-\frac{1}{2F}f(R)+2\left(\frac{\dot{A}_2}{A_2}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}+\frac{p}{F}=0 \quad \text{............(13)}$$

$$\frac{\ddot{A}_2}{A_2}+\frac{\dot{A}_1\dot{A}_2}{A_1A_2}+\frac{\ddot{A}_2^2}{A_2^2}-\frac{1}{2F}f(R)+\left(\frac{\dot{A}_1}{A_1}+\frac{\dot{A}_2}{A_2}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}+\frac{p}{F}=0 \quad \text{............(14)}$$

$$\frac{\ddot{A}_2}{A_2}+2\frac{\ddot{A}_1^2}{A_1^2}-\frac{1}{2F}f(R)+\left(\frac{\dot{A}_1}{A_1}+2\frac{\dot{A}_2}{A_2}\right)\frac{\dot{F}}{F}-\frac{\rho}{F}=0 \quad \text{...........(15)}$$

Here the overhead dots represent a differentiation with respect to t.

The plytropic equation of state followed by perfect fluid (4) is given by Adhav *et al.* [28]

$$p=\omega\rho^n-\rho \quad \text{...................(16)}$$

Where ω & n are polytropic constant and polytropic index respectively.

On subtracting equation (12), (11) and solving them, we get

$$\frac{A_1}{A_2}=\exp\left(\int\frac{cF}{A_1A_2^2}dt\right) \text{................(17)}$$

The power law relation between F and a presented by Sharif and Shamir [27], is given by

$$F=\varsigma\, a^k, \text{........(18)}$$

Where k is arbitrary constant and ς is proportionality constant.

Without loss of generality we take $\varsigma=1$. Therefore equation (18) reduces to

$$F=a^k \text{ ,...........(19)}$$

where k is constant.

Using equation (7) and (19), we get

$$F=\left(A_1A_2^2\right)^{\frac{k}{3}}, \quad \text{........................(20)}$$

Using equation (17) and equation (20), we get

$$\frac{A_1}{A_2}=\exp\left(\int cV^{\frac{k-3}{3}}dt\right) \text{...(21)}$$

Using equation (13), (14) and (16), we get

$$\rho^n=\frac{1}{\omega}\left[2\left(\frac{\ddot{A}_2}{A_2}-\frac{\dot{A}_1\dot{A}_2}{A_1A_2}\right)-\left(\frac{\dot{A}_1}{A_1}\right)\frac{\dot{F}}{F}+\frac{\ddot{F}}{F}\right], \text{..........(22)}$$

The system of linearly independent set of equations contains three equations and six unknowns $A_1, A_2, F, f(R), p, \rho$. Hence to solve the above system of non-linear independent equations we consider the special law of variation of Hubble parameter given by Berman [31] which gives the constant deceleration parameter of model.

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=\text{constant},\text{.....................} \quad (23)$$

where a is average scale factor.

Here the deceleration parameter is taken negative for purpose of accelerating model of the universe.

On solving equation (23), it gives

$$a=(\beta t+\beta_1)^{\frac{1}{1+q}},\text{.............}(24)$$

Where $\beta \neq 0$ and β_1 are integral constants. $(1+q)>0$ is the condition of expansion given by equation (24).

Using equation (21) and (24) the values of $A_1(t)$ and $A_2(t)$ is given by

$$A_1(t)=(\beta t+\beta_1)^{\frac{1}{1+q}}\exp\left[\left(\frac{2c(1+q)}{3\beta(k+q-2)}\right)(\beta t+\beta_1)^{\frac{k+q-2}{1+q}}\right]\text{..........} \quad (25)$$

$$A_2(t)=(\beta t+\beta_1)^{\frac{1}{1+q}}\exp\left[\left(\frac{c(1+q)}{3\beta(2-k-q)}\right)(\beta t+\beta_1)^{\frac{k+q-2}{1+q}}\right]\text{..........} \quad (26)$$

From above equations it is seen that product of power and exponential exist in $A_1(t)$ and $A_2(t)$ respectively and it increases infinitely exponentially with cosmic time t .

Using equation (25) and (26) the metric in equation (4) can be re-written as

$$\begin{aligned} ds^2=dt^2&-\left\{(\beta t+\beta_1)^{\frac{1}{1+q}}\exp\left[\left(\frac{2c(1+q)}{3\beta(k+q-2)}\right)(\beta t+\beta_1)^{\frac{k+q-2}{1+q}}\right]\right\}^2 dx^2 \\ &-\left\{(\beta t+\beta_1)^{\frac{1}{1+q}}\exp\left[\left(\frac{c(1+q)}{3\beta(2-k-q)}\right)(\beta t+\beta_1)^{\frac{k+q-2}{1+q}}\right]\right\}^2 \left(dy^2+dz^2\right) \end{aligned} ,\text{.........} \quad (27)$$

On taking constant of integration $\beta_1=0$ the above metric has initial singularity at cosmic time $t=0$ and approaches to isotropy.

5. Properties of cosmological model

Using equation (25) and (26) the Ricci scalar R of model is given by

$$R=\frac{2}{3}\left[c^2(\beta t+\beta_1)^{\frac{2(k-3)}{1+q}}-\frac{4\beta c}{(1+q)}(\beta t+\beta_1)^{\frac{k-4-q}{1+q}}+\frac{6\beta^2(1-q)}{(1+q)^2(\beta t+\beta_1)^2}\right],\text{..........}(28)$$

Using equation (28) the value of function $f(R)$ is given by

$$f(R)=\frac{2}{3}\left[c^2(\beta t+\beta_1)^{\frac{3(k-2)}{1+q}}-\beta_2(\beta t+\beta_1)^{\frac{2k-4-q}{1+q}}+\beta_3(\beta t+\beta_1)^{\frac{k-2-2q}{1+q}}\right], \quad (29)$$

where,

$$\beta_2=\frac{4\beta c}{(1+q)}\frac{(k-4-q)}{(2k-4-q)}, \quad \beta_3=\frac{6\beta^2(1-q)}{(1+q)(k-2-2q)}$$

Above equation (29) shows that the $f(R)$ is function of Ricci scalar of studied line element and is positive and decresing function of cosmic time t. This nature of function is equivalent to the function found by Sharif *et al.* [32].

The special volume of model by using equation (6) is given by

$$V=(\beta t+\beta_1)^{\frac{3}{1+q}} \quad (30)$$

on using equation (7) the mean Hubble parameter H of model is given by

$$H=\frac{\beta}{(1+q)(\beta t+\beta_1)}, \quad (31)$$

On using equation (7) the anisotropic parameter Δ of model is

$$\Delta=0, \quad (32)$$

This means that the model shows the completely homogeneous universe. Also this outcome of parameter gives the full support to cosmological principle stated as "on large scales, the universe is both homogeneous and isotropic."

Using equation (9) the expansion parameter θ is given by

$$\theta=\frac{3\beta}{(1+q)(\beta t+\beta_1)} \quad (33)$$

Here in the throughout discussion the value of q is taken as negative constant term.

The state finding parameters are given by using equation (12),

$$r=q+2q^2, \quad (34)$$

$$s=\frac{2(q+2q^2-1)}{3(2q-1)}, \quad (35)$$

Using equation (22), (25) & (26), the energy density of obtained model is

$$\rho=\left\{\frac{1}{\omega}\left[\frac{2}{3}c^2(\beta t+\beta_1)^{\frac{3(k-2)}{1+q}}+\beta_4(\beta t+\beta_1)^{\frac{2k-4-q}{1+q}}+\beta_5(\beta t+\beta_1)^{\frac{k-2-q}{1+q}}+\beta_6(\beta t+\beta_1)^{\frac{k-2-2q}{1+q}}\right]\right\}^{1/n} \quad (36)$$

Using equation (16) and (36), the pressure of obtained model is

$$p=\left[\frac{2}{3}c^2(\beta t+\beta_1)^{\frac{3(k-2)}{1+q}}+\beta_4(\beta t+\beta_1)^{\frac{2k-4-q}{1+q}}+\beta_5(\beta t+\beta_1)^{\frac{k-2-q}{1+q}}+\beta_6(\beta t+\beta_1)^{\frac{k-2-2q}{1+q}}\right] -\left\{\frac{1}{\omega}\left[\frac{2}{3}c^2(\beta t+\beta_1)^{\frac{3(k-2)}{1+q}}+\beta_4(\beta t+\beta_1)^{\frac{2k-4-q}{1+q}}+\beta_5(\beta t+\beta_1)^{\frac{k-2-q}{1+q}}+\beta_6(\beta t+\beta_1)^{\frac{k-2-2q}{1+q}}\right]\right\}^{1/n}, \quad \text{.........(37)}$$

here

$$\beta_4=\frac{4c\beta}{3(1+q)},\qquad \beta_5=\frac{\beta^2[(q-1)+k]}{3(1+q)^2},\qquad \beta_6=\frac{k\beta^2(k-1-q)}{(1+q)^2}$$

6. Graphical representation of dynamical parameter

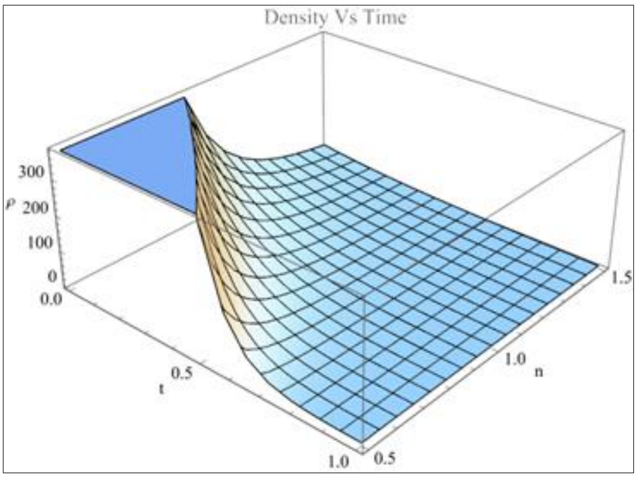

Figure 1 Density Vs Time plotted by assuming $\boldsymbol{\beta_1=1,\beta=-0.5,c=1,k=2,\omega=2,c=1,q=-0.9}$

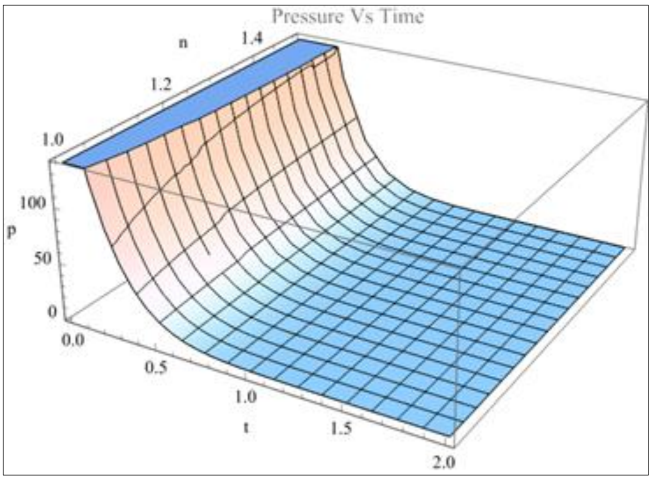

Figure 2 Pressure Vs Time plotted by assuming $\boldsymbol{\beta_1=1,\beta=-0.5,c=1,k=2,\omega=2,c=1,q=-0.9}$

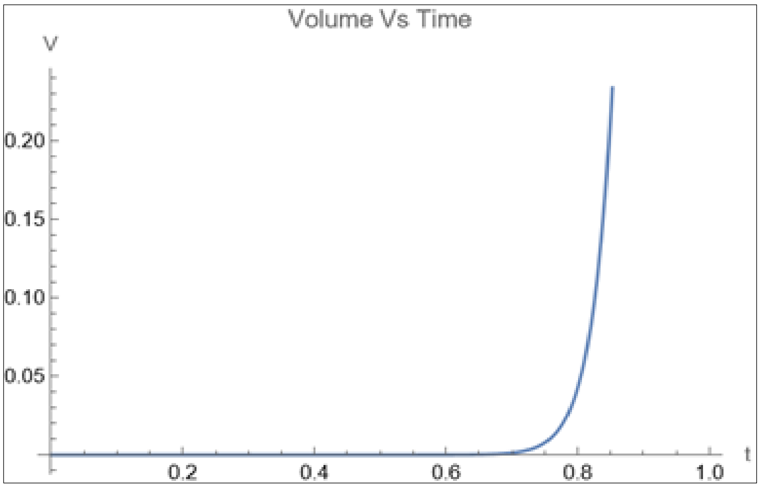

Figure 3 Volume Vs Time plotted by assuming $\beta_1=0.1,\beta=1,q=-0.9$

7. Conclusion

In current study the examination of LRS Bianchi type-I line element has been done with constant deceleration parameter q and polytropic EoS in the context of $f(R)$ gravity. Throughout the work, the deceleration parameter is assumed to be negative having a constant value which is in line with the accelerating expansion of the universe. It can be observed that the function of Ricci scalar R is gradually decreasing with cosmic time t and approaching to zero. The graphical representations show that, the energy density ρ of universe is decreasing with cosmic time t and also that with cosmic time t the pressure is reducing gradually and as $t \rightarrow \infty$, the pressure $p \rightarrow 0$. It is seen that the volume of the universe is under exponential expansion and hence universe has cosmic expansion. The observed result is consistent with the present cosmological observations. Hence, we feel that the obtained result is very useful for the researchers to analyze the characteristics of the universe in the context of $f(R)$ gravity.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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