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(Research Article)

Comparative analysis of Mamdani, Larsen and Tsukamoto methods of fuzzy inference system for students' academic performance evaluation

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Abstract

Over the last few years, the use of the fuzzy logic technique for evaluating performance in the teaching-learning process is growing rapidly. In this research work, three different fuzzy inference methods: Mamdani fuzzy inference method, Larsen fuzzy inference method and Tsukamoto fuzzy inference method have been proposed for students' academic performance appraisal for multi-input variables. To obtain a degree of satisfaction, the Triangular membership function is used. The results of experiments showed the best fuzzy inference method among Mamdani, Larsen and Tsukamoto. We have also compared the results with the existing statistical method.

Keywords: Students' academic performance evaluation; Fuzzy logic techniques; Mamdani inference method; Larsen inference method; Tsukamoto inference method; Membership Functions; Learning evaluation

1. Introduction

The most popular techniques for measuring the student's performance in education is, to assigning numerical numbers according to their achievements in the respective scale. Sometimes, this method often results with fix numbering and ignoring the vagueness in the system.

Recently various fuzzy set theory-based evaluation methods have been published, which support the whole evaluation process. Some studies are there, which are focus on the investigation of student achievement in education. Saleh and Kim proposed a method for the evaluation of students' answer scripts using a fuzzy system [1]. System proposed by Saleh and Kim applies fuzzification, fuzzy inference, and defuzzification in considering the difficulty, the importance and the complexity of questions [1]. Hamam and Georganas worked on the fuzzy logic, which previously simulated and examined, by comparing results from two different and well established fuzzy systems: Mamdani and Sugeno and the results analytically validate the vital differences between that two systems, they worked on the fuzzy logic, which previously simulated and examined, by comparing results from two different and well established fuzzy systems: Mamdani and Sugeno [2]. Goodarzi and Amiri developed a new fuzzy inference system for evaluating the learning progress is proposed; based on difficulty, importance and complexity of a question [3]. Patil *et. al.* developed a fuzzy model to evaluate the best student for the award, they focused on the fuzzy-based approach to circumvent the performance evaluation of the student based purely upon the numerical grading without entailing the human judgmental [4]. They compared the result with the existing traditional method. Figueiredo *et.al.* worked on fuzzy reasoning method for fuzzy control which made comparison with the most useful fuzzy control schemes, for a first-order with time delay process [5]. The results confirmed that Yager's method was more advisable from both computational burden and control system behavior [5]. Mizumoto proposed some fuzzy implications, such as the arithmetic rule and max-min rule, linguistic control rules and compares control results for a plant model with first order delay under various approximate reasoning methods [6]. Saepullah and Wahono conducted investigation and

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comparison between Mamdani, Sugeno and Tsukamoto method on fuzzy inference systems to find a best method in terms of reduction in electrical energy consumption of air conditioner [7].

Aim of this research work is to conduct examination and compare three different fuzzy inference methods; Mamdani fuzzy inference method [8], Larsen fuzzy inference method [9] and Tsukamoto fuzzy inference method [10] to evaluate students' performance for multiple input data.

2. Fuzzy logic

The fuzzy logic was introduced by Zadeh [11][12][13]. Fuzzy logic is beneficial for real-world problems, which include a degree of uncertainty. In real life, many situations are more or less undecided, uncertain and vague. Fuzzy logic is a powerful mathematical tool for modelling uncertain systems in industry, education and nature and provide sensible reasoning in decision making in the absence of comprehensive and accurate information.

The diagram of Fuzzy logic process is shown in figure 1.

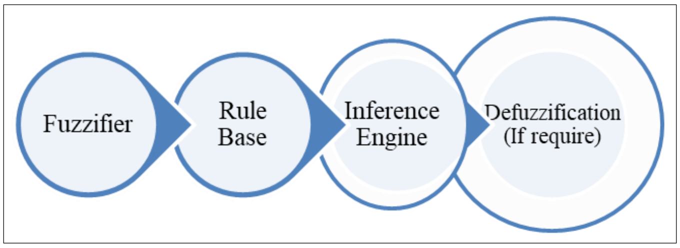

Figure 1 Elements of fuzzy modelling

Fuzzy modelling contains the following four parts:

- **Rule base:** It contains the set of rules and the IF-THEN conditions provided by the experts to supervise the decision-making system, based on linguistic data. Nowadays development in fuzzy reasoning offers several effective methods for the design of fuzzy controllers. **Fuzzification:** It is used to convert a crisp input number into a fuzzy number. Crisp inputs are the exact inputs measured by the evaluator and gave into the control system for processing.
- **Inference engine:** Determines the matching degree of the current fuzzy input concerning with each rule and determines rules which are discharged according to the input range. Later, the active rules are coupled to create control actions.
- **Defuzzification:** In this step, the calculated fuzzy output is converted into a crisp value. There are different types of defuzzification techniques are available and the best-suited one is used with a specific expert system.

3. Inference system

Fuzzy inference system is a nonlinear mapping that determines its output based on fuzzy reasoning and a set of fuzzy if-then rules. The domain and range of the mapping could be a fuzzy number or crisp number in a multidimensional space. Fuzzy inference system matches human reasoning in its use of approximate information to generate decisions. It includes rules, facts and conclusions. The fuzzy inference is also known as Fuzzy models, Fuzzy associate memory, Fuzzy rule-based systems, Fuzzy expert systems, Fuzzy logic controller, etc. Fuzzy inference system includes three parts: 1) A rule base, 2) A database and 3) A reasoning tool.

In this research paper, evaluation of students' academic performance by using three different types of inference system. The inference systems are depend on the composition rule of inference and the result is obtained from a set of fuzzy rules. Three different methods of fuzzy inference are used for evaluation of students' performance.

3.1. Mamdani method

This method was introduced by E. H. Mamdani, he had investigated the possibility of human interaction with the leaning controller [4]. The minimum operator R_c is used as an implication in this method and the max operator is used for the composition. In this method, rule is given in the below manner,

$$R_i = if\ x_1\ is\ A_i, and\ x_2\ is\ B_i\ then\ z\ is\ C_i, i = 1,2, \ldots\ldots, n$$

Then membership value, $\mu_{R_i} = \mu_{(A_i\ and\ B_i \to C_i)}(x_1, x_2, z)$

Mamdani method uses minimum operator (∧) as fuzzy implication:

$$\mu_{C_{i'}} = \alpha_i \wedge \mu_{C_i}(z), where, \alpha_i = \mu_{A_i}(z) \wedge \mu_{B_i}(z)$$

Here, α_i is called "matching degree". For multiple rules (e.g. for two rules)

$$\mu_{C_{i'}} = \mu_{C_1}(z) \vee \mu_{C_2}(z)$$

$$where, \mu_{C_1}(z) = \alpha_1 \wedge \mu_{A_i}(z)\ and\ \mu_{C_2}(z) = \alpha_2 \wedge \mu_{A_i}(z)$$

3.2. Larsen method

In this method, a product operator is used as a fuzzy implication and the max operator for the composition. In Larsen method, rule is given in the below manner ,

$$\mathrm{R_i = if\ x_1\ is\ A_i, and\ x_2\ is\ B_i\ then\ z\ is\ C_i, i = 1,2, \ldots\ldots, n}$$

Then membership value, $\mu_{\mathrm{R_i}} = \mu_{(\mathrm{A_i\ and\ B_i \to C_i})}(\mathrm{x_1, x_2, z})$

Larsen method uses product operator (·) as fuzzy implication:

$$\mu_{\mathrm{C_{i'}}} = \alpha_\mathrm{i} \cdot \mu_{\mathrm{C_i}}(\mathrm{z}), \mathrm{where}, \alpha_\mathrm{i} = \mu_{\mathrm{A_i}}(\mathrm{z}) \wedge \mu_{\mathrm{B_i}}(\mathrm{z})$$

For multiple rules (e.g. for two rules)

$$\mu_{C_{i'}} = \alpha_i \cdot \mu_{C_i}(z)$$

$$where, \alpha_i = \min\left[\max_{x_1}\left(\mu_{A_i}(z) \wedge \mu_{B_i}(z)\right), \max_{x_2}\left(\mu_{A_i}(z) \wedge \mu_{B_i}(z)\right)\right]$$

3.3. Tsukamoto method

In this method, the outcome of each fuzzy rule is represented by a fuzzy set with a monotonic membership function, as shown in figure 4.

In Tsukamoto method, rule is given as,

$\mathrm{R_i = if\ x_1\ is\ A_i, and\ x_2\ is\ B_i\ then\ z\ is\ C_i, i = 1,2, \ldots\ldots, n}$ Where, $\mu_{\mathrm{C_i}}(\mathrm{z})$ is a monotonic function.

The deduced output of a rule is represented as a crisp value induced by the rule's matching degree. The set $\mathrm{C_i}$ has a monotonic membership function $\mu_{\mathrm{C_i}}(\mathrm{z})$ and that α_i is the matching degree of $\mathrm{i^{th}}$ rule.

For fuzzy set input, $\alpha_\mathrm{i} = \min\left[\max_{\mathrm{x_1}}\left(\mu_{\mathrm{A_i}}(\mathrm{z}) \wedge \mu_{\mathrm{B_i}}(\mathrm{z})\right), \max_{\mathrm{x_2}}\left(\mu_{\mathrm{A_i}}(\mathrm{z}) \wedge \mu_{\mathrm{B_i}}(\mathrm{z})\right)\right]$

Then the result is evaluated by $\mathrm{z_i} = \mu_{\mathrm{C_i}}^{-1}(\alpha_\mathrm{i})$

The concluding result is obtained from the weighted average similar as the following when there are two rules.

$$z_0 = \frac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}$$

Since each rule infers a crisp result.

4. Case study

In this case study twenty one students chosen from College of Agriculture, Navsari Agricultural University, Waghai. For this calculation two subjects marks: Elementary mathematics (say exam 1) and Agricultural informatics (say exam 2) of 1stsemester's students were collected and that have been used as input and evaluated the students' performance by using above fuzzy inference system.

Table 1 Student's score of exam 1 and exam 2

Sr. no.	Exam 1	Exam 2	Sr. no.	Exam 1	Exam 2	Sr. no.	Exam 1	Exam 2
(I)	(II)	(III)	(I)	(II)	(III)	(I)	(II)	(III)
1	45	50	8	28	30	15	70	60
2	74	70	9	82	70	16	30	25
3	89	80	10	38	70	17	25	40
4	45	65	11	50	60	18	15	50
5	11	15	12	52	42	19	20	50
6	34	60	13	70	90	20	25	75
7	65	35	14	80	85	21	90	90

Define two fuzzy linguistic variable ranges for two exams. Each input variable has five triangle membership functions, which are shown in table 2.

Table 2 Fuzzy set input variables for triangular membership function

Linguistic expression	Symbol	Interval
Very poor	VP	(0,0,30)
Poor	P	(10,30,50)
Fair	F	(30,50,70)
Good	G	(50,70,90)
Very good	VG	(70,100,100)

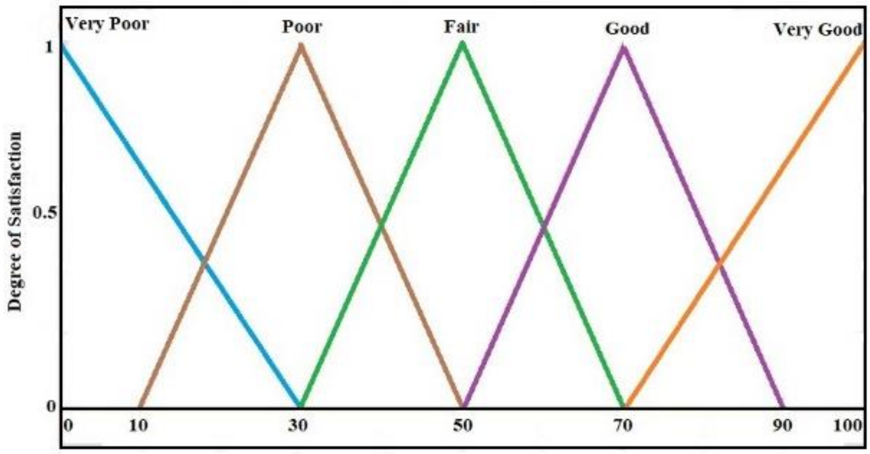

Figure 2 Triangular membership function for Exam 1 and Exam 2

One output fuzzy membership range is defined for student's performance and has five linguistic expression. Table 3 and figure 3 shows the fuzzy membership range for students' performance.

Table 3 Fuzzy membership range for student's performance

Linguistic expression	Symbol	Interval
Poor	P	(0,0,0.10,0.30)
Satisfactory	S	(0.10,0.30,0.50)
Remarkable	R	(0.30,0.50,0.70)
Very good	VG	(0.50,0.70,0.90)
Outstanding	O	(0.70,0.90,1.0,1.0)

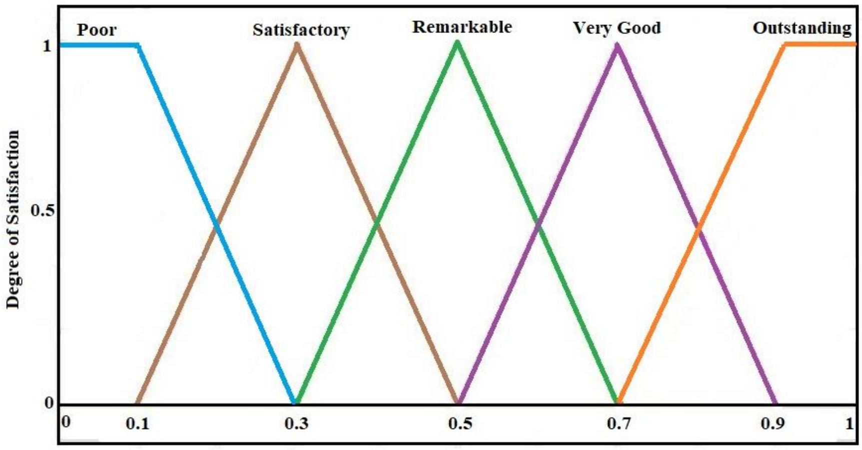

Figure 3 Fuzzy membership representation for student's performance

In the inference process, the rules are defined by using input and output membership functions. These rules are linguistic and also entitled as "if-then" rules. Rules which are used in this evaluation process is shown table 4.

Table 4 Fuzzy if-then rules

Sr. no	Exam 1	Exam 2	Output	Sr. no	Exam 1	Exam 2	Output
1	VP	VP	P	14	F	G	VG
2	VP	P	P	15	F	VG	VG
3	VP	F	S	16	G	VP	S
4	VP	G	S	17	G	P	R
5	VP	VG	R	18	G	F	VG
6	P	VP	P	19	G	G	VG
7	P	P	S	20	G	VG	O
8	P	F	S	21	VG	VP	R
9	P	G	R	22	VG	P	VG
10	P	VG	R	23	VG	F	VG
11	F	VP	S	24	VG	G	O
12	F	P	S	25	VG	VG	O
13	F	F	R				

Started with Mamdani inference method to evaluate student's performance. For each student, both exam scores were fuzzified by applying the membership function, which described in table 2. By using the rule table, active membership functions were determined. During Mamdani's approach the output was defuzzifed by using centroid method. After Mamdani's method students' performance was evaluated by using Larsen's fuzzy inference method. In Larsen's method, the same input membership functions and rule table are used for the evaluation process. To calculate output we used bisector method for defuzzification. At last, Tsukamoto inference method was used for student's performance evaluation. Yet again same input membership functions and rule table are used, but this method gives a crisp output value as an aggregated result and thus there is no need to defuzzify it. For each student, this sequence was repeated with their both exam scores.

5. Results and discussion

In this research paper, the proposed fuzzy model for the evaluation of student's performance based on two different examinations is given. For evaluation of student's performance, three different fuzzy inference methods such as Mamdani method [8], Larsen method [9] and Tsukamoto method [10] are used. Performance of students' based on the proposed fuzzy inference methods and statistical average are compared in table 5.

Table 5 Comparison of the performance of students on the basis of statistical average method with proposed fuzzy model

Sr. no.	**Exam1**	**Exam2**	**Statistical average**	**Fuzzy -1 (Mamdani Inference)**	**Fuzzy -2 (Larsen Inference)**	**Fuzzy – 3 (Tsukamoto Inference)**
1	45	50	0.48	0.44	0.45	0.45
2	74	70	0.72	0.72	0.74	0.74
3	89	80	0.85	0.86	0.88	0.85
4	45	65	0.55	0.57	0.58	0.57
5	11	15	0.13	0.14	0.11	0.15
6	34	60	0.47	0.45	0.45	0.41
7	65	35	0.50	0.50	0.50	0.45
8	28	30	0.29	0.29	0.28	0.28
9	82	70	0.76	0.77	0.82	0.82
10	38	70	0.54	0.58	0.58	0.58
11	50	60	0.55	0.60	0.60	0.60
12	52	42	0.47	0.44	0.45	0.42
13	70	90	0.80	0.88	0.50	0.90
14	80	85	0.83	0.81	0.83	0.82
15	70	60	0.65	0.70	0.70	0.70
16	30	25	0.28	0.28	0.25	0.25
17	25	40	0.33	0.27	0.23	0.32
18	15	50	0.33	0.30	0.30	0.30
19	20	50	0.35	0.30	0.30	0.30
20	25	75	0.50	0.46	0.45	0.52
21	90	90	0.90	0.88	0.90	0.90
Correlation				0.9888	0.9442	0.9866

6. Conclusion

It can be concluded that the method of Mamdani, Larsen and Tsukamoto showed to be useful in student performance evaluation in education. Accuracy of the results of this model was analyzed by using correlation with statistical average. Correlation of statistical average with fuzzy 1 is 0.9888, for statistical average with fuzzy 2correlation is 0.9442 and correlation of fuzzy 3 with statistical average is 0.9866. Both Mamdani fuzzy inference and Tsukamoto fuzzy inference methods have obtained almost same correlation coefficient, but only based on time consumption we can say that, due to no defuzzification process Tsukamoto fuzzy inference method is more efficient than the rest of the two method for sample data-based fuzzy modelling.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declare that they have no conflicts of interest.

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