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Marichev-Saigo-Maeda Fractional Integration of Product of the K- Function and the $\overline{\mathrm{H}}$ -Function

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Abstract

Fractional calculus is an important branch of mathematical analysis which deals with investigations of integrals and derivatives of arbitrary order. It has been applied to almost every field of Science, Engineering and Mathematics. The aim of this paper is to study the generalized Marichev- Saigo- Maeda fractional integral operators. We will establish two theorems which give the image of the product of $\overline{H}$ -function and K-function in Saigo operators. Certain Special cases of main results are also discussed.

Keywords: Marichev-Saigo-Maeda fractional integral operators; K-function; $\overline{\mathrm{H}}$ function; Generalized hypergeometric function

1. Introduction

1.1. Generalized Hypergeometric function

Gauss' hypergeometric function ${}_2F_1(.)$ is given by;

$$ {}_2F_1(a,b;c;\ x)=\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(c)_n}\,x^n\ , \qquad \text{.......................} (1)$$

where c is neither zero nor a negative integer. The series (1) is absolutely convergent within the circle of convergence $|x|<1$, on the circle of convergence the series is absolutely convergent if, Re (c-a-b) > 0.

Also if Re (c-a-b) > 0, Re (c) > Re (b) >0, then;

$$ {}_2F_1(a,b;c;\ 1)\ =\ \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\ ,$$

The generalized hypergeometric function ${}_pF_q$ defined by Rainville [3], is given by ;

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$${}_pF_q\left[\begin{matrix}a_1,a_2,......,a_p;\\ b_1,b_2,.......,b_q;\end{matrix}\ z\right]=\sum_{m=0}^{\infty}\frac{(a_1)_m,........,(a_p)_m}{(b_1)_m,........,(b_q)_m}\cdot\frac{z^m}{m!}\quad\text{..................}\quad(2)$$

where the parameters b_j's (j=1,........,q) is a non-positive integer. If any of the parameters $a_j \leq 0$, then the series terminates.

2. The Generalized M-Series and K-function

- Let $z,\alpha,\beta \in \mathbb{C}$ and $\mathrm{Re}(\alpha) > 0$; then the M-Series [4] is defined as:

$${}_pM_q^{\alpha,\beta}(z) = {}_pM_q^{\alpha,\beta}\left(\begin{smallmatrix}a_p\\ b_q\end{smallmatrix};z\right) = \sum_{n\geq 0}\frac{(a_1)_n,........,(a_p)_n}{(b_1)_n,........,(b_p)_n}\cdot\frac{z^n}{\Gamma(\alpha n+\beta)}\quad\text{................}\quad(3)$$

- Let $z,\alpha,\beta,\gamma \in \mathbb{C}$ and $\mathrm{Re}(\alpha) > 0$; then the K-function [5] is defined as:

$${}_pK_q^{\alpha,\beta;\gamma}(z) = {}_pK_q^{\alpha,\beta;\gamma}\left(\begin{smallmatrix}a_p\\ b_q\end{smallmatrix};z\right) = \sum_{n\geq 0}\frac{(a_1)_n,........,(a_p)_n}{(b_1)_n,........,(b_p)_n}\cdot\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\cdot\frac{z^n}{n!}\quad\text{.............}\quad(4)$$

Both series are defined provided that non of the parameters b_j's (j=1,........,q) is a non-positive integer. If any of the parameters $a_j \leq 0$, then the series terminates. These series are convergent for all z if p ≤ q and divergent if p>q+1.

They are absolutely convergent when $R\left(\sum_{j=1}^{q} b_j - \sum_{i=1}^{p} a_i\right) > 0$ and they are conditionally convergent if $-1 < R\left(\sum_{j=1}^{q} b_j - \sum_{i=1}^{p} a_i\right) \leq 0$ at z = -1 and are divergent if $R\left(\sum_{j=1}^{q} b_j - \sum_{i=1}^{p} a_i\right) \leq -1$

If $\gamma = 1$, ${}_pK_q^{\alpha,\beta;1}(z)$ reduces to ${}_pM_q^{\alpha,\beta}(z)$

2.1. The H-function

The H-function introduced by C. Fox [6], in terms of Mellin-Barnes type of contour integral is defined as follows;

$$H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,\alpha_1),.......,(a_p,\alpha_p)\\ (b_1,\beta_1),.......,(b_q,\beta_q)\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi i}\int_L z^s\,\phi(s)\,ds\ ,\qquad (z\neq 0)\quad\text{....................}\quad(5)$$

Where,

$$\phi(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)},\quad\text{..................}\quad(6)$$

Here m, n, p, q are integers satisfying $0 \leq m \leq q,\ 0 \leq n \leq p$; $a_j(j=1,\ldots\ldots,p)$ and $b_j(j=1,\ldots\ldots\ldots,q)$ are complex parameters, $\alpha_j \geq 0(j=1,\ldots\ldots\ldots,p)$, $\beta_j \geq 0(j=1,\ldots\ldots,q)$ are positive numbers. The contour integral (5) converges absolutely if,

$$T = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j > 0 \quad \text{and} \quad |\arg z| < \frac{1}{2}\pi T$$

2.2. The $\bar{H}$ -function

The $\bar{H}$ -function was introduced by Inayat Hussain [1] and studied by Bushman and Shrivastava [2] is defined and represented in the following manner,

$$\bar{H}^{m,n}_{p,q}[z] = \bar{H}^{m,n}_{p,q}\left[z \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},\ (a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m},\ (b_j,\beta_j;B_j)_{m+1,q} \end{matrix}\right]$$

$$= \frac{1}{2\pi i}\int_L z^s\, \bar{\phi}(s)\,ds\ , \qquad (z \neq 0) \ \ldots\ldots\ldots\ldots\ldots\ldots \qquad (7)$$

where,

$$\bar{\phi}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\ \prod_{j=1}^{n}\left\{\Gamma(1-a_j+\alpha_j s)\right\}^{A_j}}{\prod_{j=m+1}^{q}\left\{\Gamma(1-b_j+\beta_j s)\right\}^{B_j}\ \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s)} \quad \ldots\ldots\ldots\ldots\ldots\ (8)$$

Here L is a contour starting at the point c-i∞ and terminating at the point c+i∞, $a_j(j=1,\ldots\ldots,p)$ and $b_j(j=1,\ldots\ldots\ldots,q)$ are complex parameters, $\alpha_j \geq 0(j=1,\ldots\ldots\ldots,p)$, $\beta_j \geq 0(j=1,\ldots\ldots,q)$, (not all zero simultaneously) and the exponents $A_j(j=1,\ldots\ldots\ldots,n)$, $B_j(j=m+1,\ldots\ldots,q)$ can take integer values.

Sufficient condition for absolute convergence of the contour integral in (7) established by Buschman and Shrivastava [2] is as follows ;

$$T = \sum_{j=1}^{m} \beta_j + \sum_{j=1}^{n} |A_j\alpha_j| - \sum_{j=m+1}^{q} |B_j\beta_j| - \sum_{j=n+1}^{p} \alpha_j > 0, \ \text{and} \quad |\arg z| < \frac{1}{2}\pi T\ ,$$

2.3. Generalized Fractional Integral Operators

Let $\alpha,\beta,\eta \in \mathbb{C}$, x > 0 and $\mathrm{Re}(\alpha) > 0$; then the generalized fractional integration operators associated with Gauss Hypergeometric function, Saigo [7] are defined by the following equations;

$$\left(I^{\alpha,\beta,\eta}_{0,+}\ f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} \times {}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right) f(t)\,dt\ , \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

$$= \frac{d^n}{dx^n}\left(I^{\alpha+n,\beta-n,\eta-n}_{0+} f\right)(x); \quad \left(\mathrm{Re}(\alpha) \leq 0;\ n = [\mathrm{Re}(-\alpha)+1]\right) \ \ldots\ldots\ldots\ldots\ldots\ (10)$$

And

$$\left(I_{-}^{\alpha,\beta,\eta} f\right)(x)=\frac{1}{\Gamma(\alpha)} \int_x^{\infty}(t-x)^{\alpha-1} t^{-\alpha-\beta} \times {}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}\right) f(t)\,dt, \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$=(-1)^n \frac{d^n}{dx^n}\left(I_{-}^{\alpha+n,\beta-n,\eta} f\right)(x); \quad \left(\operatorname{Re}(\alpha) \leq 0;\ n=[\operatorname{Re}(-\alpha]+1\right) \quad \ldots\ldots\ldots\ldots (12)$$

The generalized fractional integration operators of arbitrary order involving Appell function F_3 (.) also known as Horn function [8] in the kernel have been introduced by Marichev [9] and later extended and studied by Saigo and Maeda [10], in the following forms;
Let $\alpha,\alpha',\beta,\beta',\gamma \in \mathbb{C}$, x > 0 and $\operatorname{Re}(\gamma)>0$ then ,

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x)=\frac{x^{-\mu}}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} t^{-\alpha} \times F_3\left(\alpha,\alpha',\beta,\beta',\gamma\ ;1-\frac{t}{x},1-\frac{x}{t}\right) f(t)\,dt, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

And

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x)=\frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^{\infty} (t-x)^{\gamma-1} t^{-\alpha} \times F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right) f(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Further from Saigo and Maeda [10] we also have the following two results;

- Let $\alpha,\alpha',\beta,\beta',\gamma,\rho \in \mathbb{C}$ be such that, $\operatorname{Re}(\gamma)>0$ and $\operatorname{Re}(\rho)>\max[0,\operatorname{Re}(\alpha+\alpha'+\beta-\gamma)\ ,\ \operatorname{Re}(\alpha'-\beta')]$, then;

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x)=\Gamma\left[\begin{array}{c}\rho,\rho+\gamma-\alpha-\alpha'-\beta,\rho+\beta'-\alpha' \\ \rho+\beta',\rho+\gamma-\alpha-\alpha',\rho+\gamma-\alpha'-\beta\end{array}\right] X^{\rho-\alpha-\alpha'+\gamma-1} \quad \ldots\ldots\ldots\ldots (15)$$

where,

$$\Gamma\left[\begin{array}{c}a,b,c \\ x,y,z\end{array}\right]=\frac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(x)\Gamma(y)\Gamma(z)}$$

- Let $\alpha,\alpha',\beta,\beta',\gamma,\rho \in \mathbb{C}$ be such that, $\operatorname{Re}(\gamma)>0$ and

$\operatorname{Re}(\rho)<\ 1+\min\ [\ \operatorname{Re}(-\beta),\ \operatorname{Re}(\alpha+\alpha'-\gamma),\ \operatorname{Re}(\alpha+\beta'-\gamma)]$, then;

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x)=\Gamma\left[\begin{array}{c}1-\rho-\gamma+\alpha+\alpha',1-\rho+\alpha+\beta'-\gamma,1-\rho-\beta \\ 1-\rho,1-\rho+\alpha+\alpha'+\beta'-\gamma,1-\rho+\alpha-\beta\end{array}\right] x^{\rho-\alpha-\alpha'+\gamma-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

3. Results

3.1. Theorem 1

If $\alpha,\alpha',\beta,\beta',\gamma,\rho \in \mathbb{C}$, $x>0,\ T>0,\ |\arg z|\ <\ \frac{1}{2}\pi T$,such that $\operatorname{Re}(\gamma)>0$ and $\operatorname{Re}[\rho+n\sigma+\lambda\xi]\ >\max[0,\ \operatorname{Re}(\alpha+\alpha'+\beta-\gamma),\ \operatorname{Re}(\alpha'-\beta')]$, then;

$$\mathbf{I}_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_{p}K_{q}^{\mu,\nu;\delta}\ (\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\rho+n\sigma-\alpha-\alpha'+\gamma-1}\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}$$

$$\times\bar{H}_{P+3,Q+3}^{M,N+3}\left[\omega x^{\lambda}\left|\begin{matrix}(1-\rho-n\sigma,\lambda;1),(1-\rho-n\sigma-\gamma+\alpha+\alpha'+\beta,\lambda;1),\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-\rho-n\sigma-\beta'+\alpha',\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(1-\rho-n\sigma-\beta',\lambda;1),(1-\rho-n\sigma-\gamma+\alpha+\alpha',\lambda;1),(1-\rho-n\sigma-\gamma+\alpha'+\beta,\lambda;1)\end{matrix}\right] \quad \ldots\ldots\ldots\ldots\ldots (17)$$

3.2. Proof

Applying equation (4), and (7) to the left hand side of (17) and then interchanging the order of summation and integration we have,

$$\mathbf{I}_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_{p}K_{q}^{\mu,\nu;\delta}\ (\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$=\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}\times\frac{1}{2\pi i}\int_L\omega^{\xi}\theta(\xi)\ \left\{I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho+n\sigma+\lambda\xi-1}\right\}(x)\,d\xi$$

Now applying the Saigo Maeda operator (13) we obtain the right hand side of (17).

3.3. Corollary1

If $\alpha,\beta,\gamma,\in\mathbb{C}$, $x>0$, $T>0$, $|\arg z| < \frac{1}{2}\pi T$,such that $\mathrm{Re}(\alpha)>0$ and $\mathrm{Re}[\rho+n\sigma+\lambda\xi] > \max[0,\ \mathrm{Re}(\beta-\gamma)]$, then;

$$I_{0+}^{\alpha,\beta,\gamma}\left(t^{\rho-1}\ {}_{p}K_{q}^{\mu,\nu;\delta}\ (\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\rho+n\sigma-\beta-1}\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}$$

$$\times\bar{H}_{P+2,Q+2}^{M,N+2}\left[\omega x^{\lambda}\left|\begin{matrix}(1-\rho-n\sigma,\lambda;1),(1-\rho-n\sigma-\gamma+\beta,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},(1-\rho-n\sigma+\beta,\lambda;1),(1-\rho-n\sigma-\alpha-\gamma,\lambda;1)\end{matrix}\right.\right] \quad \ldots\ldots\ldots(18)$$

3.4. Theorem 2

If $\alpha,\alpha',\beta,\beta',\rho\in\mathbb{C}$, $x>0,\ T>0,\ |\arg z| < \frac{1}{2}\pi T$,such that $\mathrm{Re}(\gamma)>0$ and $\mathrm{Re}[\rho+n\sigma-\lambda\xi] < 1+\min[\ \mathrm{Re}(-\beta),\ \mathrm{Re}(\alpha+\alpha'-\gamma),(\alpha+\beta'-\gamma)]$, then;

$$I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_pK_q^{\mu,\nu;\delta}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\rho+n\sigma-\alpha-\alpha'+\gamma-1}\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}$$

$$\times\bar{H}_{P+3,Q+3}^{M,N+3}\left[\omega x^{-\lambda}\left|\begin{matrix}(\rho+n\sigma+\gamma-\alpha-\alpha',\lambda;1),(\rho+n\sigma-\alpha-\beta'+\gamma,\lambda;1),\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(\rho+n\sigma+\beta,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(\rho+n\sigma,\lambda;1),(\rho+n\sigma-\alpha-\alpha'-\beta+\gamma',\lambda;1),(\rho+n\sigma-\alpha+\beta,\lambda;1)\end{matrix}\right]\quad\ldots\ldots\ldots.\quad(19)$$

3.5. Proof

Applying equation (4), and (7) to the left hand side of (19) and then interchanging the order of summation and integration we have,

$$I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_pK_q^{\mu,\nu;\delta}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}\times\frac{1}{2\pi i}\int_L\omega^{\xi}\theta(\xi)\ \left\{I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho+n\sigma-\lambda\xi-1}\right\}(x)d\xi$$

Now applying the Saigo Maeda operator (14) we obtain the right hand side of (19).

3.6. Corollary 2

If $\alpha,\beta,\gamma,\in\mathbb{C}$, $x>0,\ T>0,\ |\arg z| < \frac{1}{2}\pi T$,such that $\mathrm{Re}(\alpha)>0$ and $\mathrm{Re}[\rho+n\sigma-\lambda\xi] < 1+\min[\ \mathrm{Re}(\beta),\ \mathrm{Re}(\gamma)]$, then;

$$I_{-}^{\alpha,\beta,\gamma}\left(t^{\rho-1}\ {}_pK_q^{\mu,\nu;\delta}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)$$

$$x^{\rho+n\sigma-\beta-1}\sum_{n\geq 0}\frac{(a_1)_n,\ldots\ldots,(a_p)_n}{(b_1)_n,\ldots\ldots,(b_q)_n}\cdot\frac{(\delta)_n}{\Gamma(\mu n+\nu)}\frac{\eta^n}{n!}$$

$$\times \bar{H}^{M,N+2}_{P+2,Q+2}\left[\omega x^{-\lambda}\left|\begin{matrix}(\rho+n\sigma-\beta,\lambda;1),(\rho+n\sigma-\gamma,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},(\rho+n\sigma,\lambda;1),(\rho+n\sigma-\alpha-\beta-\gamma,\lambda;1)\end{matrix}\right.\right] \quad\ldots\ldots\ldots\ldots \quad (20)$$

4. Conclusion

We have given new image formulas of the $\bar{H}$-function and the K-function under Saigo-Maeda operators, many other interesting image formulas can be derived as the specific cases of our results. Also, the special functions involved here can be reduced in simpler functions, those have a variety of applications in science and technology.

Compliance with ethical standards

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