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Köthe-Toeplitz duals of some generalized difference sequence spaces and their perfectness via α-Dual

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Abstract

The concept of difference sequence spaces have been given by M.Et.and Colak . In this paper, we defined difference sequence Spaces $\Delta_v^r(l_\infty)$, $\Delta_v^r(c)$ *and* $\Delta_v^r(c_0)$ *and* their $\alpha -$ duals or Köthe – Töeplitz dual and also discuss about some topological property of these spaces. Further, we checked perfectness of these spaces and checked some properties like monotone and solidness of these spaces also.

Keywords: Difference sequence space; Köthe-Toeplitz dual; Perfect Space; Solidness

1. Introduction

l_∞ , c *and* c_0 are the classical sequence spaces of bounded complex sequences,convergent complex sequence and null sequences respectively following via norm

$$\| x \|_\infty = sup_k | x_k |$$

Here, k ∈ N {1,2,3 … } , set of natural numbers.

H. Kizmaz [2] defined, the classical difference sequence spaces $l_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$,

$$X(\Delta) = \{x = (x_k) : \Delta x \in X\}$$

Where,X ∈ { l_∞ ,c, c_0} with $\Delta x = \Delta x_k = x_k - x_{k+1}$.

M. ET. *and R.* Colak [3] *have been* defined sequence spaces $l_\infty(\Delta^m), c(\Delta^m)$ *and* $c_0(\Delta^m)$

$$l_\infty (\Delta^m) = \{x = (x_k): \Delta^m x \in l_\infty\}$$

$$c (\Delta^m) = \{x = (x_k): \Delta^m x \in c\}$$

$$c_0(\Delta^m) = \{x = (x_k) : \Delta^m x \in c_0\}, m \in N$$

$$\Delta x = x_k - x_{k+1}, \Delta^0 x = (x_k), \Delta^m x = \Delta^m x_k$$

$$= \Delta^{m-1} x_k - \Delta^m x_{k+1}$$

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$$=\sum_{v=0}^{m}(-1)^{v}\binom{m}{v}x_{k+v}$$

And also generalized some results of Kizmaz [1] and proved that these spaces are *Banach spaces* via norm

$$\| x \|_{\Delta} = \sum_{i=1}^{m} |x_i| + \|\Delta^m x\|_{\infty}$$

M. ET and ESI [8] defined $\Delta_v^m(X)$ for X ∈ $\{l_\infty, c, c_0\}$ and find their Köthe

-Toeplitz dual.

$$\Delta_v^m(X) = \{x = (x_k) : \Delta_v^m x \in X\}$$

Where,

$$\Delta_v^0 x = v_k x_k$$

$$\Delta_v x_k = v_k x_k - v_{k+1} x_{k+1}$$

$$\Delta_v^m x_k = \Delta_v^{m-1} x_k - \Delta_v^{m-1} x_{k+1}$$

$$= \sum_{i=1}^{0} (-1)^i \binom{m}{i} v_{k+i} x_{k+i}$$

Where,m∈ N, v=(v_k) is complex sequence of non-zero numbers .

Let us defined *the* sequence spaces –

$$\Delta_v^r(l_\infty) = \{x = (x_k) : \Delta_v^r x \in l_\infty\}$$

$$\Delta_v^r(c_0) = \{x = (x_k) : \Delta_v^r x \in c_0\} \text{ ....1.1}$$

$$\Delta_v^r(c) = \{x = (x_k) : \Delta_v^r x \in c\}$$

Where r ∈ N set of natural numbers and v = (v_k) is nonzero complex sequence and

$$\Delta_v^0 x = v_k x_k$$

$$\Delta_v x_k = v_k x_k - v_{k+1} x_{k+1}$$

$$\Delta_v^2 x_k = v_k x_k - 2 v_{k+1} x_{k+1} + v_{k+2} x_{k+2}$$

$$= \Delta_v x_k - \Delta_v x_{k+1}$$

$$\Delta_v^r x_k = \Delta_v^{r-1} x_k - \Delta_v^{r-1} x_{k+1} = \sum_{p=0}^{r-1} (-1)^p \binom{r}{p} v_{k+p} x_{k+p}$$

It is clear that $\Delta_v^r(l_\infty)$, $\Delta_v^r(c)$, $\Delta_v^r(c_0)$ *are* linear spaces and *normed linear spaces* via norm

$$\| x \|_{\Delta} = \sum_{p=1}^{r} |x_p v_p| + \|\Delta_v^r x\|_{\infty}.$$

2. Main results

2.1. Theorem

The sequence spaces $\Delta_v^r(l_\infty)$, $\Delta_v^r(c)$ and $\Delta_v^r(c_0)$ *are*

Banach spaces by the norm

$$\| x \|_\Delta = \sum_{p=1}^{r} |x_p v_p| + \|\Delta_v^r x\|_\infty.$$

2.1.1. Proof

suppose, (x_n) is a Cauchy sequence in $\Delta_v^r(l_\infty)$, where x_n=$(x_{n1}, x_{n2}, x_{n3,...})$ where $x_{nk}$is the k^{th} coordinate of x_n.

Now,

$$\|x_n - x_m\|_\Delta = \sum_{p=1}^{r}|(x_{np} - x_{mp})v_p| + \|\Delta_v^r(x_n - x_m)\|_\infty = \sum_{p=1}^{r}|(x_{np} - x_{mp})v_p| + \text{Sup } |\Delta_p^r(x_{nk} - x_{mk})| \to 0 \text{as}$$

$$\text{m,n} \to \infty$$

Hence,

$$|x_{nk} - x_{mk}| \to 0 \; as \text{ m,n} \to 0$$

Therefore $(x_{nk}) = (x_k, x_{2k}, x_{3k} \dots) \to 0 \; as$ Cauchy sequence in C set of complex numbers then

$$\lim_{n\to\infty} x_{nk} = x_k$$

(x_n) is a Cauchy sequence for each $\epsilon > 0 \; \exists \; N = N(\epsilon)$ such **th**at

$$\|x_n - x_m\|_\Delta < \varepsilon \; \forall \; m, n \geq N$$

Hence,

$$\sum_{p=1}^{r}|(x_{np} - x_{mp})| \leq \varepsilon \text{ And}$$

$$\sum_{p=0}^{r} \left|(-1)^p \binom{r}{p} \; (v_{k+p}x_{n(k+p)} - v_{k+p}x_{m(k+p)})\right| \leq \in \text{For all m,n} \geq N \; \lim_{m\to 0} \sum_{p=1}^{r}|x_{np} - x_{mp}|$$

$$= \sum_{p=1}^{r}|x_{np} - x_{mp}| \leq \in$$

And

$$\lim_{m\to\infty}|\Delta_p^r(x_{nk} - x_{mk})| = |\Delta_p^r(x_{nk} - x_k)| \leq \varepsilon \text{ m,n} \in N$$

Hence,

$$\|x_n - x\|_\Delta < 2\varepsilon \; \forall \; m \geq N$$

$$\text{Or } x_n \to x \; as \; n \to \infty \text{ where x=}(x_k)$$

$$\text{Now, } |\Delta_p^r x_k| = |\sum_{p=0}^{r}(-1)^p \binom{r}{p} (v_{k+p}x_{k+p})|$$

$$|\sum_{p=0}^{r}(-1)^p \binom{r}{p} (v_{k+p}x_{k+p} - v_{k+p}x_{n(k+p)} + v_{k+p}x_{n(k+p)}|$$

$$\leq \sum_{p=0}^{r}(-1)^p \binom{r}{p} (v_{k+p}x_{k+p} - |v_{k+p}x_{n(k+p)} + |\sum_{p=0}^{r}(-1)^p \binom{r}{p} (v_{k+p}x_{k+p} - v_{k+p}x_{n(k+p)}|$$

$$\leq \|x_n - x_m\|_\Delta + |\Delta_p^r x_k| = \text{O}(1)$$

Hence, x$\in \Delta_v^r(l_\infty) \; and \; therefore \; \Delta_v^r(l_\infty)$ is Banach space.

Similarly, we can proof for $\Delta_v^r(c) \; and \Delta_v^r(c_0)$.

It can be proved that $\Delta_v^r(c)$ *and* $\Delta_v^r(c_0)$ are closed subspaces of $\Delta_v^r(l_\infty)$.

Hence , these are also Banach spaces .

2.2. Lemma

- $\Delta_v^{r+1}(\boldsymbol{l}_\infty) \supset \Delta_v^r(\boldsymbol{l}_\infty)$
- $\Delta_v^{r+1}(\boldsymbol{c}) \supset \Delta_v^r(\boldsymbol{c})$
- $\Delta_v^{r+1}(\boldsymbol{c_0}) \supset \Delta_v^r(\boldsymbol{c_0})$ **.**

2.2.1. Proof

Let x$\in \Delta_v^r(\boldsymbol{l}_\infty)$

Since

$$|\Delta_v^{r+1}x_k| = |\Delta_v^r x_k - \Delta_v^r x_{k+1}|$$

$$\leq |\Delta_v^r x_k| + |\Delta_v^r x_{k+1}| \to 0 as k \to \infty$$

We have x$\in \Delta_v^{r+1}(\boldsymbol{l}_\infty)$

Thus, $\Delta_v^r(\boldsymbol{l}_\infty) \subset \Delta_v^{r+1}(\boldsymbol{l}_\infty)$.

In the same way, we can proof for $\Delta_v^r(\boldsymbol{c}), \Delta_v^r(c_0)$ **.**

2.3. Lemma

$$\Delta_v^r(c_0) \subset \Delta_v^r(\boldsymbol{c}) \subset \Delta_v^r(\boldsymbol{l}_\infty).$$

2.3.1. Proof

Proof **is** similar to lemma 2.2.

Now define an operater,

$$F: \Delta_v^r(X) \to \Delta_v^r(X)$$

Where X $\in \{\boldsymbol{l}_\infty, \boldsymbol{c}, c_0\}$

Such that Fx = $(0,0,0 \ldots, x_{m+1}, x_{m+2}, x_{m+3} \ldots)$ for x $\in \Delta_v^r(\boldsymbol{X})$

Now for X=l_∞

F: $\Delta_v^r(l_\infty) \to \Delta_v^r(l_\infty)$

Such that Fx = $(0, 0,0 \ldots, x_{m+1}, x_{m+2}, x_{m+3} \ldots)$ for x$\in \Delta_v^r(\boldsymbol{l}_\infty)$

Clearly, F is bounded linear operater on $\Delta_v^r(\boldsymbol{l}_\infty)$.

The set D[$\Delta_v^r(l_\infty)$]= {x = (x_k) : $x \in \Delta_v^r(l_\infty)$; $x_1 = x_2 = x_3 \ldots = x_m = 0$} , is subspace of $\Delta_v^r(l_\infty)$ and $\| x \|_\Delta = \|\Delta_v^r\|_\infty$ in $D\Delta_v^r(l_\infty)$

Now define $\Delta_v^r x = y$

$$=\Delta_v^{r-1}x_k - \Delta_v^{r-1}x_{k+1} \quad \ldots \ldots 1.2$$

Δ_v^r *is a linear homeomorphism* [4].

Hence D[$\Delta_v^r(l_\infty)$] and $l_\infty$are equivqlent to topological spaces.

$\Delta_v^r and\ (\Delta_v^r)^{-1}$ are norm preserving of equal to

$$\|\Delta_v^r\| = \|(\Delta_v^r)^{-1}\| = 1$$

3. Dual Spaces

here, we defined $\alpha - dual$ or köthe- Toeplitz duals $of\ \Delta_v^r(l_\infty)$, Δ_v^r(c) , $\Delta_v^r(c_0)$ and checked their perfectness.

3.1. Lemma

$$sup_k|\Delta_v^r x_k| < \infty\ sup_k k^{-1}|\Delta_v^{r-1} x_k| < \infty$$

3.2. Lemma

$$sup_k k^{-i}|\Delta_v x_k| < \infty\ implies\ sup_k k^{-(k+1)}|v_k x_k| < \infty \forall i \in N$$

3.3. Lemma

$$sup_k k^{-i}|\Delta_v^{r-1} x_k| < \infty \text{Implies } sup_k k^{-(\text{i}+1)}\left|\Delta_v^{r-(\text{i}+1)} x_k\right| < \propto \forall\ \text{I} \in N\ ,\ \text{m} \in N\ , m > i \geq 1.$$

3.4. Corollary

$$\text{If x} \in \Delta_v^r(l_\infty) \text{ then } sup_k k^{-r}|v_k x_k| < \infty.$$

3.5. Definition

Let X be a sequence space, then

$X^\alpha = \{a = (a_k) : \sum_{k=1}^{\infty}|\ a_k x_k\ | < \infty\ \forall\ x \in X\}$

Is called Köthe -Toeplitz dual of X or $\alpha - dual\ of\ X$ and

$X^\beta = \{a = (a_k): \sum_{k=1}^{\infty} a_k x_k\ is\ convergent\ \forall\ x \in X\}$

Is called Generalized Köthe -Toeplitz dual of X.

3.6. Definition

Suppose, X is a sequence space if $X = X^{\alpha\alpha}$ then X, is called perfect sequence space.

3.7. Lemma

- X c Y $\Rightarrow Y^\alpha$ c X^α.
- X c $(X^\alpha)^\alpha = X^{\alpha\alpha}$.

3.8. Theorem

$$\boldsymbol{M_1} = \{\ \text{a} = (a_k) : \sum_{k=1}^{r} k^r |a_k {v^{-1}}_k| < \infty$$

$$\boldsymbol{M_2} = \{\ \text{a} = (a_k) : \sum_{k=1}^{r} k^{-r} |a_k v_k| < \infty$$

Then

$$\left(\Delta_v^r(l_\infty)\right)^\alpha = \left(\Delta_v^r(c)\right)^\alpha = (\ \Delta_v^r(c_0))^\alpha = \boldsymbol{M_1}$$

$$\left(\Delta_v^r(l_\infty)\right)^{\alpha\alpha} = \left(\Delta_v^r(c)\right)^{\alpha\alpha} = (\ \Delta_v^r(c_0))^{\alpha\alpha} = \boldsymbol{M_2}$$

And $\Delta_v^r(l_\infty)$, $\Delta_v^r(c)$, $\Delta_v^r(c_0)$ are not perfect space .

3.8.1. Proof

$$\left(\Delta_v^r(l_\infty)\right)^\alpha = \{a=(a_k) : \sum_{k=1}^{r}|a_k x_k| < \infty \ \forall\, x \in \Delta_v^r(l_\infty)\}$$

$$\text{let } a \in M_1$$

$$\sum_{k=1}^{\infty}|a_k x_k| = \sum_{k=1}^{\infty} k^r|a_k {v^{-1}}_k|\; k^{-r}|x_k v_k| < \infty \text{ for each } x \in \Delta_v^r(l_\infty)$$

$$\sum_{k=1}^{\infty}|a_k x_k| < \infty \ \forall\, x \in \Delta_v^r(l_\infty)$$

Hence,

$$M_1 \subset [\Delta_v^r(l_\infty)]^\alpha \text{ …….. } 1.3$$

Now let a $\notin M_1$ *then for some k*

$$\sum_{k=1}^{\infty} k^r|a_k {v^{-1}}_k| = \infty$$

Now define a sequence

$$x_k = \begin{cases} 0 & 1 \le k \le n_i \\ \frac{v_k^{-1} k_i^r}{i} & n_i + 1 < k < n_{i+1} \end{cases} \quad i = 1,2,3 \ldots$$

Then,

$$|\Delta_v^r v_k x_k| = \frac{r!}{i} \ n_i + 1 < k \le n_{i+1}, \ i = 1,2,3,4\ldots$$

Hence,

$$x \in [\Delta_v^r(l_\infty)]^\alpha \text{ and } \sum_{k=1}^{\infty}|a_k x_k| = \sum_{i=1}^{\infty} 1 = \infty$$

a$\in [\Delta_v^r(l_\infty)]^\alpha$ and hence ,

$$[\Delta_v^r(l_\infty)]^\alpha \subset M_1 \text{ …….. } 1.4$$

From 1.3 and 1.4

$$[\Delta_v^r(l_\infty)]^\alpha = M_1 \text{ ………….. (2)}$$

Since

$$\Delta_v^r(c_0) \subset \Delta_v^r(\boldsymbol{c}) \subset \Delta_v^r(\boldsymbol{l}_\infty)$$

$$\Rightarrow [\Delta_v^r(l_\infty)]^\alpha \subset [\Delta_v^r(c)]^\alpha \subset [\Delta_v^r(c_0)]^\alpha \text{ …………. } 1.5$$

From 1.2, 1.3 and 1.5

$$M_1 \subset \left(\Delta_v^r(l_\infty)\right)^\alpha \subset \left(\Delta_v^r(c)\right)^\alpha \subset (\Delta_v^r c_0)^\alpha \subset M_1$$

$$\Rightarrow \left(\Delta_v^r(l_\infty)\right)^\alpha = \left(\Delta_v^r(c)\right)^\alpha = (\Delta_v^r c_0)^\alpha = M_1 \text{ …….. (3)}$$

Now, let a $\in M_2$

$$\sum_{k=1}^{\infty}|a_k x_k| \le \sup k^{-r}\,|a_k x_k| \sum_{k=1}^{\infty} k^r\,|x_k v_k| < \infty \text{ for each } x \in (\Delta_v^r(c_0))^\alpha = M_1$$

Hence,

$$M_2 \subset [\Delta_v^r(l_\infty)]^{\alpha\alpha} \text{ .... } 1.6$$

Conversaly, let a$\notin M_2$ then

$$sup_k k^{-r}|v_k x_k| = \infty$$

Hence, there is a sequence of k(i) of positive integers which is strictly increasing , such that

$$[\mathrm{k(i)}]^{-r}\left|a_{k(\mathrm{i})} v_{k(\mathrm{i})}\right| > i^r$$

Let us define a sequence,

$$x_k = \begin{cases} 0 \; k = \mathrm{k(i)} \\ \left|a_{k(\mathrm{i})}\right|^{-1} \; k \neq k(\mathrm{i}) \end{cases}$$

Then,

$\sum_{k=1}^{r} k^r \, |x_k {v_k}^{-r}|$

$$= \sum_{i=1}^{\infty} \; [\mathrm{k(i)}]^r \left|a_{k(\mathrm{i})} v_{k(\mathrm{i})}\right|^{-1} \leq \sum_{i=1}^{\infty} i^{-r} < \infty$$

Hence,

$$\mathrm{x} \in [\Delta_v^r(l_\infty)]^{\alpha}$$

And

$$\sum_{k=1}^{\infty}|a_k x_k| = \sum_{i=1}^{\infty} 1 = \infty$$

$$\mathrm{a} \notin [\Delta_v^r(l_\infty)]^{\alpha\alpha}$$
$$[\Delta_v^r(l_\infty)]^{\alpha\alpha} \subset M_2 \text{ .... } 1.7$$

From 1.6 and 1.7

$$[\Delta_v^r(l_\infty)]^{\alpha\alpha} = M_2$$

Since

$$[\Delta_v^r(l_\infty)]^{\alpha} \subset [\Delta_v^r(c)]^{\alpha} \subset [\Delta_v^r(c_0)]^{\alpha}$$

$$\Rightarrow [\Delta_v^r(c_0)]^{\alpha\alpha} \subset [\Delta_v^r(c)]^{\alpha\alpha} \subset [\Delta_v^r(l_\infty)]^{\alpha\alpha} \text{ .... } 1.8$$

From 1.6,1.7 and 1.8

$$M_2 \subset [\Delta_v^r(c_0)]^{\alpha\alpha} \subset [\Delta_v^r(c)]^{\alpha\alpha} \subset [\Delta_v^r(l_\infty)]^{\alpha\alpha} \subset M_2$$

Hence,

$$[\Delta_v^r(l_\infty)]^{\alpha\alpha} = [\Delta_v^r(c_0)]^{\alpha\alpha} = [\Delta_v^r(c)]^{\alpha\alpha} = M_2 \text{ ....(4)}$$

Since

$$M_1 \neq M_2$$

$$[\Delta_v^r(l_\infty)]^{\alpha\alpha} = M_2$$

$$\neq [\Delta_v^r(l_\infty)]^\alpha$$

$$[\Delta_v^r(c)]^{\alpha\alpha} = M_2$$

$$\neq [\Delta_v^r(c)]^\alpha$$

$$[\Delta_v^r(c_0)]^{\alpha\alpha} = M_2$$

$$\neq [\Delta_v^r(c_0)]^\alpha$$

Hence

$\Delta_v^r(l_\infty)$, $\Delta_v^r(c)$, $\Delta_v^r(c_0)$ are not perfect space.

3.9. Theorem

- $[\Delta_v^r(l_\infty)]^\alpha$ = $[D\Delta_v^r(l_\infty)]^\alpha$
- $[\Delta_v^r(c_0)]^\alpha$ = $[D\Delta_v^r(c_0)]^\alpha$

3.9.1. Proof

Since,

$$\mathrm{D}[\Delta_v^r(l_\infty)] \subset \Delta_v^r(l_\infty)$$

Then,

$$[\Delta_v^r(l_\infty)]^\alpha \subset D[\Delta_v^r(l_\infty)]^\alpha$$

Let a∈ $[\Delta_v^r(l_\infty)]^\alpha$ and x∈ $\Delta_v^r(l_\infty)$

From corollary 3.4

$$\sum_{k=1}^{\infty}|a_k x_k| = \sum_{k=1}^{\infty} k^r |a_k| (k^{-r}|x_k|) < \infty$$

Hence, a ∈ $[D\Delta_v^r(l_\infty)]^\alpha$

Therefore, $[\Delta_v^r(l_\infty)]^\alpha$ = $[D\Delta_v^r(l_\infty)]^\alpha$.

We can proof 2. in the same way.

3.10. Lemma

$$[D\Delta_v^r(l_\infty)]^\alpha = [D\Delta_v^r(c)]^\alpha.$$

3.10.1. Proof

Since, D[$\Delta_v^r(c)$] c $\Delta_v^r(l_\infty)$

$$[\Delta_v^r(l_\infty)]^\alpha \subset [D\Delta_v^r(c)]^\alpha$$

Let, a ∈ $[D\Delta_v^r(c)]^\alpha$

Then, $\sum_{k=1}^{\infty}|a_k x_k| < \infty$ for each x ∈ $\Delta_v^r(c)$

Now define a sequence

$$x_k = \begin{cases} 0 \; if k \leq m \\ k^r \; if k > m \end{cases}$$

$$\sum_{k=1}^{\infty} \mathrm{k}^{r}|a_k| = \sum_{k=1}^{m} \mathrm{k}^{r}|a_k| + \sum_{k=1}^{\infty} |a_k x_k| < \infty$$

This implies a $\in [D\Delta_v^r(l_\infty)]^\alpha$

Hence, $[D\Delta_v^r(l_\infty)]^\alpha = [D\Delta_v^r(c)]^\alpha$.

3.11. Lemma

$$\boldsymbol{l}_\infty \cap \Delta_v^r(\boldsymbol{c_0}) = \boldsymbol{l}_\infty \cap \Delta_v^r(\boldsymbol{c}).$$

3.11.1. Proof

Let x $\in l_\infty \cap \Delta_v^r$(c)

x $\in l_\infty$ and x $\in \Delta_v^r(c)$

x $\in l_\infty$ and x $\in$ $\Delta^{r-1} v_k x_k - \Delta^{r-1} v_{k+1} x_{k+1} \to l \text{ as } k \to \infty$

$$\Delta^{r-1} v_k x_k - \Delta^{r-1} v_{k+1} x_{k+1} = l + \varepsilon_k$$

This implies l=0

Hence,

x $\in \boldsymbol{l}_\infty \cap \Delta_v^r(\boldsymbol{c_0})$.

4. Definition

A difference sequence Δ_v^r(X) is solid for $(x_m) \in \Delta_v^r$(X) and for all (α_k) of scalars with $|\alpha_k| \leq 1$ ∀ k∈ N ,$(\alpha_k x_m) \in \Delta_v^r$(X).

4.1. Definition

A difference sequence Δ_v^r(X) is monotone if for $(x_k) \in \Delta_v^r$(X) and $(y_k) \in$ E where E is the sequence consisting of only 0 and 1, $(x_k y_k) \in$ E.

4.2. Theorem

A difference sequence space is solid then it is monotone.

4.3. Lemma

The spaces $\Delta_v^r(\boldsymbol{l}_\infty)$, Δ_v^r(c) are not monotone.

4.3.1. Proof

We can see this by the following example

Let us consider a sequence,

$$x_k = ik \;\forall\; \mathrm{k} \in N$$

$$y_k = \begin{cases} 1, if\ k\ is\ odd \\ 0, otherwise \end{cases}$$

Then

$(x_k y_k) \notin \Delta_v^r(\boldsymbol{l}_\infty)$.

4.4. Lemma

The spaces $\Delta_v^r(\boldsymbol{l}_\infty)$, Δ_v^r(c) are not solid

5. Conclusion

The spaces Δ_v^r (l_∞) , Δ_v^r (c) are not perfect and also the spaces Δ_v^r (l_∞) , Δ_v^r (c) are not monotone and solid.

Compliance with ethical standards

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