International Journal of Science and Research Archive

eISSN: 2582-8185

Cross Ref DOI: 10.30574/ijsra

Journal homepage: https://ijsra.net/

(Research Article)

Check for updates

Generalised fuzzy δ-ideals and δ -semi-ideals of hemirings

J Regala Jebalily *

Mathematics, Ethiraj College for Women, Chennai - 600 008, Tamilnadu, India.

International Journal of Science and Research Archive, 2023, 08(01), 383–387

Publication history: Received on 29 November 2022; revised on 08 January 2023; accepted on 10 January 2023

Article DOI: https://doi.org/10.30574/ijsra.2023.8.1.0014

Abstract

A new type of generalised fuzzy δ-ideals in a hemiring namely $(\in, \in V_\delta) -$ fuzzy δ-ideals (resp., δ- semi ideals) is observed and the relationship between the two ideals are described. The main purpose of this study is to observe the concepts of -fuzzy δ ideals and fuzzy δ semi-ideals using basic definitions and results of hemirings and investigate some of the properties and characterizations of the proposed concept.

Keywords: Hemiring S; Fuzzy leftδ-ideal; Fuzzy right δ -ideal; Semiring (S,+, *); Interval-valued fuzzy δ -ideals

1. Introduction

Semirings which are regarded as a generalization of rings have been found useful in solving problems in different disciplines of applied mathematics and information sciences because semiring provides an algebraic framework for modelling. By a hemiring, we mean a special semiring with a zero and with a commutative addition. In applications, hemirings are useful in automata and formal languages (see [16]). Many-valued logic has been considered by model phenomena in which uncertainty and vagueness are both involved. In the literature, Henriksen [13] first defined a kind of more restricted ideals in a semiring *S*, namely, the δ-ideals of *S* having the property that if *S* is a ring then a complex in *S* is a δ -ideal if and only if it is a ring ideal. However, in an additively commutative semiring *S*, the ideals of a semiring coincide with the "ideals" of a ring provided that the semiring is a hemiring. We now call this ideal an δ -semi ideal of a hemiring *S*. The properties of δ -ideals and also δ -semi ideals of a hemiring were thoroughly investigated by Torre in [24]. Recently, Jun [17] considered the fuzzy setting of δ -ideals of hemirings. Also, Ma et al. discussed the properties of generalized interval-valued fuzzy δ -ideals of hemirings in [20]. In particular, we describe the regular hemirings and irregular hemirings by using some of the fuzzy δ- ideals. Finally the implication based on fuzzy δ- semi ideals of a hemiring are discussed.

2. Preliminaries

Recall that a *semiring* is an algebraic system $(S, +, *)$ consisting of a non-empty set *S* together with two binary operations on *S* called addition and multiplication (denoted in the usual manner) such that $(S, +)$ and $(S,)$ are semigroups and the following distributive laws $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ are satisfied for all $a, b, c \in S$.

By *zero* of a semiring (S,+, *) we mean an element $0 \in S$ such that $0*x = x*0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. A semiring with zero and a commutative semigroup $(S, +)$ is called a *hemiring.*

A left ideal of a semiring is a subset *A* of *S* closed with respect to the addition such that $SA \subseteq A$.

* Corresponding author: J Regala Jebalily

A left ideal A of a hemiring S is called a *left δ-ideal* if for any $x, z \in S$ and $a, b \in A$

from $x + a + z = b + z$, it follows $x \in A$. A *right δ-ideal* is defined analogously.

We now recall some fuzzy logic concepts. A fuzzy set is a function $\mu : S \rightarrow [0, 1]$.

2.1. Definition

A fuzzy set F of a hemiring S is called a fuzzy left (resp., right) δ -ideal if it satisfies:

(F_1) $\forall x, y \in S,\ \delta(x + y) \geq \min\{\delta(x), \delta(y)\}$,

(F_2) $\forall x, y \in S,\ \delta(xy) \geq \delta(y)$ (*resp.*, $\delta(xy) \geq \delta(x)$),

(F_3) $\forall a, b, x, z \in S, x + a + z = b + z \rightarrow \delta(x) \geq \min\{\delta(a), \delta(b)\}$.

Note that a fuzzy left (resp., right) δ -ideal F of a hemiring S satisfies the inequality $F(0) \geq F(X)$ for all $X \in S$. From the Transfer Principle in fuzzy set theory, it follows that a fuzzy set F defined on X can be characterized by level subsets. i.e. by sets of the form ≥

$U(F ; t) = \{x \in X \mid F(x) \geq t\}$, *where* $t \in [0,1]$. ∈

For any algebraic system A = (X, F), where F is a family of operations (also partial) defined on X, the Transfer Principle can be formulated in the following way. ∈

2.2. Lemma

A fuzzy set F defined on A *has the property* P *if and only if all non-empty level subsets U (F ; t) have the property* P.

As a simple consequence of the above property, we obtain the following theorem.

2.3. Theorem

A fuzzy set F of a hemiring S is a fuzzy left (resp., right) δ-ideal of S if and only if each non-empty level subset U (F ; t) is a left (resp., right) δ-ideal of S.

Note: For any subset A of a hemiring S, χ_A will denote the characteristic function of A.

2.4. Theorem

A non-empty subset A of a hemiring S is a left (resp., right) δ-ideal of S if and only if χ_A is a fuzzy left (resp., right) δ-ideal of S.

2.5. Definition

[29] *A hemiring S is said to be δ- semi ideal if for each $a \in S$, there exist $x_1, x_2, z \in S$ such that $a + ax_1a + z = ax_2a + z$.*

The *h-closure* $\bar{A}$ of A in a hemiring S is defined as

$\bar{A} = \{x \in S \mid x + a_1 + z = a_2 + z\}$ for some $a_1, a_2 \in A$, $z \in S$.

It is clear that if A is a left ideal of S, then $\bar{A}$ is the smallest left δ ideal of S containing A. We also have $\bar{\bar{A}} = A$ for each $A \subseteq S$. Moreover, $A \subseteq B \subseteq S$ implies $A \subseteq B$.

2.6. Lemma

If A and B are, respectively, right and left δideals of a hemiring S, then AB ⊆ A ∩ B.

3. Implication based on Fuzzy δ-Ideals

Fuzzy logic is an extension of set theoretic variables in terms of the linguistic variable truth.

In the fuzzy logic, we denote the truth value of fuzzy proposition P by $[P]$. In the following, we display the fuzzy logical and corresponding set-theoretical notions:

$[x \quad F] \equiv F(x)$;

$[x F] = 1 \quad \in \quad F(x)$;

$[P \quad Q] = \wedge \min\{[P], [Q]\}$;

$[P \quad Q] = \vee \max\{[P], [Q]\}$;

$[P \quad Q] = \min\{1, 1 - [P] + [Q]\}$;

$[xP(x)] = \forall \inf[P(x)]$;

$= P$ if and only if $[P] = 1$ for all valuations.

3.1. Definition

A fuzzy set F of S is called a fuzzifying left (resp., right) δ-ideal of S if it satisfies:

(F_4) For any x, y ∈ S, [x ∈ F] ∧ [y ∈ F] → [x + y ∈ F],

(F_5) For any x, y ∈ S, [y ∈ F] → [xy ∈ F] (resp., [x ∈ F] → [xy ∈ F]),

(F_6) For any a, b, x, z ∈ S with x + a + z = b + z, [a ∈ F] ∧ [b ∈ F] → [x ∈ F].

Clearly, Definition 4.1 is equivalent to Definition 2.1. Hence, a fuzzifying left (resp., right) δ -ideal is an ordinary fuzzy left (resp., right) δ -ideal.

Next, we can extend the concept of implication-based fuzzy left (resp., right)

δ -ideals in the following way.

3.2. Definition

Let F be a fuzzy set and t∈[0,1] is a fixed number. Then F is called a t-implication-based fuzzy left (resp., right) - δ ideal of S if the following conditions hold:

(F_7) For any x, y ∈ S, $|=_t$ [x ∈ F] ∧ [y ∈ F] → [x + y ∈ F],

(F_8) For any x, y ∈ S, $|=_t$ [y ∈ F] → [xy ∈ F] (resp., |= [x ∈ F] → [xy ∈ F

(F_9) For any a, b, x, z ∈ S with x +a +z = b + z, $|=_t$ [a ∈ F] ∧ [b ∈ F] → [x ∈ F].

4. Conclusion

To investigate the structure of an algebraic system, it is clear that (fuzzy) ideals with special properties play an important role. In this paper, we considered the relationship among these generalized fuzzy left (resp., right) δ-ideals of hemirings. Finally, we investigated the concept of implication-based fuzzy left (resp., right) δ - ideals of hemirings. It is our hope that this work would serve as a foundation for further study of the theory of hemirings. The future study of fuzzy sets in hemirings can be connected with

- focusing on other types and their relationships among them;
- investigating prime (semiprime) (α, β)-fuzzy left (resp., right) δ -ideals;
- establishing a fuzzy spectrum of a hemiring;
- Considering quotient hemirings via (α, β)-fuzzy left (resp., right) δ -ideals.

The obtained results can be used to solve some social networks problems and to decide whether the corresponding graph is balanced or cluster able.

Compliance with ethical standards

Acknowledgments

Hereby I am acknowledging that this work is done by me and I have no objection in publishing this paper in your journal.

References

[1] S. K. Bhakat and P. Das, (∈, ∈ ∨ q)-fuzzy subgroup, Fuzzy Sets and Systems 80 (1996), no. 3, 359–368.

[2] G. Cohen, S. Gaubert and J.-P. Quadrat, Algebraic system analysis of timed Petri nets, in Idempotency (Bristol, 1994), 145–170, Publ. Newton Inst., 11 Cambridge Univ. Press, Cambridge.

[3] B. Davvaz, (∈, ∈ ∨q)-fuzzy subnear-rings and ideals, Soft Computing 10 (2006), 206–211.

[4] B. Davvaz and P. Corsini, Redefined fuzzy Hv -submodules and many valued implications, Inform. Sci. 177 (2007), no. 3, 865–875.

[5] D. Dubois and H. Prade, Fuzzy Sets and Systems, Mathematics in Science and Engineering, 144, Academic Press, New York, 1980.

[6] W. A. Dudek, Special types of intuitionistic fuzzy left h-ideals of hemirings, Soft Computing 12 (2008), 359–364.

[7] W. Dudek, Intuitionistic fuzzy h-ideals of hemirings, WSEAS Trans. Math. 5 (2006), no. 12, 1315–1321.

[8] K. G-lazek, A Auide to the Literature on Semirings and Their Applications in Mathematics and Information Sciences, Kluwer Acad. Publ., Dordrecht, 2002.

[9] J. S. Golan, Semirings and Their Applications, Kluwer Acad. Publ., Dordrecht, 1999.

[10] H. Hedayati, Generalized fuzzy k-ideals of semirings with interval-valued membership func- tions, Bull. Malays. Math. Sci. Soc. (2) 32 (2009), no. 3, 409–424.

[11] M. Henriksen, Ideals in semirings with commutative addition, Amer. Math. Soc. Notices 6 (1958), 321.

[12] J. E. Hopcroft and J. D. Ullman, Introduction to Automata Theory, Languages and Compu- tation, Addison-Wesley Publishing Co., Reading, MA, 1979.

[13] X. Huang, H. Li and Y. Yin, The h-hemiregular fuzzy duo hemirings, Int. J. Fuzzy Syst. 9(2007), no. 2, 105–109.

[14] S. K. Bhakat and P. Das, (∈, ∈ ∨ q)-fuzzy subgroup, Fuzzy Sets and Systems 80 (1996), no. 3, 359–368.

[15] G. Cohen, S. Gaubert and J.-P. Quadrat, Algebraic system analysis of timed Petri nets, in Idempotency (Bristol, 1994), 145–170, Publ. Newton Inst., 11 Cambridge Univ. Press, Cambridge.

[16] B. Davvaz, (∈, ∈ ∨q)-fuzzy subnear-rings and ideals, Soft Computing 10 (2006), 206–211.

[17] B. Davvaz and P. Corsini, Redefined fuzzy Hv -submodules and many valued implications, Inform. Sci. 177 (2007), no. 3, 865–875.

[18] D. Dubois and H. Prade, Fuzzy Sets and Systems, Mathematics in Science and Engineering, 144, Academic Press, New York, 1980.

[19] W. A. Dudek, Special types of intuitionistic fuzzy left h-ideals of hemirings, Soft Computing 12 (2008), 359–364.

[20] W. Dudek, Intuitionistic fuzzy h-ideals of hemirings, WSEAS Trans. Math. 5 (2006), no. 12, 1315–1321.

[21] K. G-lazek, A Auide to the Literature on Semirings and Their Applications in Mathematics and Information Sciences, Kluwer Acad. Publ., Dordrecht, 2002.

[22] J. S. Golan, Semirings and Their Applications, Kluwer Acad. Publ., Dordrecht, 1999.

[23] H. Hedayati, Generalized fuzzy k-ideals of semirings with interval-valued membership func- tions, Bull. Malays. Math. Sci. Soc. (2) 32 (2009), no. 3, 409–424.

[24] M. Henriksen, Ideals in semirings with commutative addition, Amer. Math. Soc. Notices 6 (1958), 321.

[25] J. E. Hopcroft and J. D. Ullman, Introduction to Automata Theory, Languages and Compu- tation, Addison-Wesley Publishing Co., Reading, MA, 1979.

[26] X. Huang, H. Li and Y. Yin, The h-hemiregular fuzzy duo hemirings, Int. J. Fuzzy Syst. 9 (2007), no. 2, 105–109.

[27] K. Iizuka, On the Jacobson radical of a semiring, Tˆohoku Math. J. (2) 11 (1959), 409–421.

[28] Y. B. Jun, M. A. O¨ ztu¨rk and S. Z. Song, On fuzzy h-ideals in hemirings, Inform. Sci. 162 (2004), no. 3–4, 211–226.

[29] M. Kondo and W. A. Dudek, On the transfer principle in fuzzy theory, Mathware Soft Comput. 12 (2005), no. 1, 41–55.

[30] D. R. LaTorre, On h-ideals and k-ideals in hemirings, Publ. Math. Debrecen 12 (1965), 219– 226.