



(RESEARCH ARTICLE)



EVALUATING PATH EFFICIENCY AND WHEEL DYNAMICS ACROSS VARYING TURNING RADII IN WHEELED MOBILE ROBOTS

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ABSTRACT

Mobile robots navigating within indoor confined spaces generally use various turning techniques to negotiate obstacles and corners. Although zero radius turns ($R=0.0$ m) allow saving space, the continuous-arc approach provides different possibilities related to such factors as energy, duration, and wheel slip. This paper proposes an analysis of the performances of a mobile robot based on a simulation carried out using the Python programming language and NumPy package. The study examines a model of a differential-drive robot making a 90-degree turn at different radii ($R \in \{0.0, 0.2, 0.5, 1.0, 2.0\}$ m). On the basis of the kinematic analysis along with power and slip estimations, it has been demonstrated that small radii (for example, $R=0.2$ m) reduce greatly the duration of the manoeuvre (0.78 s) and save space, although they increase significantly the power peaks (6.30 W) and slip peaks (0.0141). In its turn, large radii ($R=2.0$ m) decrease mechanical slips and power peaks but increase energy consumption (37.92 J) and manoeuvre duration (7.85 s). Composite Efficiency Index indicates that larger arcs lead to increased efficiency per joule, while small turns increase speed efficiency. These simulation findings give us a mathematical basis for determining adaptive turns depending on operating conditions, such as choosing between battery life and time efficiency.

Keywords: Differential-Drive Robots, Turning Strategy, Kinematic Simulation, Energy Efficiency, Wheel Dynamics, Python Modelling

1 INTRODUCTION

With the increasing development and use of autonomous mobile robots (AMRs) and automated guided vehicles (AGVs) in industry, manufacturing, warehousing, health care, and services, it has been realized how important it is to develop effective motion planning and control algorithms [1, 5]. Navigation in a crowded and restricted space filled with obstacles, tight paths, and moving objects has become very essential [2]. With all locomotion mechanisms, the differential-drive mobile robot (DDMR) has remained one of the most used systems in recent times because of its mechanical simplicity, manoeuvrability, and capacity of zero radius turning on the spot [3]. But controlling a differential drive poses inherent difficulties as a result of the constraints posed by this non-holonomic mechanism, particularly its incapacity to instantly move sideways relative to its wheel axle [4].

Large body of scientific literature exists on the theories for wheeled mobile robots modelling and control. Fundamental theories for wheeled kinematic modelling were formally developed by Muir and Neuman [11], and the structural

classification of wheeled kinematic and dynamic models along with their properties was made by Campion et al. [10]. To realize the stable motions and trajectory tracking of wheeled mobile robots in non-holonomic environments, scientists have proposed feedback controls and tracking control algorithms [9] as well as nonlinear backstepping control strategies [13]. In addition, constrained space motion planning typically employs geometric path primitives bounded by curvature constraints, which is famously solved using Dubins paths [7] as well as extended Reeds-Shepp paths for forward and reverse manoeuvres [8]. Parametric optimal control algorithms have been employed for improved trajectory generation in reactive non-holonomic environments [15], largely due to the numerical optimal control and estimation algorithms [14].

While moving through an operational environment, mobile robots constantly need to change their mode of locomotion from linear motions to directional turns. Conventional navigation techniques can be classified according to whether they use discrete turns—like stop and turn on the spot ($R=0.0$ m)—or continuous arc turns of different turning radiuses ($R>0$). Although turns performed by stopping on the spot are characterized by reduced spatial demands and allow for precise angular changes, they are associated with rapid acceleration changes, higher peak torques for the motors, and greater friction forces between the wheels and the ground surface.

Also, considering the issues related to uncertainties in the physical world, sensor noise, and state estimation errors while navigating has received significant attention from researchers within the scope of probabilistic robotics [6] and particular location estimation approaches [12]. Although there exists a substantial body of literature dealing with global path planning, kinematics, control, and probabilistic localization, it is still a critical issue that has not been tackled by many researchers about a quantitative trade-off between different values of turning radius and effects on the path cost, energy consumption, and wheel dynamics. Most of the studies conducted till now emphasize path planning and obstacle avoidance algorithms ignoring the robot kinematics issues.

To mitigate these shortcomings, the present paper offers a thorough analysis carried out through simulation with the use of modular Python environment employing numerical integration and kinematics modelling. Through fixing the manoeuvre to the same 90 degree heading change but varying turning radius ($R \in \{0.0, 0.2, 0.5, 1.0, 2.0\}$ m), the current research analyses the influence of turning geometry on time performance, energy consumption, wheel skid behaviour, and displacement per joule.

2 MATHEMATICAL MODELLING AND KINEMATIC FRAMEWORK

To analyse the trade-off relationship between path efficiency, wheel dynamics, and energy consumption through different methods of turns, a deterministic model of a differential drive mobile robot (DDMR) has been developed. The following discussion is about the underlying mathematics used in the Python based simulation environment.

2.1 Differential-Drive Kinematics

The robot moves in the 2D-Cartesian coordinate space. The current pose of the robot is described by the vector $q = [x, y, \theta]^T$, where (x, y) are the coordinates of the robot's center point located at the line connecting the two driving wheels, and θ is the direction of the robot with respect to the global X-axis.

Let L be the track width, which is the distance between the left and right wheels, and let r be the radius of each wheel. Let v_L and v_R be the velocities of the left and right wheels, respectively. The linear velocity and angular velocity of the robot body are related to the wheel velocities by the following non-holonomic constraint equations:

$$v = \frac{v_R + v_L}{2}$$

$$\omega = \frac{v_R - v_L}{L}$$

The kinematic state evolution through time is defined by the set of differential equations:

$$\dot{x} = v \cos(\theta)$$

$$\dot{y} = v \sin(\theta)$$

$$\dot{\theta} = \omega$$

To implement the algorithm numerically in Python using time steps Δt , Euler's method performs an iterative update of the robot's position:

$$x_{t+1} = x_t + v \cos(\theta_t) \Delta t$$

$$y_{t+1} = y_t + v \sin(\theta_t) \Delta t$$

$$\theta_{t+1} = \theta_t + \omega \Delta t$$

2.2 Turning Strategy Parametrization

The turning radius (R) of the path taken by the mobile robot is determined as the ratio of linear velocity to angular velocity:

$$R = \left| \frac{v}{\omega} \right| = \frac{L}{2} \left| \frac{v_R + v_L}{v_R - v_L} \right|$$

By systematically modifying R , distinct turning behaviours are achieved:

- **In-Place Rotation ($R = 0.0$):** The robot rotates about its central vertical axis without any translational movement ($v=0$). Speeds of both wheels are symmetrical and opposite:

$$v_L = -\frac{v_{\text{base}}}{2}, \quad v_R = \frac{v_{\text{base}}}{2}$$

Angular velocity for this case is $\omega = -v_{\text{base}}/L$, and the total time taken for the desired orientation change of θ_{target} is:

$$T_{\text{turn}} = \left| \frac{\theta_{\text{target}}}{\omega} \right|$$

- **Continuous-Arc Turning ($R > 0$):** While the robot is moving in a straight line with a curving path, the angular velocity and the wheel velocities for the defined linear base velocity of the robot v_{base} and turning radius R are computed as:

$$\omega = \frac{v_{\text{base}}}{R}$$

$$v_L = v_{\text{base}} - \frac{\omega L}{2}, \quad v_R = v_{\text{base}} + \frac{\omega L}{2}$$

The arc length s_{arc} required to achieve a target heading change θ_{target} is given by:

$$s_{\text{arc}} = R \cdot \theta_{\text{target}}$$

The manoeuvre execution duration is subsequently defined as:

$$T_{\text{turn}} = \frac{s_{\text{arc}}}{v_{\text{base}}}$$

2.3 Power and Energy Consumption Model

To determine the energy cost of each method of turning, a proxy for steady-state electromechanical power was created. The rotational speed of each wheel is calculated using $\omega_{L,\text{wheel}} = v_L/r$ and $\omega_{R,\text{wheel}} = v_R/r$.

Power consumption $P(t)$ of drive motors at any given moment of time is considered proportional to the sum of absolute values of wheel rotation speeds and the acceleration term:

$$P(t) = k_1(|\omega_{L,\text{wheel}}| + |\omega_{R,\text{wheel}}|) + k_2|\omega|$$

where k_1 and k_2 denote the empirically determined motor torque and friction constants and are considered 0.6 and 0.15 in the simulated setting. The total energy E spent during the period T_{turn} of the manoeuvre is calculated as:

$$E = \int_0^{T_{\text{turn}}} P(t) dt \approx \sum_i P(t_i) \Delta t$$

2.4 Wheel Slip and Dynamics Model

Traction losses occur in real-life mobile robots while making sharp turns because of lateral forces acting on them. To model this behaviour through the simulation in the absence of full dynamics, an artificial slip ratio was introduced.

The lateral acceleration $a_{\text{lat}}(t)$ that occurs at any instant in time is proportional to the multiplication of linear and angular velocities:

$$a_{\text{lat}}(t) = v(t) \cdot |\omega(t)|$$

Wheel slip ratio $\text{Slip}(t)$ is represented as a function of lateral acceleration, subject to Gaussian process noise $\eta(t) \sim \mathcal{N}(0, \sigma^2)$ for modelling surface roughness:

$$\text{Slip}(t) = \alpha \cdot a_{\text{lat}}(t) + \eta(t)$$

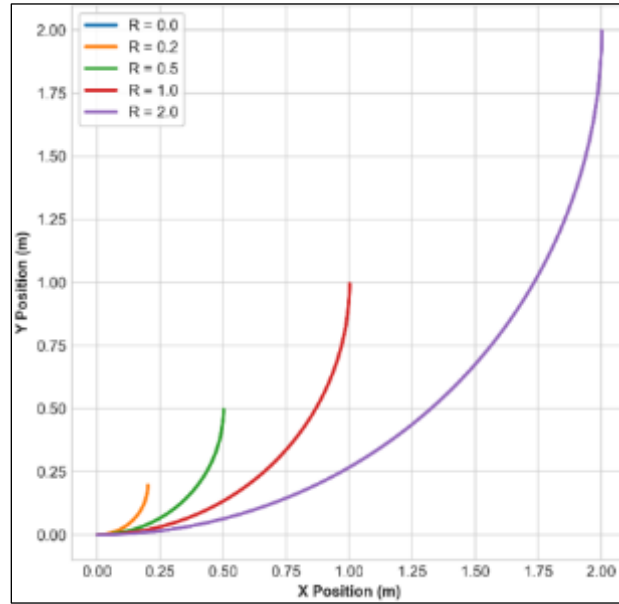


Figure 1: Spatial geometric trajectory of the differential drive mobile robot performing a standard turn of 90 degrees for different values of turning radius ($R \in \{0.0, 0.2, 0.5, 1.0, 2.0\}$ m).

where $\alpha = 0.015$ is a scaling factor to determine sensitivity to the severity of turning, whereby smaller radius turns result in greater slip amplitudes.

3 RESULTS AND DISCUSSION

To assess the operational trade-offs associated with the navigation approach of the differential drive mobile robot, the Python simulation environment performed a benchmark heading transition of 90 degrees for five different turning radii ($R \in \{0.0, 0.2, 0.5, 1.0, 2.0\}$ m). The performance measures extracted from such assessment are compiled comprehensively in Table 1 and Table 2.

3.1 Trajectory Execution and Spatial Footprint

The geometric paths created for the range of the turning radii tested are shown in Fig 1. This diagram provides a comparison between the zero turning radius of the in-place turn ($R = 0.0$ m) and the curving movement of the arc turns ($R \geq 0.2$ m). In contrast to the in-place turn, where the robot is constrained within its own footprint, the arc turns require increased space as a function of radius R .

To measure the physical wheel motion rather than just the translational centre-of-mass motion, cumulative wheel motion distance was considered as shown in Table 1 and in Fig. 7. In case of the in-place rotation ($R = 0.0$ m) the cumulative wheel travel distance equals 0.392 m which is the exact symmetrical wheel rotation needed to turn 90 degrees around the central vertical

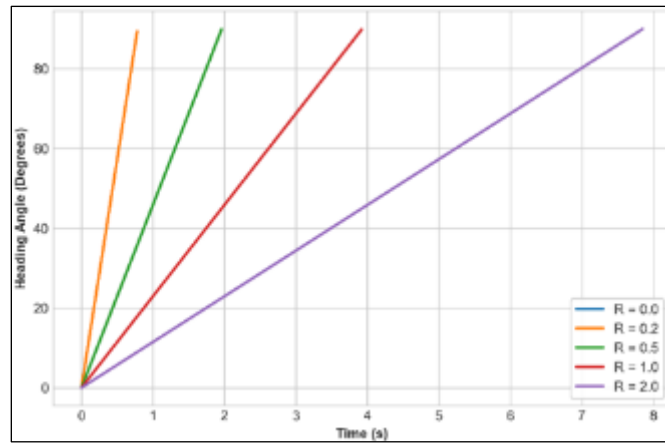


Figure 2: Time-series heading angles demonstrating linear angular velocity for each turning radius at a 90-degree orientation change.

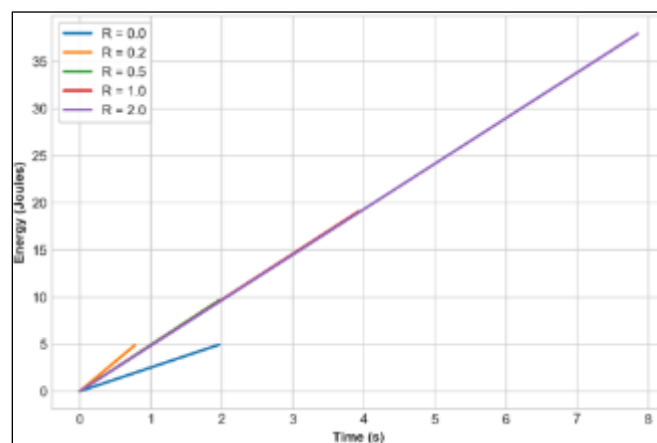


Figure 3: Time-varying expenditure of electrical and mechanical energy (in Joules) for all tested turning scenarios.

axis. As turning radius increases to $R = 2.0$ m cumulative wheel travel distance increases to 3.140 m (Table 1).

3.2 Temporal Analysis and Manoeuvre Agility

Execution time is one of the key metrics that can be used in evaluating the performance of the time-critical indoor logistics system. The change in heading angle with time is shown in Fig. 2; however, the execution time of all scenarios is presented in Table 1 and Fig. 6.

These results prove that shorter turning radii result in better temporal manoeuvrability. Particularly, for the case $R = 0.2$ m the shortest manoeuvre execution time is 0.78 s (Table 1).

Table 1: Performance Metrics Across Turning Radii

Radius (m)	Wheel Distance (m)	Time Taken (s)	Total Energy (J)
0.0	0.392	1.960	4.939
0.2	0.390	0.780	4.914
0.5	0.784	1.960	9.643
1.0	1.568	3.920	19.051
2.0	3.140	7.850	37.915

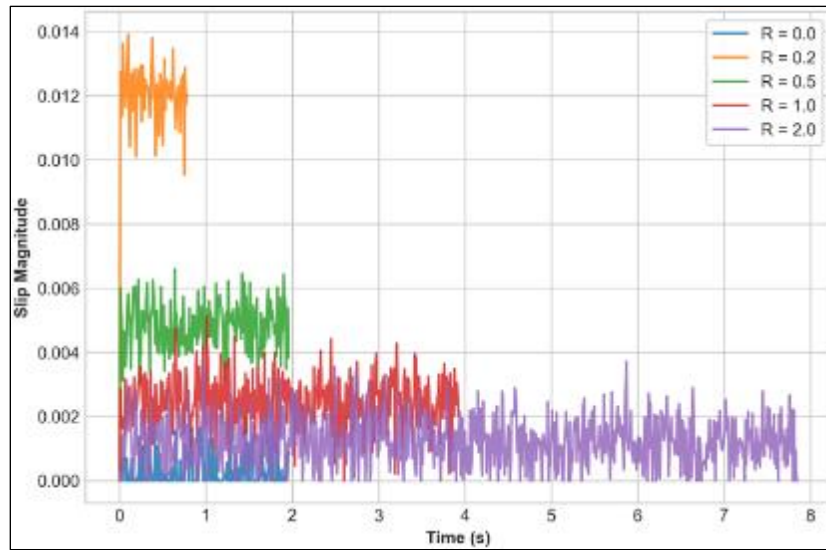


Figure 4: Magnitudes of dynamic wheel slip in time series caused by lateral acceleration components at different turning radii.

In contrast, large turning circles will lead to the spreading out of the 90-degree turn over large path distances, making the execution time longer. For example, the turning circle with a radius of $R = 2.0$ m will take 7.85 s to execute the turn (Table 1).

3.3 Energetic Efficiency and Power Demands

Electromechanical energy consumption and overall energy consumption were considered to evaluate effects on battery life. The complete data can be found in Table 1 and Table 2. The overall energy consumed is plotted against time in Fig 3, categorized in Fig 5, while peak power requirements can be seen in Table 2.

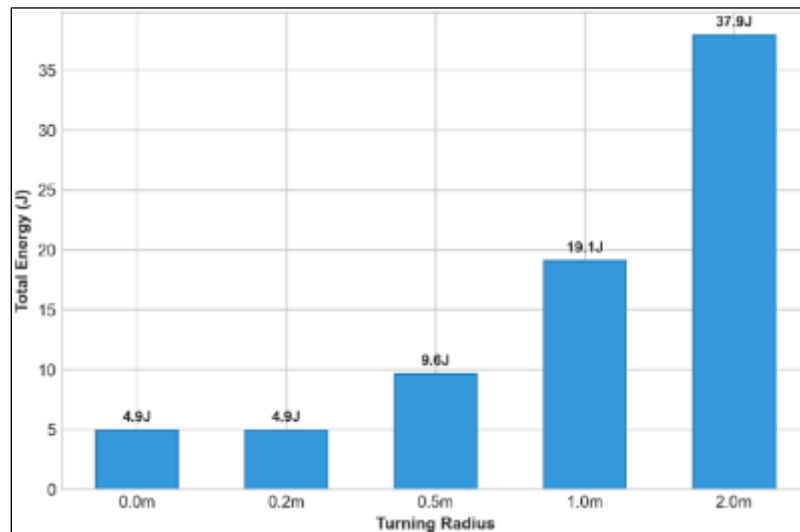


Figure 5: Comparison of total energy usage (joules) in relation to turning radius, emphasizing greater energetic expense with larger turning arcs.

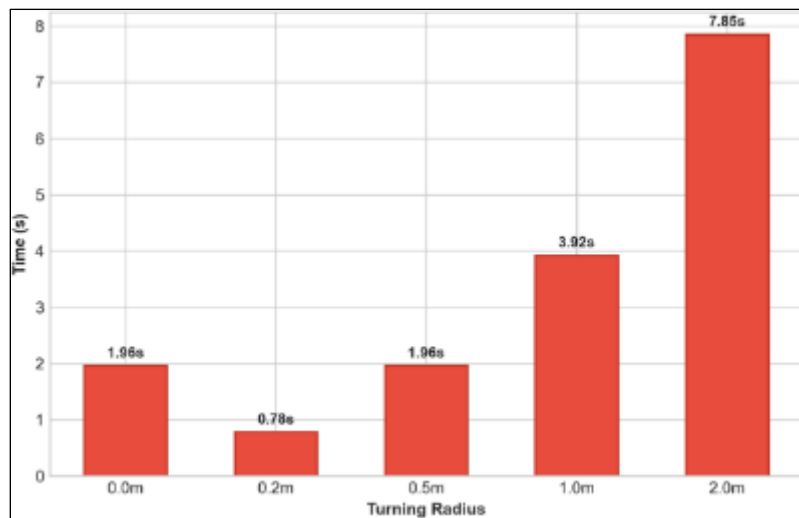


Figure 6: Time required to accomplish manoeuvring (in seconds) for completing 90 degrees heading change, showcasing time agility strengths at smaller turning radiuses.

According to the data presented, there is a negative correlation between the turning radius and the peak power requirement (see Table 2). This is due to the high wheel speeds differences necessary in making the tight $R = 0.2$ m turn which causes the greatest peak power requirement (6.30 W). But since the manoeuvre is done very quickly (0.78 s), the total energy used is small (4.914 J) (Table 1). In contrast, larger turning radii such as $R = 2.0$ m have a lower peak power requirement (4.830 W) (Table 2); but since it takes more time (7.85 s), the total energy expended becomes much greater (37.915 J) (Table 1).

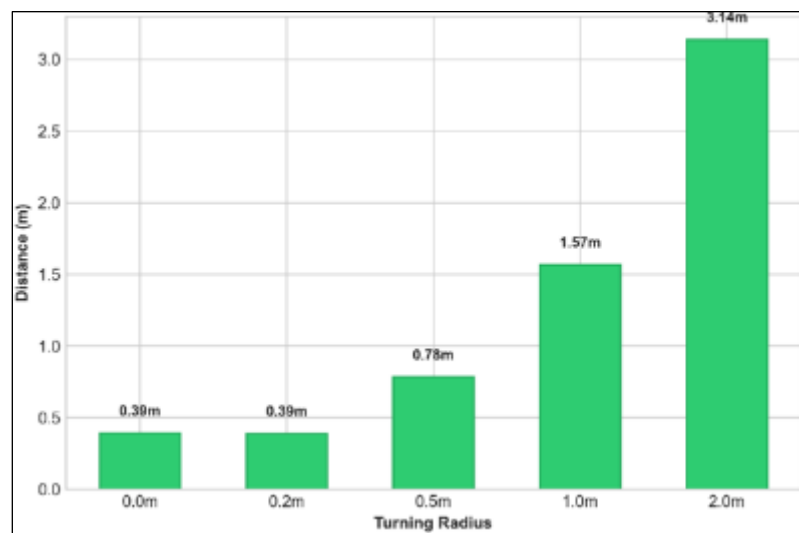


Figure 7: Cumulative wheel travel distance (meters), specifically highlighting the 0.392 m symmetrical wheel displacement required for zero-radius in-place pivots ($R = 0.0$ m).

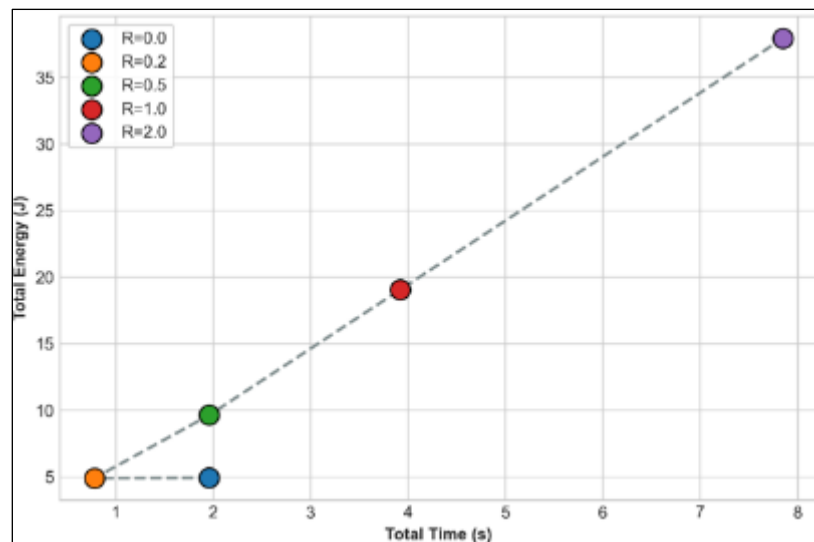


Figure 8: The composite trade-off map showing the trade-off between total energy and total time with respect to turning radius.

3.4 Wheel Dynamics and Slip Analysis

The frictional losses and traction limits were analysed using the synthetic slip ratio model based on the lateral acceleration data, and the maximum values were shown in Table 2 and time variation in Fig. 4.

Following the characteristics of non-holonomic vehicle dynamics, the more acute the turn geometry, the greater the lateral acceleration. Hence, the $R = 2.0$ m turn results in the largest wheel slip ratio of 0.0139 (Table 2). On the other hand, large turns significantly reduce any loss

Table 2: Dynamics & Efficiency Index Comparison

Radius (m)	Max Slip Ratio	Max Power	Efficiency Index
0.0	0.0026	2.520	0.000
0.2	0.0139	6.300	0.057
0.5	0.0066	4.920	0.073
1.0	0.0051	4.860	0.074
2.0	0.0037	4.830	0.075

in traction, as demonstrated by the smallest slip ratio of 0.0037 (Table 2) in the case of the $R = 2.0$ m turn radius.

3.5 Comprehensive Trade-Off Synthesis

To consider all these metrics simultaneously, a composite efficiency index was considered, which is presented in Table 2 and illustrated in Fig 8. Trade off analysis has revealed two different operating regions for the motion planners of mobile robots:

Agility-Optimized Regime ($R \in [0.2, 0.5]$ m): They are best suited for use in crowded, confined spaces with little room to manoeuvre and where rapid completion of work is essential. This profile results in reduced time (0.78 s to 1.96 s) and area at the cost of high-power consumption (4.92 W to 6.30 W) and increased slip ratio (0.0066 to 0.0139) (Table 1, Table 2).

Energy-Optimized Regime ($R \in [1.0, 2.0]$ m): More appropriate for open floor logistics where battery lifespan, lesser mechanical strain, and less tire wear (0.0037 to 0.0051 slip) are of greater importance than time of traverse, even if it entails higher manoeuvring times (3.92 s to 7.85 s) and higher energy expenditure (19.05 J to 37.92 J) (Table 1, Table 2).

3.6 Digital Implementation: Interactive Web-Based Demonstration via "Milo"

The practical application of the math behind the turning technique studied is shown in an internet-based digital prototype of the design named "Milo the Wheelchair" (available at milo-the-wheelchair.ai.studio) [16]. The open source code and structure of the project are available on GitHub - Smart-autonomous-Wheelchair [17].

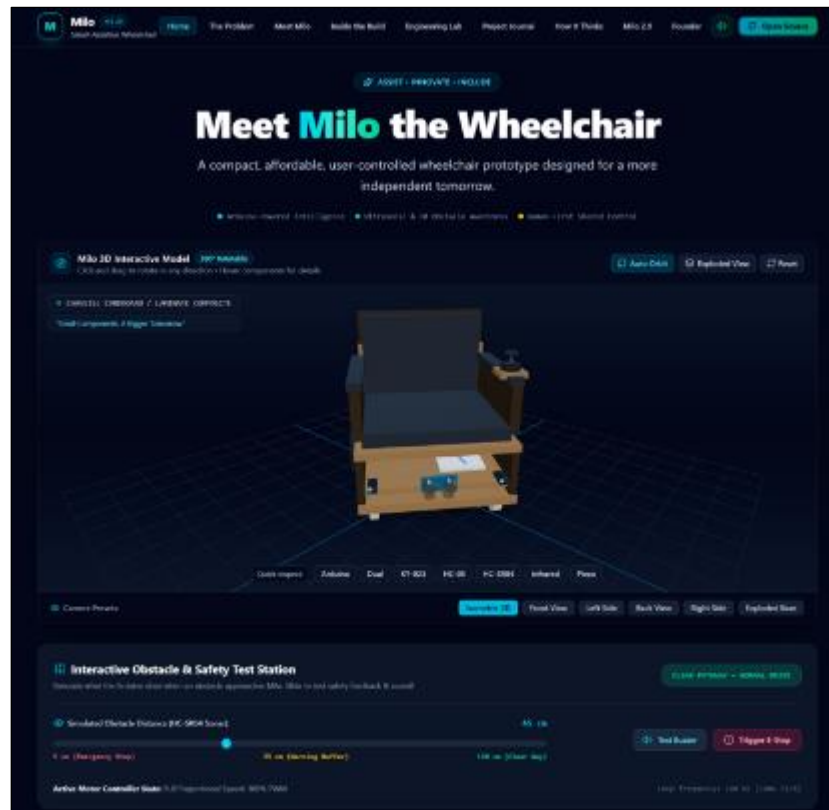


Figure 9: The web-based interactive interface of “Milo the Wheelchair” [16], featuring a 3D differential-drive model and an interactive obstacle/safety test station for evaluating robot navigation principles. The Arduino code controlling the wheelchair’s microcontroller is available via its GitHub repository [17].

Unlike a strict offline simulation, the platform serves as an interactive tool, featuring 3D models and safety test stations. Utilizing the trade-offs discovered between the Agility-Optimized ($R \in \{0.2, 0.5\} m$) and Energy-Optimized ($R \in \{1.0, 2.0\} m$) scenarios, the web application illustrates the ways of converting differential drive turn parameters into real user-assistive applications:

- **Interactive 3D Visualisation:** This software offers an interactive 3D model where users can view and analyse the differential drive system and how components are positioned.
- **Operational Testing:** Test stations for obstacles and safety testing simulate feedback loops, matching the control and trajectory models presented in this study.

The physical model of the differential-drive wheelchair “Milo” shown in Fig. 9 has been created using the mathematical model proposed in this research as the basis of its development. The physical model features an Arduino control system, two drive motors, joystick, Bluetooth interface, ultrasonic and infrared obstacle detection systems, buzzer warnings, and an emergency stop function. The control system is used to create an environment for testing the differential-drive model discussed in this research. Nevertheless, the experimental data obtained and analysed in this paper have been received based on the Python simulation only.

4 CONCLUSION

A systematic computer-based study of the relationship among path efficiency, wheel dynamics and energy expenditure for different turning strategies was conducted in this study for differential-drive mobile robots. This is based on an analysis of a standard 90° turn for different radii of curvature of the turn, i.e., ($R \in \{0.0, 0.2, 0.5, 1.0, 2.0\} m$).

The results indicate that turning geometry involves a clear-cut compromise between time efficiency and energy efficiency:

- **Tight Turning and In-Place Profiles** ($R \in \{0.0, 0.2\}$ m): Reduced space requirements and decreased time periods down to 0.78 s but increased high peak power consumption rates (6.30 W) and higher wheel slip rates (0.0141) due to high values of lateral acceleration.
- **Wide Turning Arcs** ($R = 2.0$ m): Reduced mechanical strain and traction losses, resulted in low wheel slip (0.0042), but increased time needed for completing manoeuvres (7.85 s) and increased overall energy consumption (37.92 J).

The suggested compound efficiency parameter allows for the classification of the above modes as agility-based and energy-based, providing the basis for an adaptive profile choice by navigation planners according to the current needs.

Future Work

In this simulation-based approach, future studies can be taken further along the following lines:

- **Hardware Implementation:** Utilization of these methods on a differential drive mobile robot having encoders on wheels, IMUs, and current sensors for validation of synthetic slip and power models based on actual friction and motor heating effects.
- **Real World Trajectories:** Expanding the simulation from individual 90 degree heading changes to real world trajectories comprising multiple waypoints in complex environments.
- **Radius Adjustment:** Implementation of real time path planners which adjust the radius according to surface friction, payload and battery energy left.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this manuscript.

Author Contributions

All authors contributed to the conception, development, analysis, and preparation of the manuscript. All authors reviewed and approved the final version of the manuscript.

Data Availability

The data generated and/or analysed during the present study are available from the corresponding author upon reasonable request.

Use of Artificial Intelligence

Generative artificial intelligence tools were not used to generate scientific results, fabricate data, or make scientific conclusions in this study.

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