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(Review Article)

Exponential Expansion of Bianchi Type-II Universe in $f(R,T)$ Gravity with Quadratic Equation of State

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Abstract

The LRS Bianchi type-II space time is investigated in the presence of a perfect fluid with quadratic equation of state and the cosmological constant Λ within the framework of the modified $f(R,T)$ theory of gravity proposed by Harko *et al.* (2011). By adopting a suitable functional form $f(R,T) = R + 2f(T)$ and assuming an exponential behavior for the time dependent average scale factor $a(t)$, the cosmological solution is derived. The physical characteristics of the resulting model are further analyzed through graphical representation.

Keywords: $\boldsymbol{f(R,T)}$ gravity; LRS Bianchi type-II space time; Perfect fluid; Quadratic EoS

1. Introduction

Cosmology deals with the scientific study of the universe as a whole, including its origin, structure, composition, and evolution. In recent years, significant progress has been achieved through observational advances such as measurements of the cosmic microwave background radiation, large-scale galaxy distributions, and the discovery of the accelerated expansion of the universe. Although the standard cosmological model successfully explains many present-day observations, it remains inadequate in describing the physical conditions of the early universe, where deviations from perfect homogeneity and isotropy are expected to have played a vital role (Ryden, 2017; Peebles, 1993).

Observational studies related to the isotropy of the cosmic microwave background radiation and primordial element abundances have further increased interest in anisotropic models. While standard cosmology explains the current large-scale structure of the universe, it does not fully capture the complexity of its earliest stages. In contrast, anisotropic and inhomogeneous models provide a more realistic description of these early phases, motivating the search for exact solutions of Einstein's field equations that can describe such conditions.

Within relativistic cosmology, it is common to represent matter through the energy–momentum tensor of a perfect fluid. Homogeneous models incorporating such matter distributions, along with specified equations of state, have proven effective in studying various dynamical and physical properties of the universe. These approaches help bridge theoretical models with observational phenomena.

In recent times, modified theories of gravity have been widely explored as alternatives to explain cosmic acceleration without invoking dark energy. Among them, the $f(R,T)$ gravity theory proposed by Harko *et al.* (2011) has attracted considerable attention. In this framework, the gravitational action depends on both the Ricci scalar R and the trace of the energy–momentum tensor T, leading to a non-minimal coupling between matter and geometry. This modification

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results in altered gravitational dynamics that can account for late-time acceleration (Houndjo *et al*., 2014; Sahoo & Sarmah, 2023).

Within this context, anisotropic Bianchi-type spacetimes provide a useful setting for exploring early-universe dynamics. Among these, Bianchi-type II models are particularly significant due to their spatial homogeneity combined with anisotropic structure governed by a non-trivial Lie algebra. These models include important limiting cases such as Taub–NUT solutions and certain curved FRW models, thereby offering a connection between anisotropic and isotropic cosmologies (Rani & Santhi, 2019; Bhardwaj & Yadav, 2020).

Several researchers have studied Bianchi-type II cosmological models in modified gravity frameworks, especially for locally rotationally symmetric configurations with perfect fluid distributions. Reddy and Santhi (2013) obtained exact solutions by assuming a specific law for the Hubble parameter that yields a constant deceleration parameter. Their results show that the spatial volume increases with time, indicating cosmic expansion with late-time acceleration. However, physical quantities such as expansion scalar, shear, and energy density diverge at the initial epoch and decrease over time, while anisotropy persists throughout the evolution (Singh & Sharma, 2014; Mahanta *et al*., 2014).

Further investigations have extended these models by including exotic matter sources such as domain walls, massive strings, magnetic fields, and bulk viscosity. These studies indicate that although the universe undergoes accelerated expansion, anisotropy may remain a persistent feature. Moreover, cosmological parameters like redshift, luminosity distance, and distance modulus have been found to be consistent with observational data under suitable conditions (Shukla & Pradhan, 2014; Bhardwaj, 2020; Sarmah & Goswami, 2024).

Singh and Bishi extended the quadratic-EoS Bianchi I model to $f(R,T)$ gravity with a cosmological constant, analyzing two classes, $f(R,T) = R + 2f(T)$ and $f(R,T) = f_1(R) + f_2(T)$, and showed that the same EoS can generate both decelerated and accelerated phases, with the decay of shear depending sensitively on the modified-gravity parameters (Singh & Bishi, 2015). Further studied Bianchi I cosmologies with a quadratic EoS in $f(R,T)$ theory allowing a variable cosmological term, obtaining a singular power-law solution and a non-singular exponentially expanding solution, both of which become asymptotically isotropic at late times (Beesham et al., 2020). In their analysis the effective cosmological parameter decreases with time and tends toward a small constant, suggesting that quadratic-EoS anisotropic models in $f(R,T)$ gravity can address the cosmological-constant problem while accommodating transitions between decelerated and accelerated expansion.

In a higher-dimensional setting, Baro *et al*. (2021) investigated Bianchi V cosmological models in general relativity with $p = \alpha\rho^2 - \rho$, deriving exact solutions and showing that the quadratic term strongly influences the evolution of density, pressure, and kinematical quantities while allowing negative effective pressure at late times. Mahanta *et al.* (2023) analyzed a Bianchi V universe in $f(R,T)$ gravity with a time-varying cosmological constant and quadratic EoS, obtaining two exact models (volumetric and exponential expansion) and finding that the exponential case closely mimics ΛCDM while satisfying standard energy conditions. Conference work by Gupta *et al.*(2022) on Bianchi V space-time with a quadratic EoS in a generic modified-gravity framework employed a time-dependent deceleration parameter to produce realistic transitions from early deceleration to the current accelerated phase, again highlighting the flexibility of quadratic fluids in anisotropic geometries.

Recent work has also focused on applying observational constraints and dynamical system analysis to Bianchi-type cosmologies within modified gravity theories. These studies suggest that while certain forms of $f(R,T)$ gravity can reproduce behavior similar to the ΛCDM model at late times, they may exhibit deviations in the early universe. Nevertheless, Bianchi-type II models continue to provide a valuable framework for understanding the transition from an anisotropic early universe to the nearly isotropic state observed today (Kumar *et al*., 2018; Taori *et al*., 2015; Tiwari *et al*., 2017).

Motivated by these developments, the present work aims to extend and refine existing Bianchi-type II cosmological models within the framework of $f(R,T)$ gravity, with highlighting on maintaining both mathematical consistency and physical relevance in describing the evolution of the universe.

2. $\boldsymbol{f(R,T)}$ Theory of Gravity

In $f(R,T)$ gravity, the field equations are derived from a variational Hilbert–Einstein-type principle. The action for the modified $f(R,T)$ gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m\sqrt{-g}\, d^4x, \tag{1}$$

where, $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the trace of the stress-energy tensor of matter, T_{ij} is the stress-energy tensor, and L_m is the matter Lagrangian density.

The stress-energy tensor of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \tag{2}$$

and its trace is $T = g^{ij}T_{ij}$.

By assuming that L_m depends only on the metric tensor components g_{ij}, and not on their derivatives, one obtains

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}, \tag{3}$$

varying the action with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ gravity become

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\,\Box - \nabla_i\nabla_j\right)f_R(R,T) + \Lambda = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \tag{4}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}, \tag{5}$$

here,

$$f_R = \frac{\partial f(R,T)}{\partial R}, f_T = \frac{\partial f(R,T)}{\partial T}, \Box = \nabla^i\nabla_i$$

If $f(R,T) \equiv f(R)$, then Equation (4) yields the field equations of $f(R)$ gravity.

For a perfect fluid with energy density ρ, pressure p, and four-velocity u^i, the stress-energy tensor is taken as

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij}\,, \tag{6}$$

The quadratic equation of state followed by perfect fluid is given by

$$p = K\rho^2 - \rho\,,$$

here, K is a constant.

The matter Lagrangian is chosen as $L_m = -p$, with

$$u^i\nabla_j u_i = 0, u^iu_i = 1, \tag{7}$$

Then one obtains

$$\theta_{ij} = -2T_{ij} - pg_{ij}\,, \tag{8}$$

Assuming

$$f(R,T) = R + 2f(T)\,, \tag{9}$$

where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor, the field equations in the presence of cosmological constant Λ reduces to

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij}, \tag{10}$$

For the perfect-fluid source, using equation (8), the field equations (10) become

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}\,, \tag{11}$$

2.1. Metric and Field Equations

We consider a homogeneous Locally Rotationally Symmetric Bianchi type-II space-time given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + 2B^2x\,dy\,dz + (B^2x^2 + A^2)dz^2, \tag{12}$$

where A and B are the functions of cosmic time t only.

Using comoving coordinates and Equations (6) - (8), the $f(R, T)$ gravity field equation (11), with the particular choice

$$f(T) = \lambda T, \lambda = \text{constant}, \tag{13}$$

for the metric (12), the field equation (12) in the presence of cosmological constant Λ takes the form

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{B^2}{A^4} = (8\pi + 3\lambda)p - \lambda\rho - \Lambda, \tag{14}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = (8\pi + 3\lambda)p - \lambda\rho - \Lambda, \tag{15}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = -(8\pi + 3\lambda)\rho + \lambda p - \Lambda, \tag{16}$$

where an overhead dot denotes differentiation with respect to cosmic time t.

2.2. Solutions and the Model

The field equations (14)–(16) form a system of three independent equations in four unknowns A, B, ρ, Λ and p. To obtain a solution, we consider the exponential law for $n \neq 0$ proposed by Bermann (1983) as,

$$a(t) = d\exp(c\,t), \tag{17}$$

where, c, d are the constant of integration.

The scalar expansion θ and the shear scalar σ^2 for model (12) are defined by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}, \tag{18}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \tag{19}$$

The generalized Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \tag{20}$$

The average anisotropic parameter is

$$A_\alpha = \frac{1}{3}\sum \left(\frac{\Delta H_i}{H}\right)^2, \tag{21}$$

where

$$\Delta H_i = H_i - H, (i = 1,2,3)\,.$$

On solving equation (15)-(16) and equation of state for perfect fluid, the density of the model is given by

$$\rho^2 = \frac{2}{2K(4\pi+\lambda)}\left[\frac{\ddot{A}}{A} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{4}\frac{B^2}{A^4}\right], \tag{22}$$

Assuming that the expansion scalar θ is proportional to the shear scalar σ, one gets

$$A = B^m\,, \tag{23}$$

where m is a constant.

Solving the field equations with the help of (17) and (23), the metric coefficients are obtained as

$$A = d\,[\exp(c\ t)]^{\frac{3m}{(2m+1)}},\ B = d\,[\exp(c\ t)]^{\frac{3}{(2m+1)}}, \tag{24}$$

On using equation (24), the metric (12) can be written as

$$ds^2 = -dt^2 + \left(d\exp(c\ t)^{\frac{3m}{(2m+1)}}\right)^2(dx^2+dz^2)$$

$$+\left(d\exp(c\ t)^{\frac{3}{(2m+1)}}\right)^2(dy^2+2x\,dy\,dz+x^2dz^2), \tag{25}$$

2.3. Physical and Kinematical Properties of the Model

The physical and kinematical parameters for the equation (25) which represents a perfect fluid Bianchi type-II cosmological model in $f(R,T)$ gravity are given as follows,

The spatial volume is

$$V = A^2B = d^3\,[\exp(c\ t)]^{\frac{3(2m+1)}{(2m+1)}}, \tag{26}$$

which shows that the volume expands exponentially with time. Hence, the Universe begins with a smaller spatial volume in the past and grows continuously as t increases. This behavior is consistent with an ever-expanding cosmological model.

Using equation (18) and (19), the scalar expansion θ and the shear scalar σ^2 are given by

$$\theta = 3\mathrm{c}, \tag{27}$$

$$\sigma^2 = \frac{3c^2(m-1)^2}{(1+2m)^2}, \tag{28}$$

Since θ is constant, the model exhibits a uniform rate of expansion. In particular, the expansion does not depend explicitly on time, indicating a steady kinematical evolution of the universe. Also σ^2 is also constant in time. It measures the deviation of the model from isotropic expansion. For $m \neq 1$, the shear remains nonzero, so the model is anisotropic. However, when $m = 1$, the shear scalar vanishes and the model reduces to isotropic expansion.

Using equation (20), the generalized Hubble parameter is given by

$$H = \mathrm{c}, \tag{29}$$

Here, the Hubble parameter is constant, confirming the exponential nature of the expansion. A constant H implies that the expansion rate of the universe does not change with cosmic time in this model.

Using equation (21), the average anisotropic parameter is given by

$$\Delta = \frac{3(1+2m^2)}{(1+2m)^2} - 1, \tag{30}$$

Since this expression does not contain t explicitly, the anisotropy parameter is time-independent. This means that the degree of anisotropy remains fixed throughout the cosmic evolution. The model is therefore not isotropizing with time unless the constant m is chosen such that the anisotropy vanishes

On solving equation (22), the density of the model is given by

$$\rho = \left[\frac{1}{2K(4\pi+\lambda)}\left(\frac{9c^2(-1+m)m}{(1+2m)^2} - \frac{\left(e^{ct}\right)^{-6+\frac{12}{1+2m}}}{4d^2}\right)\right]^{\frac{1}{2}}, \tag{31}$$

By using quadratic EoS, the pressure corresponding to above density in equation (31) is given by

$$p = \frac{1}{2K(4\pi+\lambda)}\left(\frac{9c^2(-1+m)m}{(1+2m)^2} - \frac{\left(e^{ct}\right)^{-6+\frac{12}{1+2m}}}{4d^2}\right) - \left[\frac{1}{2K(4\pi+\lambda)}\left(\frac{9c^2(-1+m)m}{(1+2m)^2} - \frac{\left(e^{ct}\right)^{-6+\frac{12}{1+2m}}}{4d^2}\right)\right]^{\frac{1}{2}}, \tag{32}$$

Using Equations (15), (24), (31) and (32), the cosmological constant Λ is given by

$$\Lambda = \frac{27c^2m^2}{(1+2m)^2} + \frac{\sqrt{2}(2\pi+\lambda)}{\sqrt{K(4\pi+\lambda)}}\sqrt{\frac{6^2c^2(-1+m)m}{(1+2m)^2} - \frac{e^{\frac{6ct(1-2m)}{1+2m}}}{d^2}} + \left(\frac{(8\pi+3\lambda)}{8(4\pi+\lambda)}\left(\frac{e^{\frac{6ct(1-2m)}{1+2m}}}{d^2} - \frac{36c^2(-1+m)m}{(1+2m)^2}\right) - \frac{3}{4}e^{\frac{6ct(1-m)}{1+2m}}\right), \tag{33}$$

From equation (31) it is evident that the energy density ρ is controlled by the competition between the geometric contribution involving m and the exponentially decaying term in t, so that ρ remains positive as long as the bracketed term stays non-negative. The square-root dependence implies that even moderate variations in the combination of parameters $K, (4\pi + \lambda), c, d$ and m can lead to significant changes in the magnitude of the density, allowing one to tune early and late-time energy scales in the model. As t increases, the exponential factor weakens the second term, so the effective density gradually saturates towards a constant value determined mainly by the anisotropy parameter m this mimics a transition from a high-density phase to a quasi-de Sitter regime. Using the quadratic equation of state, the corresponding pressure (32) acquires both a linear and a nonlinear dependence on ρ, so that the pressure is no longer a simple barotropic multiple of the density, but encodes higher-order fluid effects relevant at high density. At early times, when the square-root contribution is large, the quadratic correction in (32) can drive the pressure to substantially negative values, which is compatible with an accelerating phase similar to those discussed in quadratic-EoS dark energy and dark matter models. As the universe expands and ρ decreases, the nonlinear term becomes subdominant and the pressure gradually approaches an effectively linear law, so that the fluid behaves more like standard matter or dark energy, depending on the choice of parameters.

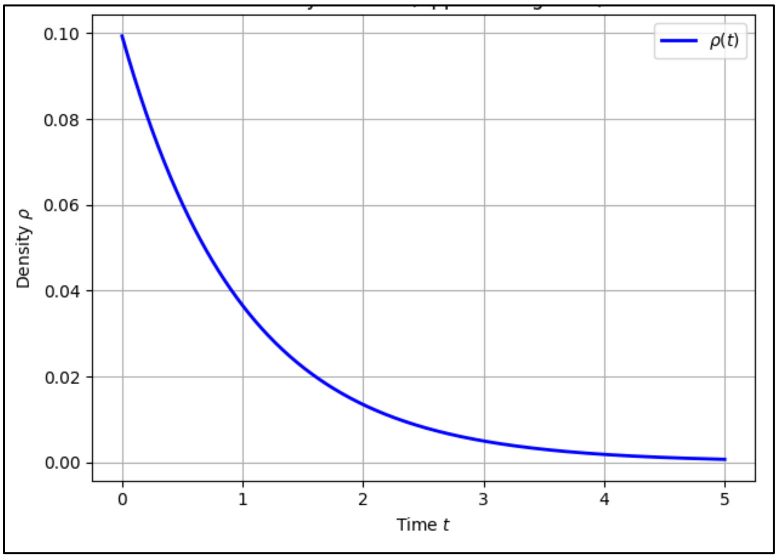

Figure 1 Density verse cosmic time $\boldsymbol{t}$ for $\boldsymbol{c = 1, d = 1, m = 1, K = -1, \lambda = 0.1}$

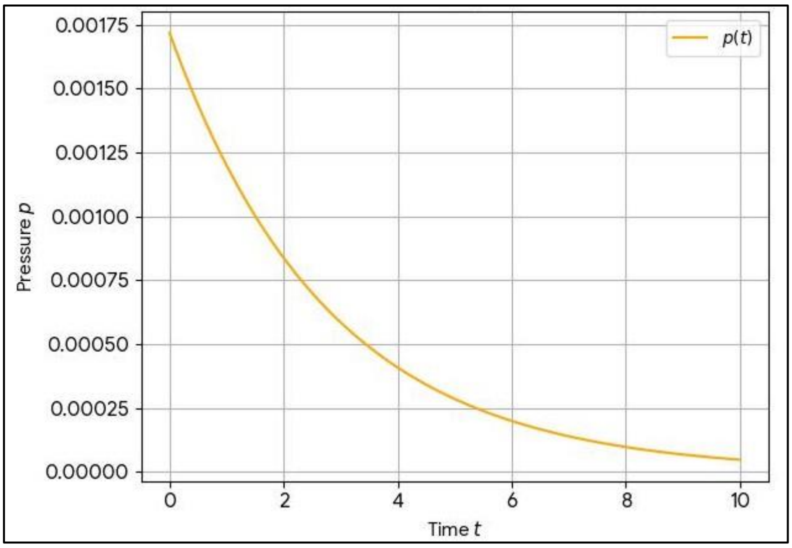

Figure 2 Pressure verse cosmic time $\boldsymbol{t}$ for $\boldsymbol{c = 1, d = 1, m = 1, K = -1, \lambda = 0.1}$

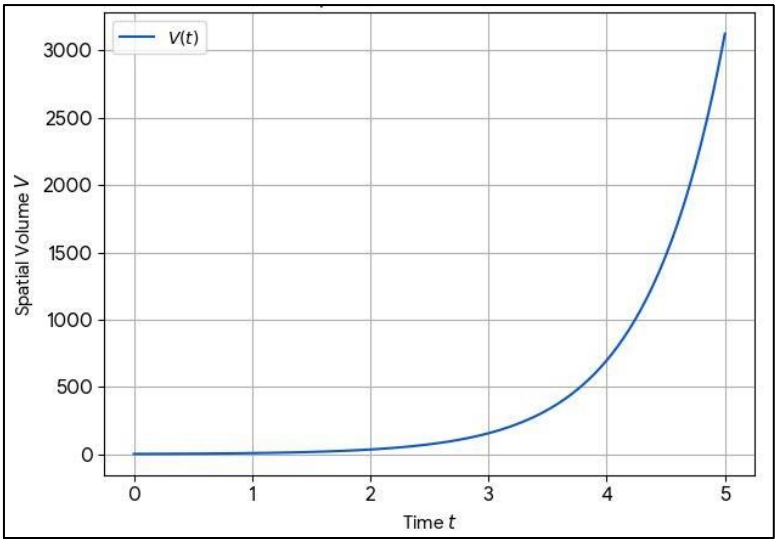

Figure 3 Spatial Volume verse cosmic time $\boldsymbol{t}$ for $\boldsymbol{c = 1, d = 1.2, m = 1}$

From the figure – 1 and 2, it is observe that the as the volume of the universe increases, both density and pressure decrease over time. This drop aligns with thermodynamic principles in cosmology as the spatial volume expands, matter and radiation are diluted, leading to a reduction in both the mass density and the outward pressure exerted by the fluid. From the figure – 3 it is observe that the volume of the model grows exponentially over time. This rapid expansion corresponds to a continuously expanding universe, which is driven by the expansion rate parameter c.

3. Conclusion

The LRS Bianchi type-II cosmological model in the presence of a perfect fluid has been studied in $f(R,T)$ theory of gravity. In this work, we have obtained a cosmological model in which the key physical and kinematical parameters are expressed explicitly as functions of cosmic time t only. The resulting model exhibits an exponentially expanding universe with constant Hubble parameter, constant expansion scalar, and nonzero shear scalar for $m \neq 1$, showing that the universe remains anisotropic during its evolution. The spatial volume increases continuously with time, indicating an expanding cosmological scenario.

The energy density and pressure are time-dependent and the model provides a viable framework to simulate a unified cosmic evolution scenario in which an initially dense, rapidly evolving universe naturally evolves toward a late time accelerating phase without appealing separate dark components.

The average anisotropic parameter remains constant, which implies that the model does not approach isotropy at late times unless special parameter choices are made. Thus, the model provides an exact anisotropic cosmological solution with steady expansion and persistent directional dependence. Such solutions are useful for understanding the possible role of anisotropy in the early and late stages of cosmic evolution. The model may be useful for studying the large-scale dynamics of the early universe in $f(R,T)$ theory of gravity.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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