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(Short Communication)

The Problem of Time in Canonical Quantum Gravity: A Model Study

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Abstract

The incorporation of quantum mechanics with general relativity has remained one of the greatest challenges in theoretical physics, fundamentally manifest in the problem of time. This problem appears primarily in the canonical quantization of general relativity. For example, in Wheeler-DeWitt equation the resulting dynamics are invariant under time reparameterizations which leads to a static, time-independent description of the universe. In this paper we briefly outline the origin of the problem of time and then discuss the deparametrization scheme for a system whose motion is non-linear and coupled between two variables.

Keywords: General relativity; Quantum gravity; Problem of time; Deparametrization

1. Introduction

The canonical formulation of general relativity [1] treats the spacetime metric, $g_{\mu\nu}$, as the fundamental variable. When quantized, this leads to the Wheeler-DeWitt equation [2]. In this equation the total Hamiltonian is a constraint. In fact, the total Hamiltonian is the sum of Hamiltonian and diffeomorphism constraints [3]. The Hamiltonian constraint is in the core of the problem of time in quantum gravity. It so happens that in quantum version of this constraint we set the quantum Hamiltonian operator to zero, which means that the wave function $|\psi\rangle$ does not evolve with respect to an external time parameter *t.* The Wheeler-DeWitt equation describes a state which is 'frozen' in time. This is due to inherent reparametrization invariance of general relativity. Time here is simply a coordinate level having no physical meaning intrinsically [4]. In this paper we will discuss about a system which shows a sort of reparametrization invariance and propose its deparametrization to see if the problem of time for this model appears or not.

2. Primary Constraint

We are discussing about a non-standard physical system in relativistic regime in which dynamical variables are coupled.

The action is

$$S(\xi,\eta) = \int d\tau \sqrt{\frac{m}{2}\left(\dot{\xi}^2 + \dot{\eta}^2\right)\left(E - \frac{1}{2}k(\xi - \eta)^2\right)}\ , \qquad (1)$$

where $\dot{\xi} = \frac{d\xi}{d\tau}$ and $\dot{\eta} = \frac{d\eta}{d\tau}$, and τ is a parameter. In this expression ξ and η are coordinates of two particles, E is a constant energy term and m is mass of each particle. It can be proved that under some approximation this action is reparametrization invariant. Due this invariance the canonical Hamiltonian for this action is identically zero. This reparametrization invariance is an indication that the dynamics of this system is governed by the constraints.

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Now the total Hamiltonian of the system will be

$$H_T = \lambda\left(p_\xi{}^2 + p_\eta{}^2 - {}^m/_2\left(E - {}^1/_2\, k(\xi - \eta)^2\right)\right), \tag{2}$$

where λ is Lagrange multiplier, p_ξ and p_η are canonical momenta.

The primary constraint for this theory is

$$p_\xi{}^2 + p_\eta{}^2 - {}^m/_2\left(E - {}^1/_2\, k(\xi - \eta)^2\right) = 0. \tag{3}$$

Due to this constraint the dynamics is restricted to the surface in phase space where this constraint is satisfied [5]. Since our action is parametrized by an arbitrary parameter τ, and the Hamiltonian does not explicitly depend on any external time parameter, the evolution is generated by the constraint itself.

3. Deparametrization

To deparametrize this theory we have to express the dynamics as the evolution with respect to an intrinsic degree of freedom, thus getting rid of the external and unphysical parameter. In fact, the aim of deparametrization [6] is to identify one of the physical variables within the system that changes monotonically. This physical time variable then serves as a physical clock. Under this approximation all other variables are expressed as the function of this time variable. In the context of quantum gravity deparametrization is performed to recover a notion of time evolution.

We note that reparametrization invariance of our action indicates that our arbitrary parameter τ is not physical and the dynamics should be expressible in terms of physical variables themselves. Moreover, the presence of the term $\left(\dot{\xi}^2 + \dot{\eta}^2\right)$ under the square root indicates that there is associated a notion of speed in configuration space. Since this speed is to remain non-zero along physical trajectories, it might be possible to select one of the coordinates to parametrize the path.

4. Problems of Deparametrization

Though our model is reparametrization invariant and a sort of deparametrization can be achieved, still there are some potential problems associated with it. The potential term depends on the difference of the coordinates, strongly coupling the motion of ξ and η. this coupling might lead to the situation where neither of the coordinates is guaranteed to be monotonic. The term $E - {}^1/_2\, k(\xi - \eta)^2$ under square root introduces a constraint on the allowed regions of the configuration space. At the boundaries of this region the speed might become zero. In this situation we can not use coordinates as time variables.

When we derive Euler-Lagrange equations for this system, these come out to be non-linear and coupled as

$$\frac{d}{d\tau}\left[\frac{m\dot{\xi}\sqrt{E - {}^1/_2 k(\xi - \eta)^2}}{\sqrt{2m\left(\dot{\xi}^2 + \dot{\eta}^2\right)}}\right] - \frac{-mk(\xi - \eta)\left(\dot{\xi}^2 + \dot{\eta}^2\right)}{4\sqrt{{}^m/_2\left(\dot{\xi}^2 + \dot{\eta}^2\right)\left(E - {}^1/_2 k(\xi - \eta)^2\right)}} = 0, \tag{4}$$

for variable ξ. Similarly for variable η we also get these complicated equations. It seems that without a further simplification deriving the explicit equations of motion by directly applying Euler-Lagrange equations can be mathematically very involved.

5. Conclusion

Our proposed action conceptually resembles the action of general relativity as it is reparametrization invariant. It is possible that our proposed action might correspond to an effective Lagrangian arising from a more fundamental theory after some degrees of freedom have been integrated out. But it surely describes motion on a manifold with a metric that depends on both velocities and positions in a particular way. Therefore, finding a solution for the problem of time in this model might give some insight. However, based on the coupled nature of the dynamics, as is obvious from Eq. 4, it is not guaranteed that our model is simply deparametrizable with respect to either ξ or η globally for all possible motions. Still, a sort of local deparametrization might be possible in those regions of phase space where one of the coordinates behaves monotonically. Unless we find a way to choose a monotonical variable for the evolution of the system we cannot proceed for its quantization and the problem of time will remain there. We will discuss these issues in next paper.

Compliance with ethical standards

Disclosure of conflict of interest

There is no conflict of interest to be disclosed.

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