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Viscous teleparallel gravity with FLRW space-time

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Abstract

This work explores the formulation of cosmological models within the framework of viscous teleparallel gravity, where gravity is described by torsion rather than curvature. Incorporating a bulk viscous fluid in a flat Friedmann-Robertson-Walker (FRW) universe, the study investigates modified Friedmann equations derived from a non-equilibrium thermodynamic approach. Exact solutions for different cosmic epochs, including accelerated expansion, are obtained using a specific ansatz for the Hubble parameter. The impact of viscous effects on cosmic dynamics is analyzed, showing that such models can mimic dark energy behavior. These results suggest that viscous teleparallel gravity provides a viable alternative description of late-time cosmic acceleration.

Keywords: Modified $f(T)$ gravity; Viscous fluid; FLRW Space Time; Hubble Parameter

1. Introduction

Observations of distant supernovae, the distribution of galaxies, and the cosmic microwave background suggest that the universe is geometrically flat and transitioned from a slowing to an accelerating expansion [1-4]. Explanations for this acceleration generally fall into two categories. One approach involves introducing a hypothetical substance with negative pressure, known as dark energy, within the standard framework of general relativity [5-8]. The other approach considers modifications to Einstein's theory of gravity itself. An alternative framework for explaining the universe's accelerating expansion, without invoking dark energy, is teleparallel gravity, often expressed as $f(T)$ theory [9-10]. This approach, while fundamentally equivalent to general relativity, offers a different perspective: it interprets gravity as arising from torsion rather than curvature. In teleparallel gravity, the fundamental dynamical variables are tetrad fields, and the Weitzenböck connection, which lacks curvature, replaces the Levi-Civita connection of standard general relativity, effectively substituting torsion for curvature. This formulation results in field equations that are second-order, which can be advantageous in certain calculations. Researchers have explored various aspects of $f(T)$ gravity. For example, the energy within an expanding universe has been analyzed [11], and its dynamical behaviour has been examined [12]. Spherically symmetric solutions [13] and the existence of relativistic stars [14] have also been investigated within this framework. More recently, studies have focused on interacting dark energy models with specific $f(T)$ functions [15], and the evolution of dark energy models using exponential and logarithmic $f(T)$ functions [16]. The distinction between ΛCDM and $f(T)$ gravity based on Noether symmetry has been explored [17]. The evolution from an early to a dark energy dominated universe, using a LRS Bianchi type-I model, has revealed an isotropization process, where initial anisotropic expansion transitions to isotropy at late times, and suggested that nonlinear models better align with observations [18]. Furthermore, the generalized second law of thermodynamics has been considered within $f(T)$ modified gravity [19]. Recently, Furthermore, foundational issues within $f(T)$ gravity theory remain a key focus, with researchers examining perturbative and non-perturbative approaches, causality, and degrees of freedom. In

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[20] the investigations aim to solidify the theoretical underpinnings of $f(T)$ gravity and address potential inconsistencies. There is ongoing research into the application of modified gravity mentioned in [21, 22]

Bulk viscosity is considered significant in the universe's early evolution, particularly around neutrino decoupling, where matter behaved as a viscous fluid. Additionally, it has been proposed that a cosmic fluid with bulk viscous pressure could drive the observed accelerated expansion [23]. The possibility of bulk viscosity causing the current accelerated expansion was explored by Fabris *et al*. [24]. Exact solutions incorporating bulk viscosity, where the viscous coefficient is a power function of mass density, have been derived [25]. Studies have also investigated LRS Bianchi type-I and Bianchi type-III cosmological models with cloud strings and bulk viscosity [26, 27]. The integrability of cosmic strings in Bianchi type-III space-time, with bulk viscous fluid, has been analyzed [28]. Furthermore, the impact of bulk viscosity, with a time-varying coefficient, on the evolution of isotropic Friedmann–Robertson–Walker (FRW) models within an open thermodynamic system has been examined [29].

Hence by inspiring the above discussion in the present work we consider the bulk viscosity in $f(T)$ gravity also, the motivation to study bulk viscous fluid within the framework of $f(T)$ gravity stems from a confluence of theoretical and observational considerations. Firstly, $f(T)$ gravity, as a modified theory of gravity based on torsion, offers a compelling alternative to dark energy for explaining the universe's accelerated expansion. Integrating bulk viscosity into this framework allows researchers to explore the interplay between modified gravity and dissipative processes in the cosmic fluid. Secondly, bulk viscosity is believed to have played a significant role in the early universe, particularly during neutrino decoupling, and its inclusion can provide a more realistic description of the universe's evolution. Combining these elements allows for the examination of how modified gravitational dynamics interact with the non-ideal fluid characteristics of the early and late-time universe.

2. A brief review of $f(T)$ cosmology

The line element of the Riemannian manifold is given by

$$dS^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \qquad (1)$$

This line element can be converted to the Minkowskian description of the transformation called tetrad, as follows

$$dS^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = \eta_{ij}\theta^i\theta^j, \qquad (2)$$

$$dx^{\mu} = e_i^{\mu}\theta^i,\ \ \theta^i = e_{\mu}^i dx^{\mu}, \qquad (3)$$

where $\eta_{ij} = diag[1,-1,-1,-1]$ and $e_i^{\mu}e_{\mu}^i = \delta_{\nu}^{\mu}$ or $e_i^{\mu}e_{\mu}^j = \delta_i^j$.

The root of metric determinant is given by $\sqrt{-g} = det[\,e_{\mu}^i] = e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbocks connection components are defined as

$$\Gamma_{\mu\nu}^{\alpha} = e_i^{\alpha}\partial_{\nu}e_{\mu}^i = -e_{\mu}^i\partial_{\nu}e_i^{\alpha} \qquad (4)$$

Through the connection, we can define the components of the torsion and contorsion tensors as

$$T_{\mu\nu}^{\alpha} = \Gamma_{\mu\nu}^{\alpha} - \Gamma_{\mu\nu}^{\alpha} = e_i^{\alpha}\left(\partial_{\mu}e_{\nu}^i - \partial_{\mu}e_{\mu}^i\right), \qquad (5)$$

$$K_{\alpha}^{\mu\nu} = \left(-\frac{1}{2}\right)\left(T^{\mu\nu}{}_{\alpha} + T^{\nu\mu}{}_{\alpha} - T_{\alpha}^{\mu\nu}\right). \qquad (6)$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor $S_{\alpha}^{\mu\nu}$ from the components of the torsion and contorsion tensors, as

$$S_{\alpha}^{\mu\nu} = \left(\frac{1}{2}\right)\left(K^{\mu\nu}{}_{\alpha} + \delta_{\alpha}^{\mu}T^{\beta\nu}{}_{\beta} - \delta_{\alpha}^{\nu}T_{\beta}^{\beta\mu}\right). \qquad (7)$$

The torsion scalar T is

$$T = T^{\alpha}_{\mu\nu} S^{\mu\nu}_{\alpha}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Now, we define the action by generalizing the Teleparallel Theory i.e. f(T) theory as [18]

$$S = \int [T + f(T) + L_{matter}]\ e\ d^4x. \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Here, f(T) denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (9) with respect to the tetrads, we get the following equations of motion

$$S^{\nu\rho}_{\mu} \partial_{\rho} T f_{TT} + \left[e^{-1} e^{i}_{\mu} \partial_{\rho}\left(e e^{\alpha}_{i} S^{\nu\rho}_{\alpha}\right) + T^{\alpha}{}_{\lambda\mu} S^{\nu\lambda}_{\alpha}\right](1 + f_T) + \frac{1}{4}\delta^{\nu}_{\mu}(T + f) = 4\pi T^{\nu}_{\mu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where T^{ν}_{μ} is the energy, momentum tensor, $f_T = df(T)/dT$and, and by setting $f(T) = a_0 =$ constant this is dynamically equivalent to the General Relativity.

3. Metric and Field Equations

The spatially homogeneous and isotropic FLRW line element is

$$ds^2 = -dt^2 + a^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where $d\Omega^2 = d\theta^2 + sin^2\theta\ d\varphi^2$. The angle $\theta$and φ is the usual spherical polar coordinates, with $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$. The coordinates (t, r, θ, φ) are called co-moving coordinates. In cosmology, a comoving coordinate system is employed (t, r, θ, φ), where the coordinates expand along with the universe itself, ensuring that objects at rest relative to the cosmic background maintain fixed spatial coordinates. The observed homogeneity of the universe defines a preferred frame, known as the cosmic rest frame, which aligns with this comoving coordinate system. The spatial curvature of the universe is represented by a constant, denoted as k. Specifically, k > 0 signifies a closed universe, k = 0 corresponds to a flat universe, and k < 0 indicates an open universe.

The energy momentum tensor T^{ν}_{μ} for bulk viscous fluid distribution is taken as

$$T^{\nu}_{\mu} = (\bar{p} + \rho)\ u^{\nu} u_{\mu} + \bar{p}\ g^{\nu}_{\mu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

together with co-moving co-ordinates

$$u^{\nu} = (0, 0, 0, 1) \text{ and } u^{\nu} u_{\nu} = -1, \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\bar{p} = p - \xi\ u^{\nu}_{;\nu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where $u^{\nu}$is the four-velocity vector of the cosmic fluid, $\bar{p}, p$ and ρ are the effective pressure, isotropic pressure and energy density of the matter respectively, ξ is the coefficient of bulk viscosity which is a function of time t. Total effect of bulk viscosity is to reduce the pressure p of the perfect fluid by an amount $\xi\theta$, so that the effective pressure of the viscous fluid turns out to be $\bar{p} = (p - \xi\theta)$.

From the equation of motion (10), the line element (11) for the viscous fluid of the stress energy tensor (12) can be written as

$$\frac{\dot{a}}{a}\ \dot{T}\ f_{TT} + \left\{\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right\}(1 + f_T) + \frac{1}{4}(T + f) = (4\pi)\bar{p}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$3(1 + f_T)\frac{\dot{a}^2}{a^2} + \frac{1}{4}(T + f) = (4\pi)\ (-\rho). \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The overhead dot represents the differentiation with respect to time t.

4. Solution of the field equations

To solve the system of field equations in the context of $f(T)$ gravity with bulk viscosity, a specific scale factor, $a = (e^{\alpha k t} - 1)^{\frac{1}{\alpha}}$, (where α and k be the constant parameters) has been chosen. This choice is motivated by the desire to find analytical solutions that can describe an expanding universe with both exponential and power-law contributions to the expansion rate. This form is particularly appealing because it can capture different cosmological epochs, including early-time power-law expansion and late-time accelerated expansion, potentially driven by bulk viscosity or modified gravity effects. By substituting this scale factor into the field equations, the system can be simplified, allowing for the determination of key cosmological parameters and a deeper understanding of the dynamics of the universe in this modified gravity setting. This approach provides a valuable tool for exploring the interplay between bulk viscosity, modified gravity, and the expansion history of the universe. For this choice of scale factor, the Hubble's parameter is obtained as

$$H = k(1 + a^{-\alpha}) \text{ .... } (17)$$

With the help of above equation (17), the kinematic parameter called deceleration parameter (q) is observed as,

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{\alpha}{k}\left(\frac{1}{a^{\alpha}+1}\right)\frac{\dot{a}}{a}. \text{ .... } (18)$$

The graphical behavior of deceleration parameter is clearly seen in the figure 1.

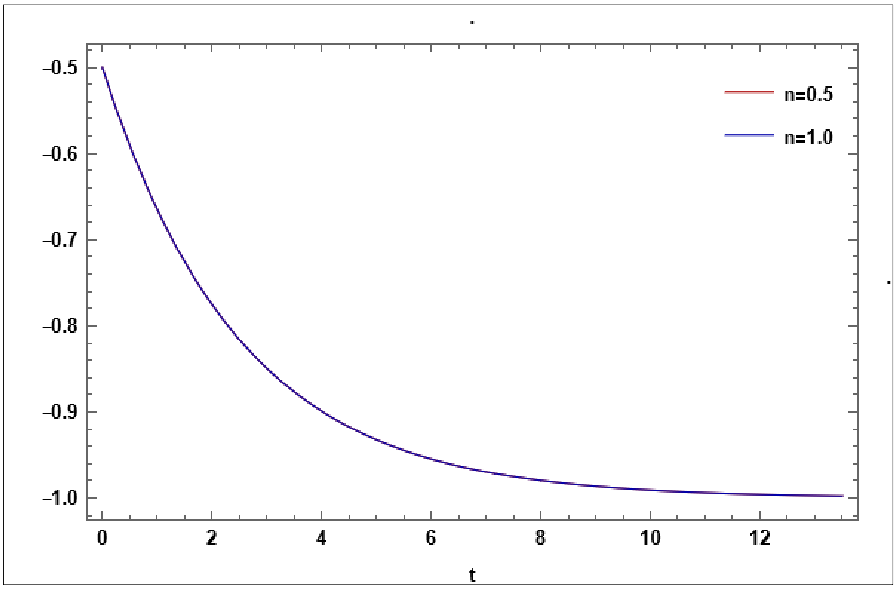

Figure 1 Behavior of q versus t.

The attached figure illustrates the evolution of the deceleration parameter q as a function of cosmic time t for two different values of the parameter n, namely $n = 0.5$ and $n = 1.0$. The deceleration parameter (q), provides insight into the acceleration status of the universe: a positive q indicates deceleration, while a negative implies acceleration. From the plot, it is evident that q remains negative throughout the evolution and gradually approaches -1 at late times, indicating that the universe is undergoing continuous accelerated expansion. This trend is consistent with current cosmological observations, such as those from Type Ia supernovae, cosmic microwave background (CMB), and baryon acoustic oscillations (BAO), which confirm that the universe transitioned into an accelerated phase approximately five billion years ago. The asymptotic behaviour toward $q = -1$ suggests that the universe is evolving toward a de Sitter-like phase dominated by dark energy, resembling the late-time behavior predicted by the standard ΛCDM model. Furthermore, the near overlap of the curves for both values of n indicates that the late-time dynamics of the model are largely insensitive to the specific choice of n, emphasizing the dominant role of dark energy in driving the cosmic acceleration. Overall, the figure supports a cosmological scenario that aligns well with observational data and theoretical expectations of an accelerating universe influenced by a cosmological constant.

With the help of above equation (17), the Torsion scalar is obtained as

$$T = -6H^2 = -6k^2(1 + a^{-\alpha})^2 \qquad \text{.......................... (19)}$$

The graphical behavior is clearly seen in the figure 2.

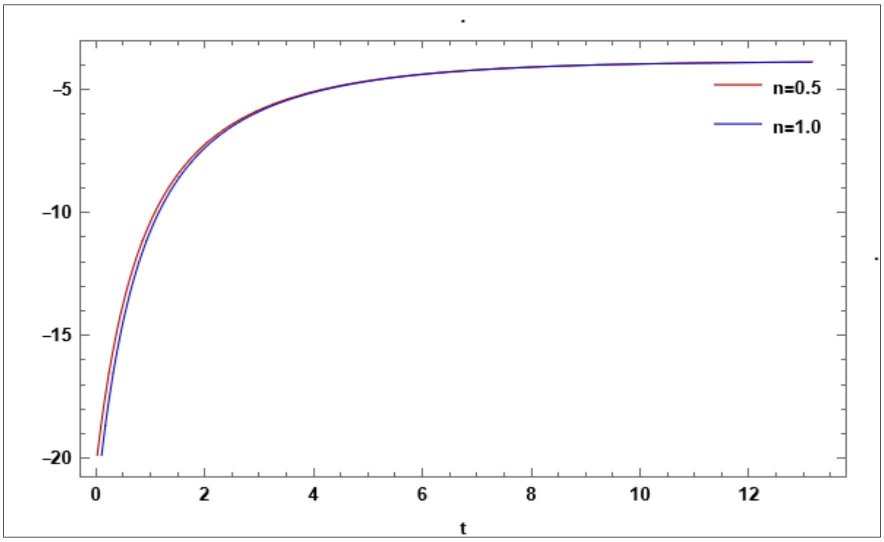

Figure 2 Behavior of T versus t.

The graph illustrates how the warping or contortion of space changes over time. We're looking at this spatial distortion for two different scenarios, labeled "$n = 0.5$" and "$n = 1.0$". Initially, at the very beginning when time is just commencing, the degree of spatial warping is very high and negative, indicating a significant deformation of space. Then, as time progresses, this spatial contortion rapidly decreases, as if space is being straightened out. Eventually, it settles into a steady state, where there's still some level of spatial distortion, but it remains constant. Comparing the two scenarios, "$n = 0.5$" and "$n = 1.0$", we observe a similar pattern in their behavior. However, the "$n = 1.0$" scenario exhibits a slightly greater magnitude of spatial warping throughout the entire time period. Thus, although both scenarios eventually reach a stable, unchanging state, the "$n = 1.0$" scenario consistently has a more pronounced deformation of space. This reveals that the "n" value influences the extent of spatial contortion. Imagine adjusting a dial that controls the amount of bending or warping in space. The graph helps us understand how this spatial distortion evolves over time and how it's affected by different parameter settings.

The depiction model of $f(T)$ gravity is obtained as

$$f(T) = T^n = (-6)^n k^{2n}(1 + a^{-\alpha})^{2n} \text{.......................... (20)}$$

The graphical behavior of the function of the torsion scalar is clearly seen in the figure 3.

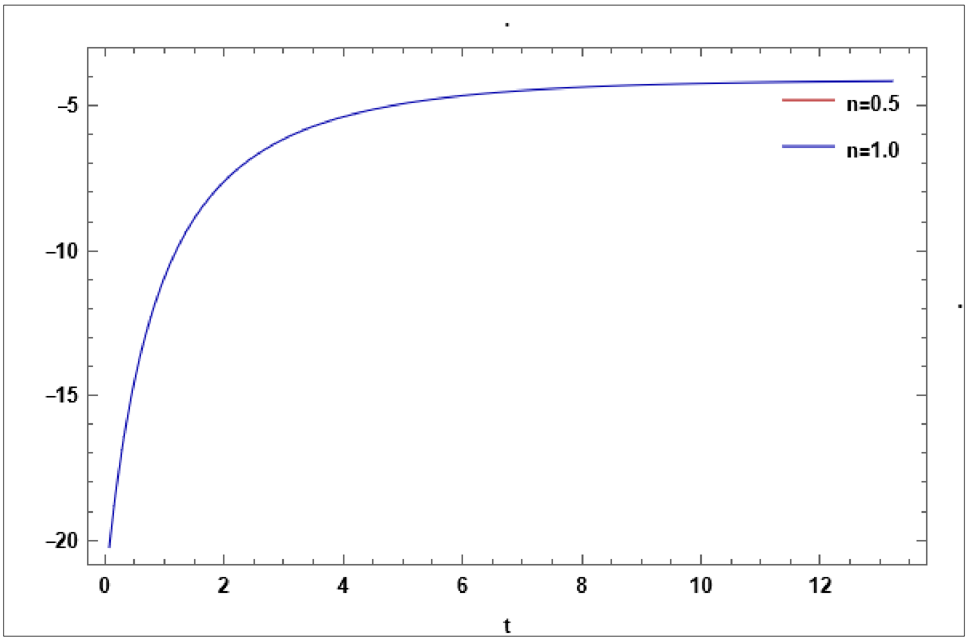

Figure 3 Behavior of $\boldsymbol{f(T)}$ versus t.

This graph illustrates the evolution of a function related to the torsion scalar in an $f(T)$ gravity model over time, comparing two scenarios where the parameter n is set to 0.5 (red line) and 1.0 (blue line). Both scenarios exhibit a similar qualitative trend: an initial, substantial negative value indicates a significant contribution from torsion in the early stages, followed by a rapid increase towards a constant, negative asymptotic value as time progresses. This suggests that while torsion plays a crucial role initially, its influence stabilizes over time, reaching a steady-state value. Quantitatively, the n = 1.0 curve consistently displays a larger (more negative) magnitude throughout the observed time interval, indicating that a higher value of n amplifies the contribution of the torsion scalar function. This difference in magnitude, while not altering the overall qualitative behavior, highlights the sensitivity of the model to the parameter n and its impact on the strength of the torsion-related function. The observed convergence to a constant value suggests the system is evolving towards a stable configuration where the influence of torsion, though present, is unchanging. This analysis is valid under the assumption that the plotted function accurately represents the torsion scalar behavior within the specified $f(T)$ gravity model, and the chosen range of time is relevant for the physical phenomena being studied.

The energy density of the model is obtained as

$$\rho = \frac{1}{16\pi a^2}\left(a^2(6(1+a^{-\alpha})^2k^2 - 6^n(-(1+a^{-\alpha})^2k^2)^n) - 12\dot{a}^2(1+6^{-1+n}(-(1+a^{-\alpha})^2k^2)^{-1+n}n)\right). \quad \ldots\ldots\ldots\ldots \quad (21)$$

The graphical behavior of the energy density of the model is clearly seen in the figure 4.

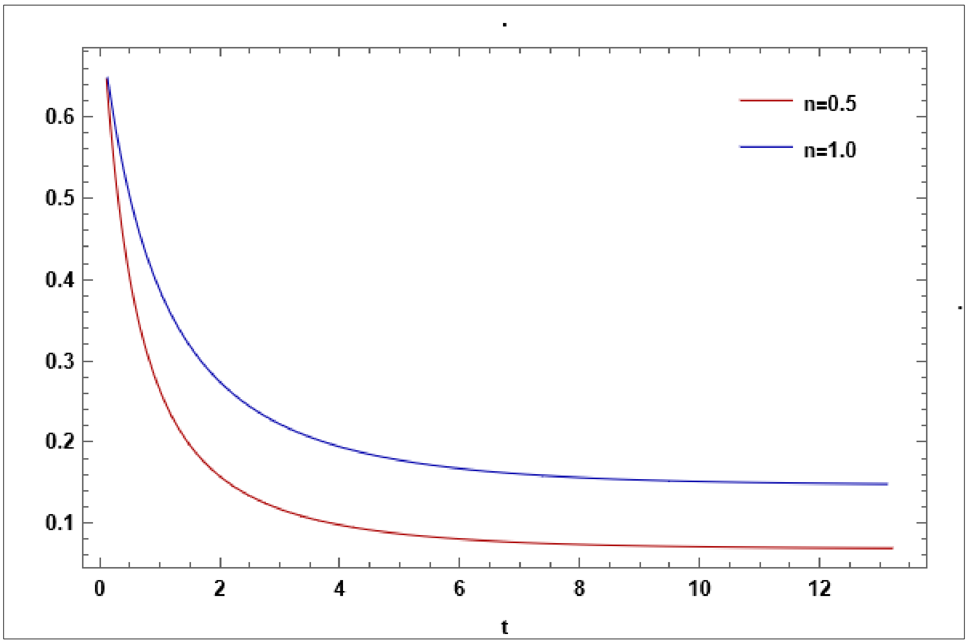

Figure 4 Behavior of $\boldsymbol{\rho}$ versus t.

The attached figure depicts the energy density's evolution over time within an $f(T)$ gravity model, comparing two scenarios with different parameter values: $n = 0.5$ (red line) and $n = 1.0$ (blue line). Both scenarios exhibit a similar qualitative trend: a rapid initial decrease in energy density followed by a gradual approach towards a constant, non-zero asymptotic value. Specifically, at the very beginning ($t \approx 0$), the energy density is relatively high for both values of n. However, it experiences a sharp decline within the first few time units, indicating a significant energy release or redistribution in the early stages of the system's evolution. This rapid decrease suggests a dynamic phase where the energy density is far from equilibrium and undergoing substantial changes.

As time progresses, the rate of decrease slows down, and the energy density asymptotically approaches a constant value. This suggests that the system is settling into a more stable configuration where the energy density remains relatively unchanged over time. However, it's crucial to note that the asymptotic value is not zero, implying that there's still a non-negligible energy density present even at late times.

The viscous pressure of the model is obtained as

$$\bar{p} = \frac{1}{16\pi a^2}((-6(1 + a^{-\alpha})^2 k^2 + 6^n(-(1 + a^{-\alpha})^2 k^2)^n) + 8\dot{a}^2(1 + 6^{-1+n}(-(1 + a^{-\alpha})^2 k^2)^{-1+n} n)). \qquad (22)$$

The graphical behavior of the viscous pressure of the model is clearly seen in the figure 5.

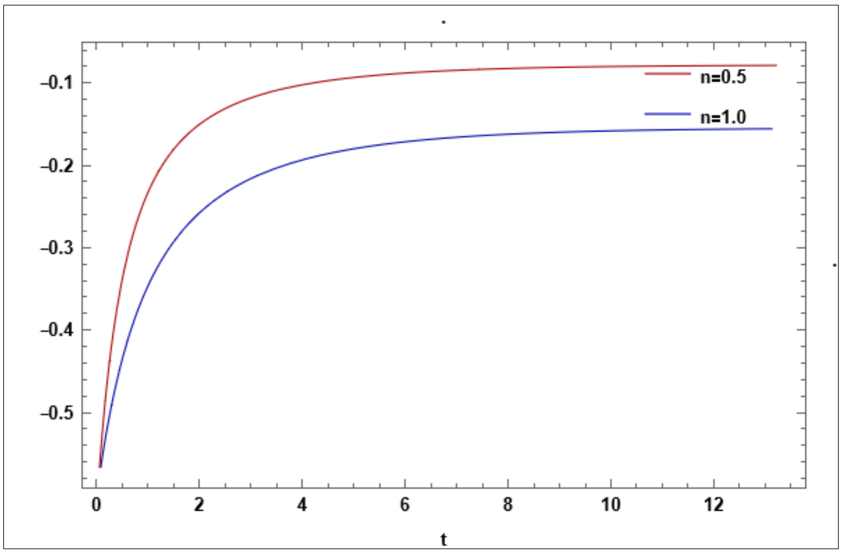

Figure 5 Behavior of $\boldsymbol{p}$ versus t.

The negative viscous pressure depicted in the $f(T)$ gravity model, evolving from a large initial negative value towards a constant negative asymptote, directly relates to the accelerating behavior of the universe. In cosmological models, negative pressure, particularly from viscosity, acts as a repulsive force driving expansion; the initial large negative viscous pressure could thus contribute to the early universe's rapid acceleration. As the viscous pressure approaches a constant negative value, it suggests a continued contribution to the overall negative pressure, potentially sustaining the observed late-time acceleration. Furthermore, the sensitivity of the viscous pressure to the parameter 'n' implies that within the $f(T)$ framework, the acceleration rate is highly dependent on parameter choices, highlighting viscosity's crucial role in shaping the universe's expansion dynamics.

The viscous equation of state parameter of the model is obtained as

$$w = \frac{\bar{p}}{\rho} = \frac{((-6(1+a^{-\alpha})^2k^2 + 6^n(-(1+a^{-\alpha})^2k^2)^n) + 8\dot{a}^2(1+6^{-1+n}(-(1+a^{-\alpha})^2k^2)^{-1+n}n))}{\left(a^2(6(1+a^{-\alpha})^2k^2 - 6^n(-(1+a^{-\alpha})^2k^2)^n) - 12\dot{a}^2(1+6^{-1+n}(-(1+a^{-\alpha})^2k^2)^{-1+n}n))\right)}$$

The graphical behavior of the viscous equation of state parameter of the model is clearly seen in the figure 6.

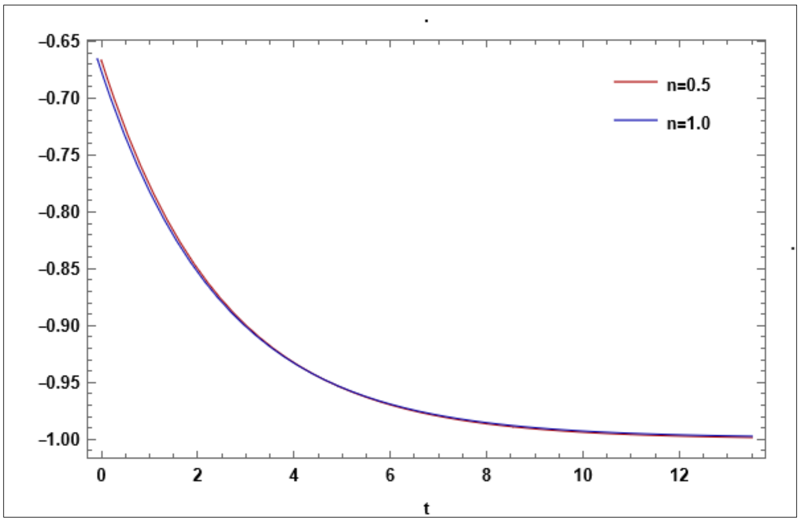

Figure 6 Behavior of $\boldsymbol{w}$ versus t.

The graph reveals that the viscous equation of state parameter in this $f(T)$ gravity model, for both $n = 0.5$ and $n = 1.0$, rapidly decreases from a slightly higher negative value towards a constant, negative asymptotic value around -1, with $n = 1.0$ showing a marginally lower value. This behavior, remaining consistently below $-1/3$ and approaching -1, directly indicates an accelerating universe, as the negative values signify a repulsive pressure driving expansion. The initial rapid decrease suggests a significant viscous contribution to early acceleration, while the late-time approach to -1 mimics a cosmological constant, sustaining the observed cosmic acceleration. Therefore, the viscous component within the $f(T)$ model effectively generates the necessary negative pressure, aligning with the observed accelerating universe, and the parameter 'n' has a minor quantitative impact on this overall trend.

The observed behavior of the viscous equation of state parameter in the $f(T)$ gravity model, as depicted in the figure, bears a striking resemblance to the Lambda-CDM model, the current standard model of cosmology. Both scenarios exhibit an equation of state parameter approaching -1 at late times, indicating a dominant component with negative pressure driving accelerated expansion. In Lambda-CDM, this component is the cosmological constant (Λ), a form of dark energy with a constant energy density. Similarly, the viscous term in the $f(T)$ model, as it evolves towards -1, effectively mimics this cosmological constant behavior, suggesting that viscosity can play a role analogous to Λ in driving the late-time acceleration of the universe. The slight quantitative differences between the $n = 0.5$ and $n = 1.0$ scenarios might correspond to variations in the effective dark energy density or its evolution within the $f(T)$ framework, similar to how alternative dark energy models in Lambda-CDM can slightly alter the cosmological constant's influence.

5. Conclusion

In this paper, we have investigated the dynamics of a flat, homogeneous, and isotropic universe within the framework of viscous teleparallel gravity. Unlike General Relativity, which describes gravitation through curvature, teleparallel gravity employs the torsion tensor of the Weitzenböck connection to account for gravitational interactions. By introducing a bulk viscous fluid as the cosmic medium, we extended the traditional perfect fluid assumption, accounting for dissipative processes that are expected to play a significant role in the evolution of the universe, particularly during periods of high energy density and rapid expansion.

Using a phenomenological approach, we assumed a linear bulk viscosity coefficient proportional to the Hubble parameter, which simplifies the analysis while still capturing the essential physics of dissipative effects. The modified Friedmann equations were derived accordingly, revealing additional terms resulting from the presence of viscosity. These new terms effectively act as a source of negative pressure, which can mimic the role of dark energy in driving the accelerated expansion of the universe.

An ansatz for the Hubble parameter was proposed to find exact solutions to the cosmological equations. These solutions encompass various cosmic epochs, including decelerated and accelerated expansion phases, depending on the parameters chosen. Notably, the solutions demonstrate that the universe can undergo a transition from deceleration to acceleration purely due to the effects of bulk viscosity, without invoking a cosmological constant or scalar field. This result aligns with observational data indicating that the universe has experienced such a transition in its recent history.

Our analysis also addressed the thermodynamic implications of introducing viscosity into the cosmic fluid. In particular, we examined the entropy production associated with viscous processes and confirmed the consistency of our model with the second law of thermodynamics. This aspect is crucial, as any viable cosmological model must not only reproduce the observed dynamics but also adhere to fundamental thermodynamic principles.

Moreover, we discussed the potential of viscous teleparallel gravity to serve as an effective framework for describing late-time cosmic acceleration. In contrast to conventional dark energy models, which often rely on exotic components with unknown physical origins, the viscous fluid approach offers a more grounded mechanism rooted in non-equilibrium thermodynamics and fluid dynamics. It provides a compelling alternative explanation for the observed acceleration without introducing additional scalar fields or modifying the gravitational action beyond torsional contributions.

In conclusion, the inclusion of bulk viscosity in teleparallel gravity yields a rich structure capable of explaining key features of cosmological evolution, including the transition to an accelerated expansion phase. These findings open up new avenues for further exploration, such as considering more general forms of viscosity, extending the analysis to anisotropic or inhomogeneous models, and comparing predictions with observational data from supernovae, cosmic microwave background, and large-scale structure surveys. The results underscore the potential of viscous teleparallel gravity as a promising candidate for a more complete and realistic description of the universe's dynamical behaviour.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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